U. S. DEPARTMENT OF COMMERCE

CHARLES SAWYER, Secretary

COAST AND GEODETIC SURVEY

ROBERT F. A. STUDDS, Director

Special Publication 283

MAGNETIC OBSERVATORY MANUAL

By

H. E. McCOMB

UNITED STATES
GOVERNMENT PRINTING OFFICE
WASHINGTON : 1952

For sale by the Superintendent of Documents, U. S. Government Printing (
Washington 25, D. C. Price $1.50

IIA Lib.,

FOREWORD

The magnetic observatories of the world, where geophysical science keeps a steady watch on the ever-changing magnetic conditions of our planet, are but little known to the general public, though they fill a vital niche in the complex organization of modern science and technology. The later editions of Hazard's Directions for Magnetic Measurements contain detailed information on the operation of certain instruments used in geomagnetic measurements and in addition a brief chapter on magnetic observatory operations. There has long been needed a supplemental manual to explain in detail the determination of instrumental constants and to give technical directions for installing and operating a magnetograph. This manual endeavors to meet that need insofar as it has become manifest in the conduct of magnetic observatory work by the United States Coast and Geodetic Survey.

In view of the multiplicity of types of magnetic instruments used, not only in the United States but in other parts of the world, it would not be practicable to treat them all in full. Certain basic principles are the same for most of the instruments now in service, and greatest attention has been given in these pages to those elements of operation which are more or less common to all observatory instruments.

In making physical measurements it is always important to know in advance the probable sources of controlling errors, and to conduct the special tests or routine operations accordingly. Consequently, the appendixes on orientation errors and scale-value errors are developed rather fully for the express purpose of showing the observer where precision in adjustments or observational procedure is vital and where attempts at high precision are not justifiable.

It is assumed that the observer who may wish to use this manual is competently trained in the operation of magnetometers, earth inductors, or similar geomagnetic instruments.

The whole-hearted cooperation of other members of the United States Coast and Geodetic Survey staff in the preparation, checking, and editing of the original manuscript has contributed substantially to the completion of this manual and is gratefully acknowledged.

Washington, May 1952

CONTENTS

Chapter		page
	Foreword	III
1.	The forces between bar magnets	1
	Magnets and magnetic fields	1
	Deflections	5
	Analysis of the field of a bar magnet	8
2.	The Earth's magnetic field	16
3.	The magnetic observatory	21
4.	Instrumental constants and corrections	25
	Refinements contributing to accuracy	25
	Scale value (magnetometer)	26
	Magnetometer equations	26
	Moment of inertia	28
	Temperature coefficients	33
	Induction factor	42
	Summary of constants	55
	Adjustment of log C	55
	Magnetic standards	57
5.	Optical systems and photographic registration	62
	Collimation	62
	Magnetograph optics	63
	Photographic registration	67
6.	Quartz-fiber techniques	70
7.	The declination variometer	74
8.	The horizontal-intensity variometer	84
9.	The vertical-intensity variometer	99
10.	Temperature coefficients of variometers	109
11.	Determination of scale values	126
	Use of deflector magnet	126
	Electromagnetic methods	132
	Scale value and geomagnetic latitude	134
12.	Orientation of variometer recording magnets	135
	Necessity for testing orientation	135
	Orientation formulas	136
	Directions for orientation tests	141
13.	Directions for installing a magnetograph	148
	Preliminary steps	148
	Installing the D variometer	150
	Installing the H variometer	154
	Installing the Z variometer	159
	Low-sensitivity magnetographs	162
	The la Cour magnetograph	163
14.	Processing of data at the observatory	165
	Program of observatory work	165
	Importance of adequate control	174
	Directions for processing records	174
	Scaling of ordinates	177

Appendix		
I.	Some approximations	183
II.	Temperature coefficient of magnetic moment	195
III.	Orientation errors	197
IV.	Variometer scale-value errors	204
V.	Change of orientation and scale value of D variometer caused by other magnets of the magnetograph	211
VI.	Nomograms	217

	Bibliography	228
	Index	229

TABLES

		Page
1.	Components of the field of a short bar magnet	11
2.	Summary of formulas; distribution factors	13
3.	Special values for distribution coefficients, P_A and Q_A	15
4.	Correction factors; lack of balance of coils in induction-coefficient tests	55
5.	Torsion constant of quartz fibers by different methods	90
6.	Variation of H scale value with sensitivity-control-magnet distance	93
7.	Computed and observed a factors at various observatories	97
8.	Variation of Z scale value with adjustment of sensitivity poise	103
9.	Variation of Z scale value with azimuth of recording magnet	107
10.	Applied fields for temperature compensation	117
11.	Recommended scale values for observatories in different latitudes	134
12.	Miscellaneous data; observatory variometers	145
13.	Orientation of variometer magnets	146
14.	D variometer: miscellaneous data	152
15.	H variometer: miscellaneous data	155
16.	H variometer: torsion in the fiber	156
17.	Z variometer: miscellaneous data	160
18.	Critical adjustments of deflector in orientation tests	203
19.	Summary of errors in D and Z scale values due to maladjustment of deflector	210
20.	Summary of errors in H scale value due to maladjustment of deflector	210
21a.	Approximate distribution coefficients for Position A	227
21b.	Approximate distribution coefficients for Position B	227
21c.	Approximate distribution coefficients for Position C	228

ILLUSTRATIONS

1.	Field of a bar magnet	2
2.	Action of a uniform magnetic field on a magnet	3
3.	Relation between length of magnet and pole distance	4
4.	Resolution of a magnetic field into two components	4
5–7.	Magnetic deflections, various positions	6–7
8.	Components of the field of a magnet	11
9.	Deflector positions	12
10.	Components of the earth's magnetic field	16
11.	The earth's magnetic field at various latitudes	19
12a.	Magnetic variation building	22
12b.	Magnetic absolute building	23
13.	Magnetograph	24
14.	Magnetism tester	25
15–17.	Observations and computations for moment of inertia	30–32
18.	Temperature bath	35
19–21.	Observations and computations for temperature coefficient of magnetic moment	35–37
22.	Variation of magnetic moment with temperature	38
23.	Observations of temperature coefficient, H-variometer method	40
24.	Positions of magnets, induction coefficient	43
25.	Induction-coefficient apparatus (Lamont's method)	45
26.	Variation of deflection angle with position of deflector	46
27.	Observations and computations, induction coefficient by Lamont's method	47
28.	Deflection angles, induction coefficient	48
29.	Apparatus for measuring induction coefficient (Nelson's method)	49
30.	Diagram of solenoid	49
31.	Relation between coil factor and distance	51
32.	Wiring diagram for induction-coefficient apparatus	52
33–34.	Observations and computations, induction coefficient by Nelson's method	53–54
35.	Constants of magnetometer	56
36.	Computation of log C for magnetometer	58
37.	The sine galvanometer	59
38–41.	Standardization cards for magnetic instruments	60–61
42.	Planoconvex lens	62

Page
43. Illustration of oculars in an auto-collimator 62
44. Optical lever 63
45. Planoconvex mirror 63
46. Origin of ghost images 64
47. 90° prism 65
48. Planoconvex prism 65
49. Planoconvex cylindrical lens 65
50. Action of cylindrical lens 65
51. Three-faced mirror 66
52. Magnetograph recorder 68
53. Parallax between time line and recording spots 68
54. Quartz fibers, la Cour type 70
55. Frame of mounted galvanometer fibers 70
56. Apparatus for mounting brass stems on quartz fibers 72
57. Coupler for attaching quartz fibers to suspended magnet 73
58. Torsion factor 77
59. Effect of fiber torsion on *D* variometer magnet 77
60. Effect of the *C* field on *D* variometer magnet 78
61. Simple *H* variometer 85
62. *H* variometer with sensitivity-control magnet 91
63. Variation of scale value 93
64. The *a* factor in the *H* scale value 95
65. Effect of change of declination on *H* scale value 98
66. Value of declination affecting *H* scale value 98
67. Diagram for *Z* variometer 100
68. *Z*-variometer magnet oriented at an angle with the magnetic meridian 101
69. The adjusting poises of the *Z*-variometer magnet 102
70. Vertical-intensity recording magnet, Schmidt type 110
71. Magnetic temperature compensation of *Z*-variometer 113
72. Scale-value observations and computations 127
73. Scale-value observations and computations; magnetic moment of deflector known 130
74. Relation between the minute and gamma *D* scale values 131
75. Horizontal-intensity variometer 133
76. *D*-variometer magnet not in magnetic meridian 136
77–78. Tests of orientation of *D*-variometer magnet 137
79. *H*-variometer magnet not in magnetic prime vertical 140
80–81. Tests of orientation of *H*-variometer magnet 140
82. Orientation bench 142
83. The magnetic meridians through the variometers 143
84. Observations for orientation tests 145
85. Plan of magnetograph installation at Tucson Magnetic Observatory 149
86. Magnetograph recorder 151
87. Mirror adjusting apparatus 156
88. Torsion in *H*-variometer fiber 158
89. La Cour vertical-intensity variometer 163
90. Observatory magnetometer 166
91. Quartz Horizontal Magnetometer (QHM) 167
92. Earth inductor 168
93. Magnetogram 169
94. Special light source and photoelectric cell 170
95–97. Base-line computations 171–173
98. Time comparisons 175
99. Magnetograph record 176
100. Magnetogram scaling, parallax test, scale-value deflections 177
101. Tabulation of scaled hourly values 179
102. Magnetic activity report 182
103–117. Errors in placement of deflector, orientation tests 199
118. Positions of deflector for scale values 204
119–130. Errors in placement of deflector, scale-value observations 205
131. Relative locations of magnets in the magnetograph at College, Alaska 211
132–136. Positions of other magnetograph magnets relative to *D* magnet 212
137. Field of a bar magnet (nomogram) 217
138. Change in the field of a bar magnet, magnetic moment known (nomogram) 218

		Page
139.	Change in the field of a bar magnet, field at a distance r known	219
140.	The D Nomogram	220
141.	The H Nomogram	221
142.	Nomogram for conversion of D scale values	222
143.	Nomogram showing spurious effects on recorded declination	223
144.	Nomogram showing spurious effects on recorded horizontal intensity	224
145.	Nomogram showing spurious effects on recorded vertical intensity	225
146.	Nomogram for distribution factors	226

CHAPTER 1. THE FORCES BETWEEN BAR MAGNETS

MAGNETS AND MAGNETIC FIELDS

1. ***Poles and axis of a magnet.***—A permanent magnet is surrounded by a magnetic field. Both the direction and magnitude of this field at any point are fixed relative to the magnet, so long as no other magnetic fields are present at the point and the magnet itself suffers no change in its magnetization.

2. The lines of force about a bar magnet appear to converge in two regions near the ends of the magnet. These regions are called the *poles* and are designated as N (north-seeking or $+$) and S (south-seeking or $-$). In the figures of this manual the north-seeking end of the magnet is indicated by a shaded area (see fig. 1).

3. It is convenient to treat the poles as points, their exact positions being determined by the pattern of the external field. The *magnetic axis* of a magnet is the line joining its poles. The *pole distance* is reckoned from either pole to the midpoint between them. It is one-half the distance between the poles, and is usually designated as l.

4. The term *unit pole* denotes the theoretical concept of a point pole which would repel a like pole with a force of one dyne at a distance of one centimeter, in a vacuum. The number, m, of unit poles concentrated at a point is called the *pole strength.*

5. ***Inverse-square law.***—If m_1 and m_2 represent the pole strengths of two poles and r their distance apart, then the force, F, between them is

$$F=\frac{m_1m_2}{\mu_{cgs}r^2} \tag{1}$$

in which μ_{cgs} is a constant depending upon the medium. In this manual we take μ_{cgs} for air as unity, since it actually exceeds unity by only about 4 parts in 10 million.

6. ***Magnetic field strength.***—The strength, f, of a magnetic field is the force that a unit pole would experience if placed in that field. That is, f is the force per unit pole.

$$f=\frac{F}{m} \tag{2}$$

and

$$F=f\,m \tag{3}$$

in which m is the pole strength, as 1 pole, 2 poles, etc., and F is the force. Other names for magnetic field strength are *magnetizing force* and *magnetic intensity.*

7. When the force, F, is equal to one dyne and $m=1$, the field intensity is unity. The unit of field intensity is called the *oersted*, or sometimes the *gauss*. Also, in a vacuum, the field at a distance of one centimeter in any direction from a unit pole is one oersted. The field, in a vacuum, at a distance r from a pole of strength m, is

$$f=\frac{m}{r^2} \tag{4}$$

Convenient units in geomagnetic work are: the *gamma* (γ), equal to 0.00001 oersted; and the *milligauss*, equal to 100γ or 0.001 oersted.

8. ***Field of a bar magnet.***—In figure 1, let P be a point on the magnetic axis, produced, of the magnet NS and at a distance r from its center. If m is the pole strength of either pole then the field, f_+, at the point P due to N is $f_+=\frac{m}{(r-l)^2}$ and that due to S is $f_-=\frac{-m}{(r+l)^2}$.

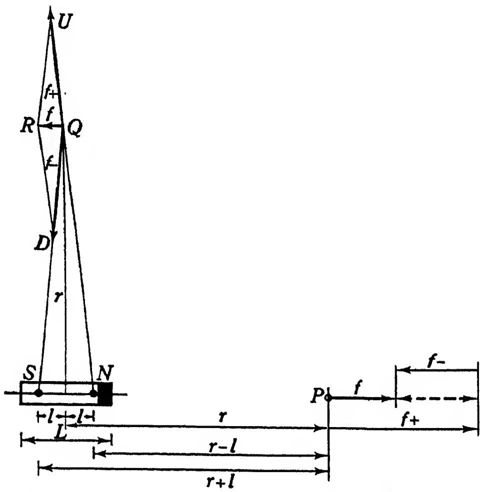

Figure 1.—Field of a bar magnet; on the axis extended, and on the perpendicular bisector of the axis.

The resultant field at P, due to both poles is equal to the sum of these fields, or

$$f_+ + f_- = f = \frac{m}{(r-l)^2} - \frac{m}{(r+l)^2}$$

$$= \frac{m(r+l)^2 - m(r-l)^2}{(r^2-l^2)^2}$$

$$= \frac{4mrl}{(r^2-l^2)^2}.$$

If l is small compared to r, l^2 is negligible compared to r^2 and

$$f \approx \frac{4ml}{r^3}. \tag{5}$$

9. For a point Q on the perpendicular bisector of the magnetic axis, the field at Q due to N is $f_+=\frac{m}{\overline{QN}^2}=\frac{m}{(r^2+l^2)}$ and it is directed away from N as QU. The field at Q due to S is $f_-=\frac{m}{\overline{QS}^2}=\frac{m}{(r^2+l^2)}$, but is directed toward S, as QD. The resultant of these two vectors

is QR. Note that the resultant, QR, is directed opposite to the line from S to N. In the similar triangles NSQ and QRU,

$$\frac{QR}{QU}=\frac{NS}{NQ}$$

$$\frac{NS}{NQ}=\frac{2l}{\sqrt{r^2+l^2}}$$

$$f=QR=QU\left(\frac{NS}{NQ}\right)=\frac{m}{r^2+l^2}\cdot\frac{2l}{\sqrt{r^2+l^2}}$$

$$f=\frac{2ml}{\sqrt{(r^2+l^2)^3}}$$

If r is large compared to l, l^2 may be neglected in computing the approximate field at Q, and

$$f\approx\frac{2ml}{r^3} \qquad (6)$$

Thus the field at Q is approximately one-half the field at P.

10. ***Uniform field.***—If r is large, in the case of a bar magnet, the change in direction and intensity of the field due to relatively small

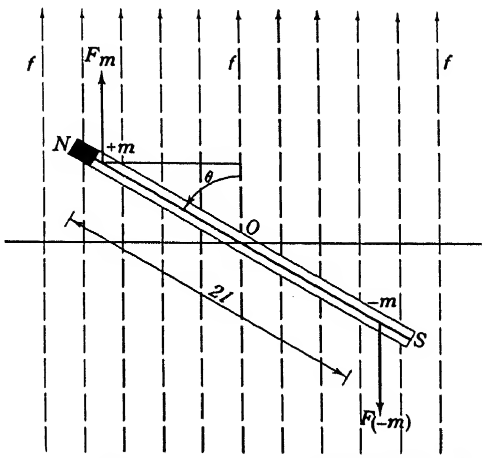

Figure 2.—Action of a uniform magnetic field on a magnet.

changes in r will be correspondingly small. If r is extremely large the changes in direction and intensity for small changes in r may become negligible in magnetic measurements. We call this a uniform field.

11. ***Magnetic moment.***—Suppose a magnet is suspended or pivoted at its center of gravity so that it is free to oscillate in a horizontal plane in a uniform horizontal field, f, figure 2. The force on the north-seeking pole ($+m$) is $+fm$ and on the pole ($-m$) it is $-fm$,

where m is the pole strength of either pole of the magnet. As these two forces are equal and oppositely directed, the clockwise turning moment or couple, L', will be

$$\begin{aligned} L' &= fm\ (2l \sin\theta) \\ &= 2ml\ (f \sin\theta) \end{aligned}$$

in which θ is the angle between the magnetic axis and the direction of the field, f. The quantity, $2ml$, is constant for a particular magnet so long as its magnetization does not change, and is called the *magnetic moment* of the magnet. That is,

$$L' = fM \sin\theta. \tag{7}$$

12. It is thus possible to measure the magnetic moment of a magnet in terms of f, θ, and L'. When θ is zero, $\sin\theta=0$, and the couple is zero. Hence a magnet freely suspended in a uniform field will come to rest with its magnetic axis parallel to the field. When $\theta=90°$, that is, when the magnet is at right angles to the field, $\sin\theta=1$ and

$$L' = fM \tag{8}$$

and

$$M = \frac{L'}{f}. \tag{9}$$

If the uniform field has unit intensity, then $M=L'$, which means that the magnetic moment may be defined as the couple (dyne-centimeters) which will maintain the magnet at right angles to a uniform field of unit strength. It is expressed in dyne-centimeters per unit field, that is, $\frac{\text{dyne-cm}}{\text{oersted}}$ or simply the cgs unit of magnetic moment.

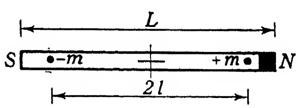

Figure 3.—Relation between length of magnet and pole distance; shaded end is *north-seeking* end of magnet.

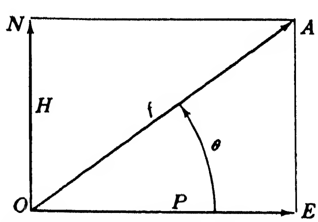

Figure 4.—Resolution of a magnetic field into two components.

13. Referring to figure 3, experiments show that, for bar magnets, the pole distance, l, and the length of the magnet, L, are related as follows:

$$l \approx 0.4\ L. \tag{10}$$

14. ***Resolution of fields.***—A field has both magnitude and direction and may be resolved into components. The resultant of two fields acting at a point may be evaluated graphically as in figure 4. Let $H=0.3$ cgs unit at O and directed along ON. Let $P=0.4$ cgs unit, directed along OE. The resultant field, $f=0.5$, is directed along OA. A small compass needle placed at O would come to rest with its axis directed along OA. The components of f along OE and ON are $OE=OA\cos\theta$ and $ON=OA\sin\theta$.

15. ***Field at a point P in terms of magnetic moment.***—Substituting M, the magnetic moment of the magnet, for $2\,ml$ in equations (5) and (6), we have the following approximate relations:

$$f\approx\frac{2M}{r^3},\ \text{parallel to } SN \tag{11}$$

for the field of a bar magnet at a distance r from the center of the magnet along its magnetic axis produced, and

$$f\approx\frac{M}{r^3},\ \text{parallel to } NS \tag{12}$$

for the field at a distance r from the center of the magnet along the perpendicular bisector of the magnetic axis. Equations (11) and (12) serve reasonably well for estimating the field of a magnet so long as l is small compared to r. For more precise values of these fields, corrections must be applied for distribution, as explained in paragraph 37.[1]

16. ***Estimation of the field from a nomogram.***—Corresponding values of f, M, and r in equation (11) are shown graphically in figure 137. The variation of the field with distance, r, is shown in figures 138 and 139.

DEFLECTIONS

17. ***Gauss' first position.***—Deflector end-on, approximate A position (see par. 32). In figure 5, let M_s be a short magnet pivoted at the point O and free to turn in a horizontal plane. If H is the only field acting on M_s the magnet will come to rest with its axis parallel to H, that is in the direction of OA. Now place a second magnet, M_a, in a fixed position at P so that its magnetic axis is on the perpendicular to H through O, and with its center at a distance r from O. M_a produces a field f in the direction OB; the resultant of f and H is OC. M_s will turn through an angle u, coming to rest with its magnetic axis in the direction OC of the resultant field. From equation (7), the clockwise couple on M_s due to H is $HM_s \sin u$ and the counter-clockwise couple on M_s due to the field f, of the deflector is $fM_s \sin(90°-u)=fM_s\cos u$.

[1] The aim and scope of this manual preclude showing explicitly the permeability of air in each of the many equations that need it under the standard conventions regarding the nature of pole strength. For full dimensional coherence, one may read into the equations the factor μ_{cgs} accompanying and qualifying every symbol that denotes a field component as derived from the pole strength or moment of the magnet that produces the field. The equations affected include those for deflection experiments (e. g. eq. 41), so what we really measure in deflections is, according to this approach, the induction B rather than the magnetic intensity H.

When equilibrium is established and the magnet is at rest these couples are equal and

$$HM_s \sin u = fM_s \cos u \tag{13}$$

$$\frac{f}{H} = \frac{\sin u}{\cos u} = \tan u \tag{14}$$

$$H = \frac{f}{\tan u} \tag{15}$$

but

$$f \approx \frac{2\ M_a}{r^3}$$

whence

$$H \approx \frac{2\ M_a}{r^3 \tan u} \tag{16}$$

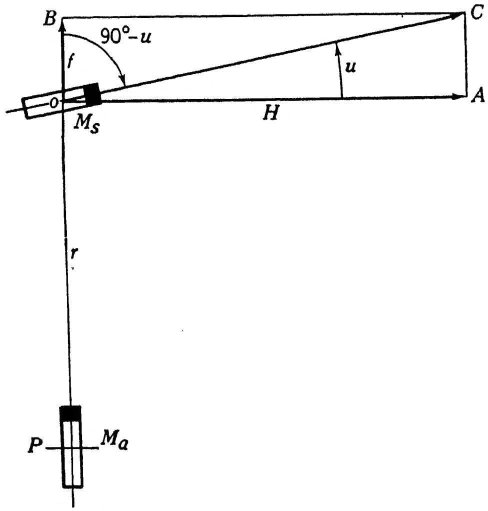

Figure 5.—Magnetic deflection, first position of Gauss.

Example: Let M_s=472 cgs; r=30 cm; and H=0.175 cgs=17500γ. Then

$$f \approx \frac{2(472)}{30^3} \approx 0.035 = 3500\gamma;$$

$$\tan u \approx \frac{0.035}{0.175} = 0.200$$

$$u \approx 11^\circ\ 19'$$

18. ***Gauss' second position.***—Deflector broadside, approximate B position. In this position the deflector axis is at right angles to H

with its center at P, PO being directed along H (see fig. 6). The field of M_a at O is directed opposite to M_a (see fig. 1) and

$$f \approx \frac{M_a}{r^3}$$

$$H \approx \frac{M_a}{r^3 \tan u} \tag{17}$$

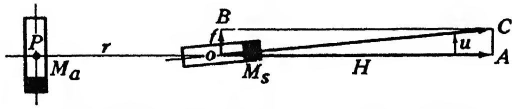

Figure 6.—Magnetic deflection, second position of Gauss.

Example as in paragraph 17:

$$f \approx \frac{472}{30^3} \approx 1750\gamma$$

$$\tan u \approx \frac{1750}{17500} = 0.100$$

Hence, $u \approx 5°\ 43'$ (approximately one-half the deflection in the A position).

19. ***Lamont's first position.***—A position, deflector end-on. In this position the deflector is placed (fig. 7) so that its magnetic axis is

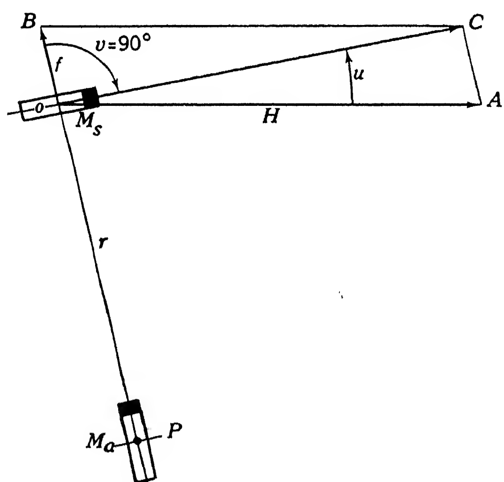

Figure 7.—Magnetic deflection, first position of Lamont.

always perpendicular to the suspended magnet when the latter comes to rest along the resultant. The couple M_s due to H is $HM_s \sin u$ and that due to f is $fM_s \sin v$, but v is 90° so that

$$HM_s \sin u = fM_s \tag{18}$$

and

$$\frac{f}{H}=\sin u \tag{19}$$

$$H=\frac{f}{\sin u} \tag{20}$$

$$H\approx\frac{2\ M_a}{r^3 \sin u} \tag{21}$$

Equation (21), with some minor correction factors, is standard in magnetometer deflections. Equations (16) and (17) are basic in variometer scale-value deflections where a permanent magnet is used as the deflector.

ANALYSIS OF THE FIELD OF A BAR MAGNET

20. ***Procedure for short magnets.***—Just as a field may be resolved into components, so the magnetic moment may be resolved, enabling us to evaluate the field of the magnet at any point P, or any component thereof. In figure 8, let M represent the magnetic moment of the magnet. We require the principal components of the field, at a point P on a line OP making an angle θ with the magnetic axis of the magnet and at a distance r from O. The various components are described below.

21. The *radial component*, f_r, is the component of f at P in the direction of OP. Let M_r=the component of the magnetic moment, M, in the direction of OP. Then

$$M_r=M\cos\theta \tag{22}$$

$$f_r=\frac{2\ M_r}{r^3}$$

$$=\frac{2\ M\cos\theta}{r^3} \tag{23}$$

22. The *tangential component*, f_θ, is the component normal to OP at P. The component of the magnetic moment normal to OP is

$$M_\theta=M\sin\theta \tag{24}$$

$$f_\theta=\frac{M_\theta}{r^3}$$

$$=\frac{M\sin\theta}{r^3} \tag{25}$$

23. The *total field*, f, at the point P, is the resultant of f_r and f_θ.

$$f^2=f_r^2+f_\theta^2$$

$$=\left[\frac{2M}{r^3}\cos\theta\right]^2+\left[\frac{M}{r^3}\sin\theta\right]^2$$

$$=\frac{M^2}{r^6}(4\cos^2\theta+\sin^2\theta)$$

$$=\frac{M^2}{r^6}[4\cos^2\theta+(1-\cos^2\theta)]$$

$$=\frac{M^2}{r^6}(3\cos^2\theta+1)$$

$$f=\frac{M}{r^3}\sqrt{3\cos^2\theta+1}. \tag{26}$$

24. The *parallel component*, $f_{||}$, is the component of f parallel to the magnetic axis of M. Consider PJ as made up of $PA-JA$. By construction,

$$PA=f_r\cos\theta$$

$$JA=FC=RF\sin\theta=f_\theta\sin\theta$$

$$f_{||}=PA-JA=f_r\cos\theta-f_\theta\sin\theta$$

Substituting values of f_r and f_θ,

$$f_{||}=\left(\frac{2M}{r^3}\cos\theta\right)\cos\theta-\left(\frac{M}{r^3}\sin\theta\right)\sin\theta$$

$$=\frac{2M}{r^3}\cos^2\theta-\frac{M}{r^3}\sin^2\theta$$

$$=\frac{M}{r^3}(2\cos^2\theta-\sin^2\theta)$$

$$=\frac{M}{r^3}(2\cos^2\theta-1+\cos^2\theta)$$

$$=\frac{M}{r^3}(3\cos^2\theta-1)$$

$$=\frac{3M}{r^3}(\cos^2\theta-0.333). \tag{27}$$

25. The *perpendicular component*, $f_\perp$, is the component at P, perpendicular to the magnetic axis of M. Consider the perpendicular component, $f_\perp$, as made up of $PB+BG$:

$$PB=f_r \sin \theta$$

$$BG=RC=RF \cos \theta=f_\theta \cos \theta$$

$$f_\perp=PB+BG=f_r \sin \theta+f_\theta \cos \theta$$

$$=\frac{2M}{r^3} \cos \theta \sin \theta+\frac{M}{r^3} \sin \theta \cos \theta$$

$$=\frac{M}{r^3} (2 \sin \theta \cos \theta+\sin \theta \cos \theta)$$

$$=\frac{M}{r^3} (3 \sin \theta \cos \theta) \qquad (28)$$

26. We now list the five equations developed above, together with values of the respective components (see fig. 8) based on assumed constants as follows: $M=10{,}000$ cgs; $r=100$ cm; $\theta=40°$. We have, then

Radial	$f_r=\frac{2M}{r^3} \cos \theta$	1532γ
Tangential	$f_\theta=\frac{M}{r^3} \sin \theta$	643γ
Total	$f=\frac{M}{r^3} \sqrt{3 \cos^2 \theta+1}$	1662γ
Parallel	$f_\parallel=\frac{M}{r^3} (3 \cos^2 \theta-1)$	760γ
Perpendicular	$f_\perp=\frac{3M}{r^3} \sin \theta \cos \theta$	1477γ

27. *Other axes and components.*—It is often convenient to compute a component of the total field, f, parallel to some other direction. In figure 8, suppose the chosen direction is PQ. Then the component of f parallel to PQ is PD, and the component perpendicular to PQ is DF. If PQ is the direction of magnetic north, PD will be denoted by f_N and DF by f_E. If PQ is in the direction of a suspended magnet, $PD=f_p$ and $DF=f_n$. If PD and DF are parallel respectively to the X- and Y-axes, then $PD=f_x$ and $DF=f_y$.

28. *Components of the field for small values of* θ.—It is interesting to note the values of the various components for small values of θ and for values of θ near 90°. This is of special importance in connection with placing the deflector in the orientation tests described in chapter 12. The values of these components are given in table 1.

TABLE 1.—***Components of the field of a short bar magnet.***

[Computed for a Uniform Distance of 100 cm, With $M=1000$, for Angles Close to 0° or 90°.]

θ	f_r	f_θ	f	$f_{\parallel}$	$f_{\perp}$
°	γ	γ	γ	γ	γ
0	200. 00	0. 00	200. 00	200. 00	0. 00
1	199. 97	1. 75	199. 98	199. 91	5. 23
2	199. 88	3. 49	199. 91	199. 63	10. 46
3	199. 73	5. 23	199. 79	199. 18	15. 68
4	199. 51	6. 98	199. 63	198. 54	20. 88
5	199. 24	8. 72	199. 43	197. 72	26. 05
85	17. 43	99. 62	101. 13	97. 72	26. 05
86	13. 95	99. 76	100. 73	98. 54	20. 88
87	10. 47	99. 86	100. 41	99. 18	15. 68
88	6. 98	99. 94	100. 18	99. 63	10. 46
89	3. 49	99. 98	100. 05	99. 91	5. 23
90	0. 00	100. 00	100. 00	100. 00	0. 00

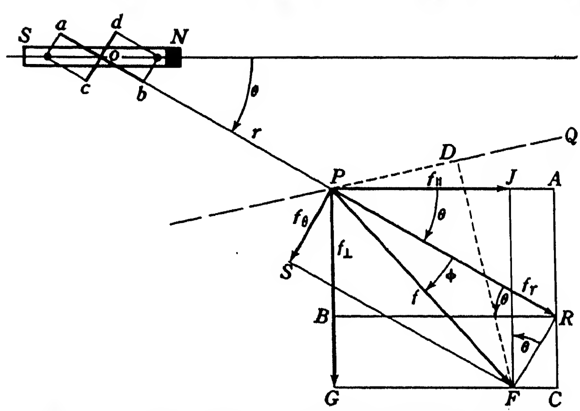

Figure 8.—**Components of the field of a magnet at any point P.**

29. *Direction of total field, f, at a point, P.*—Consider figure 8. Let the angle $RPF=\phi=$the angle between the total field vector, f, and the radial field vector, f_r.

$$\tan\phi=\frac{f_\theta}{f_r}$$

$$=\frac{\frac{M}{r^3}\sin\theta}{\frac{2M}{r^3}\cos\theta} \tag{29}$$

$$=\frac{\sin\theta}{2\cos\theta}$$

$$=\tfrac{1}{2}\tan\theta \tag{30}$$

and for small angles

$$\phi=\tfrac{1}{2}\theta. \tag{31}$$

30. ***Distribution coefficients and distribution factors.***—Equations (11) and (12) are based on the assumption that the distance, r, is great compared to the pole distance. In practice this is not always true so that in calculating the couple due to the interaction of two magnets we must apply certain factors to the equations for the couple based on (11) and (12) in order to obtain greater precision. Bergen, Schmidt, Hazard, Hartnell, and others have developed special solutions for these factors as they apply to pairs of magnets of different lengths and different orientations with respect to each other. The correction factors take the form

$$1+\frac{P}{r^2}+\frac{Q}{r^4}+\cdots,$$

in which P and Q are called the first and second distribution coefficients.

31. Figure 9 represents three mutually perpendicular planes. All axes pass through the center of a suspended magnet, M_s, the X and Y

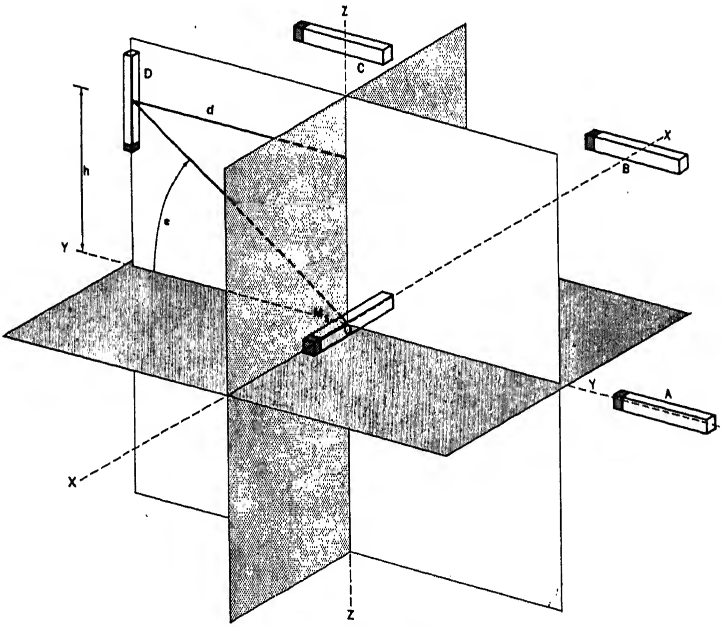

Figure 9.—Deflector positions.

axes being horizontal and the Z axis vertical. The deflector is designated M_a. (M_a and M_s are also used to designate the magnetic moments of these magnets.)

32. Four positions are considered. The axis of the suspended magnet lies along the X axis for all positions.

A Position: The center of the deflector, M_a, is on the Y-axis, and the magnetic axis of M_a is parallel to the Y-axis;

B Position: The center of the deflector, M_a, is on the X-axis, and the magnetic axis of M_a is parallel to the Y-axis;

C Position: The center of the deflector, M_a, is on the Z-axis, and the magnetic axis of M_a is parallel to the Y-axis;

D Position: The center of the deflector, M_a, is anywhere in the YZ plane and the magnetic axis of M_a is parallel to the Z-axis.

33. It should be noted that, although the positions of M_a and M_s are described above relative to the vertical Z axis and the horizontal X and Y axes, the axes may be rotated in any direction. The position of M_a relative to M_s is considered in each case, rather than the positions of the two magnets relative to a fixed set of orthogonal axes.

34. Table 2 gives the formulas for calculating the distribution coefficients in terms of the pole distances of the pair of magnets and also gives the form in which the distribution factors should be applied for the four positions. In the computations in this manual, the pole distance of a magnet is taken as 0.4 of the length of the magnet (see par. 13).

35. Table 21 gives the approximate values of P and Q in terms of the over-all lengths of the deflector and suspended magnet for various combinations of L_a up to 20 cm and L_s up to 12 cm. This table together with the nomogram, figure 146, will be found very helpful in making a quick and reasonably accurate estimate of the value of the distribution factor. With magnets of odd shapes, the pole distance may be known to be other than 0.4 of the magnet length; in such cases, enter the table with a nominal value of L equal to 2.5 l, irrespective of the actual length of the magnet, so that l will be 0.4 of the nominal L. (For table 21 see pp. 227–228.)

TABLE 2.—***Summary of formulas A, B, C, and D Positions.***

Position	P	Q	Factor
A	$2l_a^2-3l_s^2$	$3l_a^4-15l_a^2l_s^2+\frac{45}{8}l_s^4$	$A=1+\frac{P_A}{r^2}+\frac{Q_A}{r^4}$
B	$-\frac{3}{2}l_a^2+6l_s^2$	$\frac{15}{8}l_a^4-\frac{45}{2}l_a^2l_s^2+15l_s^4$	$B=1+\frac{P_B}{r^2}+\frac{Q_B}{r^4}$
C	$-\frac{3}{2}(l_a^2+l_s^2)$	$\frac{15}{8}(l_a^2+l_s^2)^2$	$C=1+\frac{P_C}{r^2}+\frac{Q_C}{r^4}$
D	$l_a^2\left(\frac{35}{6}\sin^2\epsilon-\frac{5}{2}\right)-\frac{5}{2}l_s^2$	$l_a^4\left(\frac{35}{8}-\frac{105}{4}\sin^2\epsilon+\frac{231}{8}\sin^4\epsilon\right)+l_a^2l_s^2\left(\frac{35}{4}-\frac{105}{4}\sin^2\epsilon\right)+\frac{35}{8}l_s^4$	$D=1+\frac{P_D}{r^2}+\frac{Q_D}{r^4}$

M_a=deflector or its magnetic moment;
M_s=suspended magnet or its magnetic moment;
l_a=pole distance of the deflector;
l_s=pole distance of the suspended magnet;
h=height of the center of the deflector above center of M_a;
r=distance between centers of M_a and M_s;
ϵ=angular elevation of center of M_a above (or below) center of M_s (D-position).

If d=horizontal distance for D-position, $r^2=h^2+d^2$, and $\tan\epsilon=\frac{h}{d}$.

36. In Lamont's first position, the deflector is always in the A position. The couple, L_1, tending to turn the suspended magnet out of the magnetic meridian is,

$$L_1=\frac{2M_aM_s}{r^3}\left(1+\frac{P_A}{r^2}+\frac{Q_A}{r^4}\right). \tag{32}$$

The restoring couple, L_2, due to the uniform field, H, acting on M_s, is

$$L_2=HM_s \sin u. \tag{33}$$

When M_s comes to rest these couples are equal and

$$HM_s \sin u=\frac{2M_aM_s}{r^3}\left(1+\frac{P_A}{r^2}+\frac{Q_A}{r^4}\right) \tag{34}$$

and

$$H=\frac{2M_a}{r^3 \sin u}\left(1+\frac{P_A}{r^2}+\frac{Q_A}{r^4}\right). \tag{35}$$

37. In equation (34), if r is quite large compared to the pole distance, l_a, of the deflector, and if the pole distance, l_s, of the suspended magnet is very small, this relation may be written

$$H \sin u=\frac{2M_a}{r^3}\left(1+\frac{2l_a^2}{r^2}\right). \tag{36}$$

And from equation (19),

$$f=H \sin u$$

Hence,

$$f=\frac{2M_a}{r^3}\left(1+\frac{2l_a^2}{r^2}\right). \tag{37}$$

Equation (37) will yield a reasonably precise value of the field at a point P along the magnetic axis of M_a produced. Similarly, for the B position, the field in the direction of the perpendicular bisector of M_s would be

$$f=\frac{M_a}{r^3}\left(1-\frac{3}{2}\frac{l_a^2}{r^2}\right). \tag{38}$$

38. *Special values for P_A and Q_A.*—In the equation $P_A=2l_a^2-3l_s^2$, by letting $P_A=0$ and solving for $\frac{l_a}{l_s}$, we find $\frac{l_a}{l_s}=1.225$. Likewise, in the equation $Q_A=3l_a^4-15l_a^2l_s^2+\frac{45}{8}l_s^4$, by setting $Q_A=0$ and solving for $\frac{l_a}{l_s}$, two positive values are found; $\frac{l_a}{l_s}=2.143$ for the usual case that l_a is greater than l_s; $\frac{l_a}{l_s}=.641$ for the rare case when l_a is less than l_s. The long and short magnets of a magnetometer are usually designed with the ratio of these lengths such that P_A or Q_A will be practically zero.

Table 3 summarizes these ratios in form for convenient use. Note that in solving for $P_A=0$ or $Q_A=0$, l_a is taken as 0.4 L_a.

TABLE 3.—*Special values for P_A and Q_A.*

For $P_A=0$	Condition	For $Q_A=0$ and $l_a>l_s$	Condition
(a) $l_a=1.225\ l_s$		(g) $l_a=2.143\ l_s$	
(b) $l_s=0.816\ l_a$		(h) $l_s=0.467\ l_a$	
(c) $L_a=1.225\ L_s$	$\left.\begin{matrix}\\ \end{matrix}\right\} \dfrac{L_a}{L_s}=\dfrac{l_a}{l_s}$	(i) $L_a=2.143\ L_s$	$\left.\begin{matrix}\\ \end{matrix}\right\} \dfrac{L_a}{L_s}=\dfrac{l_a}{l_s}$
(d) $L_s=0.816\ L_a$		(j) $L_s=0.467\ L_a$	
(e) $Q_A=-4.500\ l_a^4$		(k) $P_A=+1.35\ l_a^2$	
(f) $Q_A=-0.115\ L_a^4$	$l_a=0.4\ L_a$	(l) $P_A=+0.216\ L_a^2$	$l_a=0.4\ L_a$

EXAMPLES

For $P_A=0$	Case I	Case II	Equation	For $Q_A=0$	Case I	Case II	Equation
	cm	cm			cm	cm	
when $L_a=$	4.9	9.28		when $L_a=$	4.9	9.28	
$L_s=$	4.0	7.58	(d)	$L_s=$	2.29	4.33	(j)
$Q_A=$	$-66.$	$-854.$	(f)	$P_A=$	$+51.9$	18.6	(l)

CHAPTER 2. THE EARTH'S MAGNETIC FIELD

39. ***The magnetic elements.***—The direction of the earth's magnetic field at a point on the surface of the earth is the direction taken by a freely suspended magnetic needle (free to turn in any direction in space).

(a) The magnitude of the field is called the *total intensity* and is indicated by F in figure 10.

(b) The *horizontal intensity*, H, is the projection of F on the horizontal plane.

(c) The *vertical intensity*, Z, is the projection of F on the vertical.

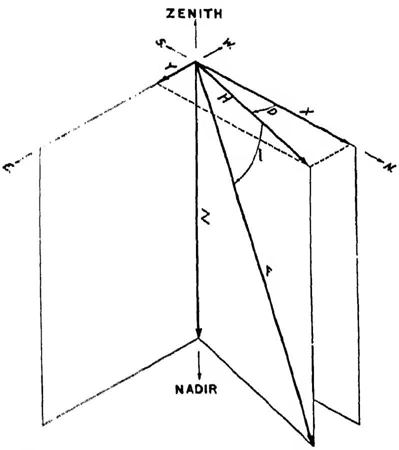

Figure 10.—Components of the earth's magnetic field.

(d) The true *north component*, X, is the projection of H on the true north direction.

(e) The true *east component*, Y, is the projection of H on the true east direction.

(f) The *magnetic declination*, D, is the angle between H and X.

(g) The *magnetic dip* or *inclination*, I, is the angle between H and F.

(h) The *magnetic meridian plane* is the vertical plane through F containing H, F, and Z.

(i) The *magnetic meridian* will be used in this manual to denote the horizontal line through a specified point, in the direction of H.

(j) The *magnetic prime vertical plane* is the plane perpendicular to H.

40. The principal magnetic elements are D, H, and I. A knowledge of their direction and magnitude at any point enables one to compute other desired components.

41. The relation between these various elements is shown in the vector diagram, figure 10. Note also these primary equations:

$$Z=H\tan I \qquad F^2=H^2+Z^2 \qquad X=H\cos D$$
$$Z=F\sin I \qquad\qquad\qquad Y=H\sin D$$
$$H=F\cos I \qquad\qquad\qquad H^2=X^2+Y^2$$

42. Approximate values of the magnetic elements for any part of the world may be scaled from world isomagnetic charts[1] with sufficient accuracy for use in designing magnetic observatory instruments for particular regions.

43. ***General pattern approximated by dipole field.***—The accrued results of past measurements show that the earth's magnetic

---

[1] Those issued by the U. S. Hydrographic Office, Navy Department, are a standard source. For most areas, adequate values may likewise be obtained from the Smithsonian Physical Tables (see item 14 of bibliography on p. 228).

field is such as could be roughly accounted for by supposing a bar magnet at its center with a magnetic moment, M, of about 8.1×10^{25} cgs. The field of such a magnet would have a configuration depending on its length—that is, on the separation of its poles. However, it is found that the shorter the magnet the better the fit. As the poles are brought closer together and their strength simultaneously increased so as to preserve a constant magnetic moment, the field pattern at remote points is but little changed, and with the pole spacing very small in comparison to the distance, the field approaches a limiting pattern which we term the field of a *magnetic dipole*, taking the pole distance as an infinitesimal quantity. The field of a centered dipole is the simplest over-all approximation to the earth's field. Note further that the dipole field would be indistinguishable from one produced by uniform magnetization (parallel to an axis) of the entire earth, or of any smaller portion thereof occupying the volume within a concentric spherical surface. A suitable distribution of electric current, flowing along circular paths in a metallic core, could also yield such a field.

44. The axis of the centered dipole that most nearly duplicates the earth's field is known as the earth's *magnetic axis*. It pierces the surface of the earth at two points known as the *geomagnetic poles*, where the dipole field would be perpendicular to the surface.[2] The places where the actual field is perpendicular to the surface are known as the earth's *magnetic poles* and do not coincide with the geomagnetic poles, since the dipole field is only a rough approximation to the actual field. The currently adopted positions of the two kinds of poles are as follows:

Pole	Location	
	Latitude	Longitude
North magnetic	73° N.	100° W.
South magnetic	68° S.	144° E.
North geomagnetic	78°.5 N.	69° W.
South geomagnetic	78°.5 S.	111° E.

45. ***Earth's field at various latitudes.***—In figure 11 the circle represents the surface of the earth and the small bar magnet represents the hypothetical magnetized portion at the center. Since the north-seeking end of a compass needle is called the North end, the magnetically active core of the earth behaves as though its geographically northernmost part were magnetically a South pole.

46. The approximate relative magnitude and direction of the components f_θ, f_r, and f, at certain points on the circumference of the circle of radius r, where r is very great compared to the length of the magnet, are shown in figure 11, for various values of θ. The angle θ is the angle between the magnetic axis and the radius through the chosen point on the circumference. If we consider the plane of the circle as a section through the earth and consider that the magnetic axis lies in the plane of this section, then the components of the earth's field, at the surface, may be computed approximately, using equations (23) and (25) of chapter 1 (p. 8), with $M=8.1\times10^{25}$ cgs.

[2] Chapman and Bartels, Geomagnetism, vol. II, pp. 644–45, 1951 (see item 2 of bibliography).

The *magnetic latitude* at a point is the angle whose tangent is ½ tan I, where I is the actual dip at the point. The *geomagnetic latitude* is the angle whose tangent is ½ tan I', where I' is the dip computed from the simple dipole approximation. It can be shown (see eq. 30) that the geomagnetic latitude is a coordinate like ordinary latitude but reckoned in relation to the geomagnetic rather than to the geographic poles.

Example: Suppose $\theta=112°.5$, corresponding to a geomagnetic latitude of 22°.5 N, taking the radius of the earth to be 6.37×10^8 cm,

$$Z=-f_r=-\frac{2M}{r^3}\cos\theta=0.241 \text{ cgs}$$

$$H=f_\theta=\frac{M}{r^3}\sin\theta=0.290 \text{ cgs}$$

$$F=\sqrt{H^2+Z^2}=0.377 \text{ cgs}$$

$$I=\tan^{-1}\frac{Z}{H}=39°.6$$

47. The correspondence between f, f_r, f_θ, ϕ, and F, H, Z, I is as follows:

FIELD OF MAGNET		EARTH'S FIELD	
Name	Symbol	Name	Symbol
Radial component	f_r	Reversed vertical intensity	$-Z$
Tangential component	f_θ	Horizontal intensity	H
Total field	f	Total intensity	F
Direction of f	--------	Direction of F	--------
Angle between f and f_θ	$90°-\phi$	Inclination (or dip)	I

The negative sign is required in the relation $f_r=-Z$ because Z is taken positive inward and f_r is positive outward.

48. ***Equation of a line of force.***—It has been shown by Chapman and Bartels,[3] and others, that a line of force due to a magnetic dipole is the locus of the equation

$$r=C\sin^2\theta \tag{39}$$

in which r=distance from a dipole magnet to a point P,
θ=the angle r makes with the magnetic axis of the dipole,
and C=a constant for the particular line of force.

49. It is obvious from equation (39) that a line of force (any line of force) must pass through the dipole since $r=0$ when $\theta=0°$. When $\theta=90°$, $\sin^2\theta=1$ and $r=C$. By assigning arbitrary values to C (such as 1, 2, 3, etc.), and 5°, 10°, 15°, etc. to θ, the corresponding values of r may be calculated from equation (39). Figure 11 shows the pattern of the lines of force about a dipole for various values of C.

[3] S. Chapman and J. Bartels, Geomagnetism, p. 11 (see item 2 of bibliography on p. 228).

50. To draw a line of force which shall pass through a given point P whose polar coordinates are known with respect to the dipole and its magnetic axis, first solve equation (39) for C using the given values of r and θ. Then using this value of C and various values of θ, solve for the corresponding values of r. Figure 11 shows the lines of force

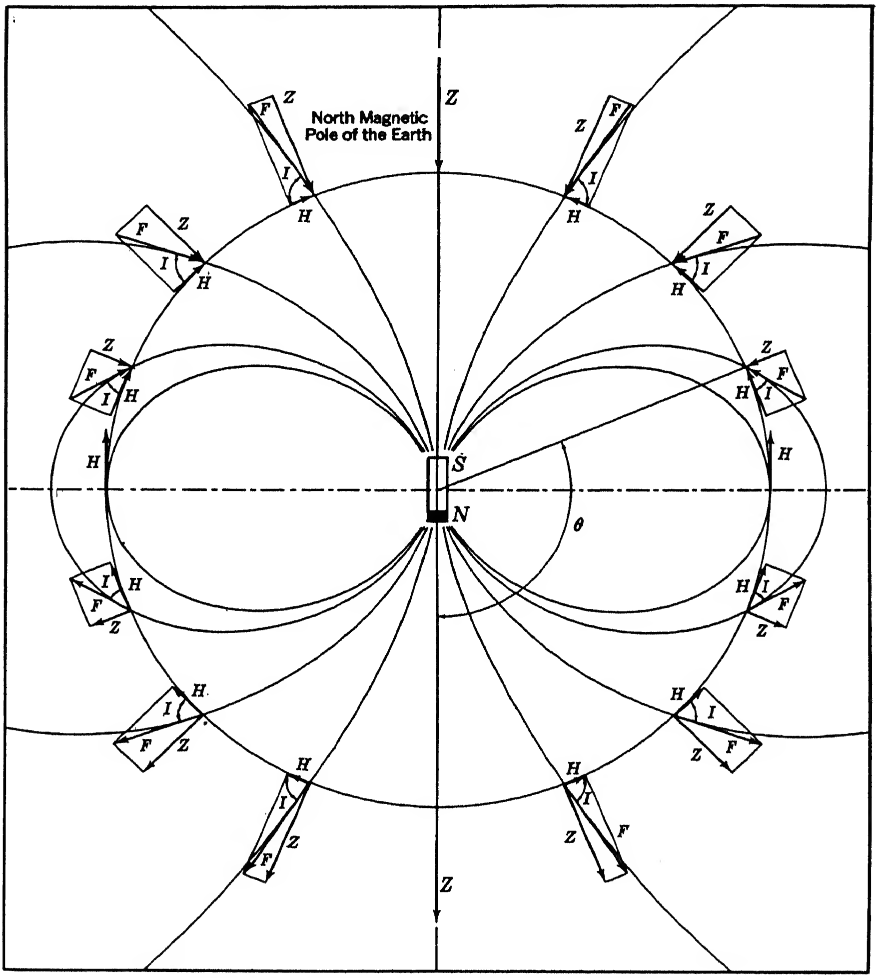

Figure 11.—The earth's magnetic field at various latitudes.

passing through several points on the circumference of a circle of radius 10 units for values of $\theta=0°$, $22°.5$, $45°$, $67°.5$, and $90°$, and for the corresponding angles in the other quadrants.

51. ***Changes in the earth's magnetic field.***—The earth's magnetic field is always changing in direction or intensity or both. These changes are described briefly in the following paragraphs.[4]

52. *Daily variation.*—There is usually a fairly systematic departure of the magnetic field from its daily mean value. This repeats itself

[4] Further details are given by A. K. Ludy and H. H. Howe, Magnetism of the Earth (see item 13 of bibliography on p. 228).

(though with somewhat variable form and amplitude) day after day. The amount of departure depends upon the time of day, the season, the magnetic latitude, and other factors not wholly understood. This systematic change is called *daily variation*.

53. *Irregular disturbance and magnetic storms.*—Superimposed on the regular daily variation, there are usually irregular changes. When they become very large, we say there is a *magnetic storm*. These storms are associated with sunspots, and are characterized by auroral displays and pronounced disturbances to radio-wave transmission and wire telegraphy. A magnetic storm may last many hours or even several days, and the more severe ones occur all over the earth at the same time.

54. *Secular and annual change.*—In general, the average value of a magnetic element changes from one year to the next, and the change usually continues in one direction for many years. This is called the *secular change*. The amount in one year is called the *annual change*.

CHAPTER 3. THE MAGNETIC OBSERVATORY

55. ***Selection of an observatory site.***—The primary objective of the operation of a magnetic observatory is the measurement of the continuous variations in the earth's magnetism in the particular region where the observatory is situated.

56. The site should be free from local disturbances (irregularities of the magnetic field), either natural or artificial. This can be ascertained only by a preliminary magnetic survey of a large area surrounding the proposed site, followed by a more detailed survey of the immediate site, say over an area of 50 meters radius. Such a survey should consist of magnetic observations made at points uniformly spaced in the form of a grid over the immediate area with, say, 10 meters between stations. It is usually sufficient to test for irregularities in magnetic declination only. Anomalies in vertical intensity may be determined by a magnetic field balance. A quartz horizontal magnetometer, QHM, is probably the most satisfactory instrument for the determination of anomalies in horizontal intensity. In general, if tests in the immediate vicinity of the proposed site show that there are gradients in declination, anywhere in the area, as great as $10'$ between stations 10 meters apart, or as great as 20γ in horizontal or vertical intensity in the same distance, the site should be considered unsatisfactory. Also, the mean value of the measured element should be about the same as the mean value for the site, as scaled from the latest available isomagnetic chart.

57. ***Buildings and equipment.***—The minimum requirements for a magnetic observatory are as follows:

(a) Satisfactory site, as described in paragraph 56.
(b) Magnetic variation building, including instrument piers.
(c) Absolute observatory building, including instrument piers.
(d) Magnetograph, consisting of H, D, and Z variometers, a photographic recorder, time-marking mechanisms, and facilities and equipment for scale-value and orientation observations.
(e) Office and photographic darkroom.
(f) Magnetometer (or equivalent) for measurement of H and D.
(g) Earth inductor (or equivalent) for measurement of I.
(h) Mean-time chronometer.
(i) Time marking clock or chronometer.
(j) True meridian line or true azimuth from declination pier in absolute observatory to a permanent azimuth mark. (Two permanent azimuth marks recommended.)
(k) True azimuth line established in the variation room.
(l) Miscellaneous equipment for processing records.

58. Figure 12a shows a plan view of a *variation building,* designed for operation of one or two complete magnetographs. For other types of magnetographs the pier plan would require alteration but the basic requirements would be practically the same as described here. The building and piers are constructed of tested, nonmagnetic materials throughout. The piers should have enlarged bases extending well below the ground surface and should rest on natural, undisturbed formation. Within the variation room and attached permanently to the walls are special shelves of heavy construction for supporting deflector holders used in orientation tests and scale-value

observations by the magnetic method. The variation rooms are thermally insulated and are suitable for photographic recording.

59. The *absolute building* is a small, nonmagnetic structure shown in figure 12b. As in the variation building, the bases of the piers

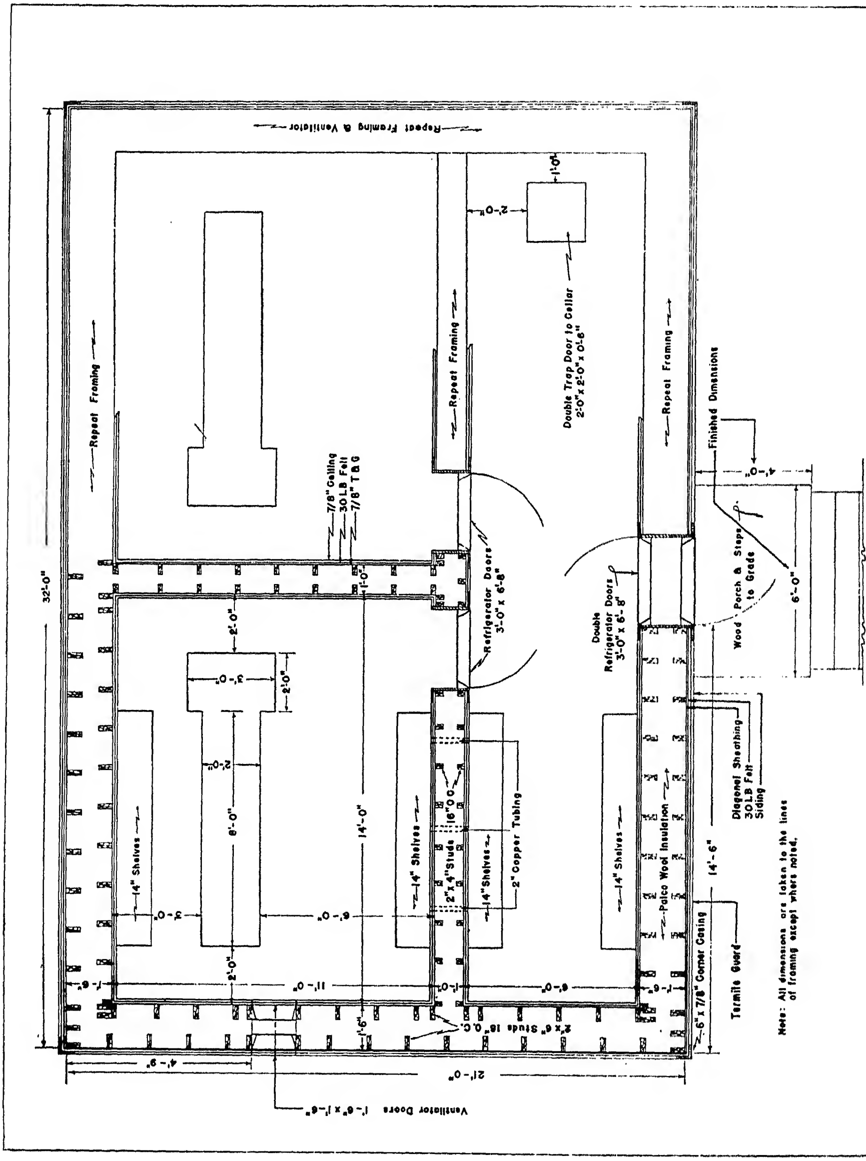

Figure 12a.—Magnetic variation building, for two magnetographs.
DIMENSIONS ARE GIVEN IN FEET (') AND INCHES (").

should rest on natural, undisturbed formation, well below the ground surface. In routine operations the magnetometer is operated on the central pier and the earth inductor and its galvanometer are operated on the other piers. The windows of this structure are so arranged as to provide an unobstructed view of one (or two) well established true azimuth marks.

60. The three magnetic *variometers* shown in figure 13 are constructed to respond to variations in magnetic declination, horizontal intensity, and vertical intensity. A recorder, a light source, and a time-flashing lamp with auxiliary reflector complete the *magnetograph*

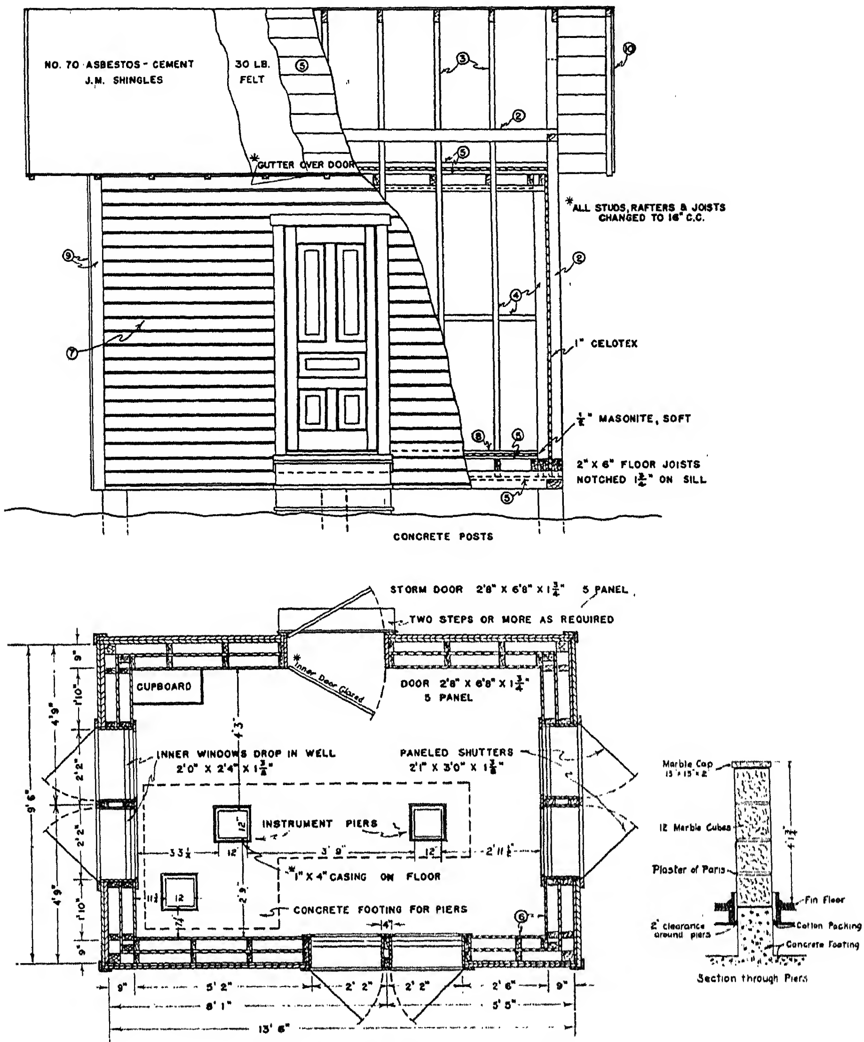

Figure 12b.—Magnetic absolute building.

assembly. If scale values are to be determined by the electrical method, each variometer must be provided with a Helmholtz-Gaugain coil (not shown). The various components of the magnetograph are described in more detail in chapters 5 and 7–9. Descriptions of magnetometers and earth inductors with detailed directions for their use are contained in "Directions for Magnetic Measurements," by D. L. Hazard.

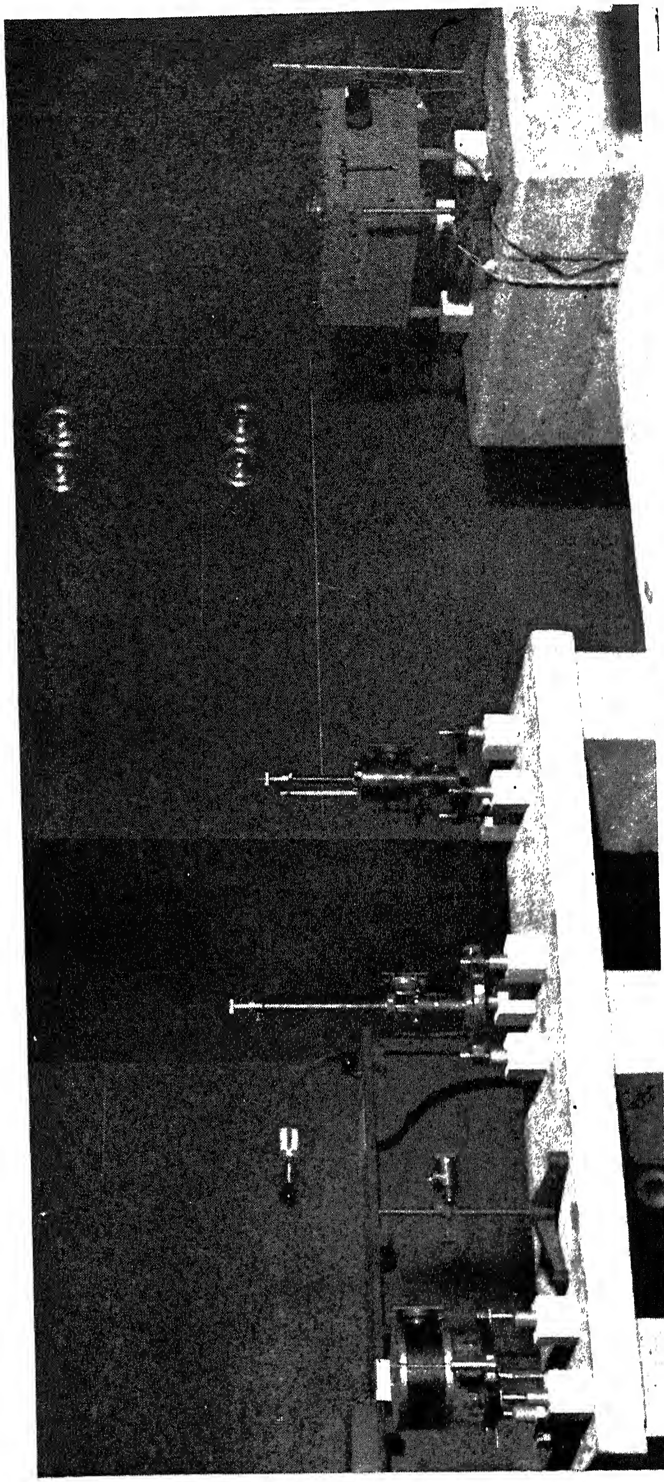

Figure 13.—Magnetograph, with modified la Cour variometers; installed in test laboratory at Cheltenham Magnetic Observatory.

On the slab in the foreground are, left to right, the Z variometer, the thermograph element, the D variometer, and the H variometer.

CHAPTER 4. INSTRUMENTAL CONSTANTS AND CORRECTIONS

REFINEMENTS CONTRIBUTING TO ACCURACY

61. ***Quality of instruments.***—In order to assure the successful accomplishment of accurate measurements of the earth's magnetic field, it is first necessary to be sure that the available instruments are of satisfactory quality and are so designed as to be capable of measurements with the required precision. Quality here refers not only to proper design and fine workmanship on the part of the instrument maker but also to freedom from hidden defects which could and often do introduce the necessity for index corrections that theoretically should not exist. Such hidden defects may often be traced to the presence of minute quantities of impurities in the materials of which the instrument is made, such as to cause erratic or spurious deflection of the suspended magnet or to distort the earth's field in the vicinity of the instrument.

62. ***Test for magnetic parts.***—Before any measurements or observations are made for the determination of any of the constants of a magnetometer, earth inductor, or other instrument used directly in measuring a component of the earth's field, the parts of the instrument should be tested carefully for magnetic properties. Any part found to show objectionable traces of magnetism should be rejected; if that part is near the moving magnet or coil of the assembled instrument, any detectable magnetic effect should be sufficient cause for rejection.

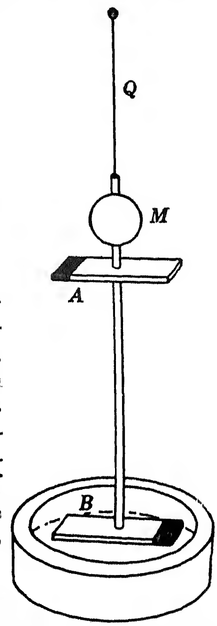

Figure 14.—Moving-magnet system of magnetism tester.

63. To test a specimen for magnetism, it is usually sufficient to place it near one of the magnets of an astatic moving-magnet system (magnetism tester) such as that shown in figure 14. The two magnets A and B having equal magnetic moments (10 to 20 cgs units) are mounted about 10 cm apart in the same plane on a stiff brass rod, but with their moments oppositely directed. The system is equipped with a mirror, M, and is suspended by a fine filament, Q, such as a magnetometer suspension fiber. A lamp and scale are provided for visual observation of the deflections of the magnets. A copper damping box for the lower magnet is necessary to reduce undesirable oscillations, and a cylindrical shield, with window, not shown in the illustration, provides protection against spurious deflections due to air currents. As long as the magnetism tester is not located too near direct-current machinery or cables, it will prove to be quite satis-

factory for use even in a machine shop for testing parts of magnetic instruments. The specimen to be tested is placed near one of the magnets of the tester, and the deflection (if any) noted *while the specimen is at rest.* It is important that the specimen remain at rest while observing the deflection of the magnets, since secondary magnetic fields will be set up by eddy currents induced in the specimen if it is moved about in the field of the magnets. Specimens should be tested in several positions.

64. Although the magnetism tester as described does not yield quantitative results, it can be made very sensitive by properly balancing the fields of the two magnets, and a little practice with the tester will enable any observer readily to become skilled at finding traces of magnetic materials in instrument parts.

65. ***Instrumental constants to be determined.***—For measurements of magnetic declination, the scale value of the scale in the telescope (or magnet) must be determined as explained in paragraphs 66 and 67. The equations used in the computation of H and M by the method of oscillations and deflections involve the moment of inertia, induction factor, temperature coefficients, deflection distances, and distribution coefficients. These instrumental constants must be determined as explained in paragraphs 69–145.

SCALE VALUE

66. ***Scale in the telescope.***—This scale value is determined as follows: Focus the telescope on a well-defined, distant object and determine the difference in circle reading corresponding to a difference of, say, 20 divisions on the scale. The ratio of the difference in circle readings to the difference in scale readings is the scale value, in minutes of arc per scale division. Repeat several times and take the mean.

67. ***Scale in the magnet.***—Proceed as follows: When magnetic conditions are quiet, suspend the magnet with a filament having a small torsion factor, and when the magnet is at rest, with the vertical cross line of the telescope bisecting some even scale division, observe the circle reading. Rotate the magnetometer through precisely 10 scale divisions to the right, repeat the circle reading; then 20 divisions to the left, etc., making several determinations. As in paragraph 66 the ratio of the change in circle reading to the change in scale reading is the scale value in minutes of arc per scale division. Note that there will be a slight restraining effect due to torsion of the filament, but if a fine filament is used, the effect will be negligible for such small angular displacements. The operation may be simplified and the effects of change in magnetic declination and torsion entirely eliminated if the suspended magnet is held in a fixed position relative to the ground by a nonmagnetic stop mounted directly on the pier and independent of the magnetometer.

MAGNETOMETER EQUATIONS

68. As shown by Hazard,[1] the two fundamental equations showing the relation between the horizontal intensity, H, of the earth's mag-

[1] Dir. for Mag. Meas. (see item 4 of bibliography).

netic field, and the magnetic moment, M, of the long magnet of a magnetometer, are

$$HM=\frac{\pi^2K}{T^2(1+0.0000116d)^2\left(\frac{f}{f-h}\right)\left(1+\mu\frac{H}{M}\right)\left(1+(t-t')q\right)} \tag{40}$$

$$\frac{H}{M}=\left[\frac{2}{r^3}\left(1+\frac{P_A}{r_2}+\frac{Q_A}{r^4}\right)\left(1-\frac{2\mu}{r^3}\right)\right]\frac{1}{\sin u} \tag{41}$$

in which M=magnetic moment of the long magnet at temperature t;
K=moment of inertia of the long magnet and its stirrup at temperature t';
T=observed time of one oscillation (half-period) in chronometer seconds;
d=rate of chronometer in seconds per day;
h=angle through which magnet turns when torsion head is turned through an angle f;
f=angle through which torsion head is turned to deflect the suspended magnet through the angle h;
μ=induction factor of the long magnet;
t=observed temperature of long magnet during deflection observations;
t'=observed temperature of long magnet during oscillation observations;
q=temperature coefficient of magnetic moment of the long magnet;
r=deflection distance at temperature t;
u=deflection angle for particular value of r;
and P_A and Q_A are distribution coefficients.

69. The moment of inertia, induction factor, temperature coefficient, deflection distances, and distribution coefficients must be determined by special observations or otherwise evaluated before equations (40) and (41) can be used in the computation of HM and $\frac{H}{M}$. The time of one oscillation, its correction for rate of chronometer, the torsion factor, and the deflection angles are determined directly at the time horizontal-intensity observations are made. For all practical purposes, the distribution coefficients, induction factor, and temperature coefficient are taken as constants at all temperatures at which observations are likely to be made, but deflection distances and moment of inertia of the long magnet change appreciably with temperature, and corrections must be applied accordingly to these terms. The torsion factor of the suspension filament is usually determined just before or just after each set of oscillation observations. The rate of the chronometer is, of course, an independent determination not involving the magnetometer itself.

70. It is customary to combine some of the factors in equation (41) and treat them as a single constant for a particular value of r at a particular temperature. Thus, equation (41) may be written

$$\frac{H}{M}=\frac{C}{\sin u} \tag{42}$$

in which

$$C=\frac{2}{r^3}\left(1-\frac{2\mu}{r^2}\right)\left(1+\frac{P_A}{r^2}+\frac{Q_A}{r^4}\right). \tag{43}$$

The constant C may be computed for various values of r and these values corrected for temperature as explained in paragraph 454, appendix I (p. 193).

MOMENT OF INERTIA

71. ***Basic relations.***—The time of one oscillation, T, of a magnet oscillating as a torsion pendulum under the influence of the horizontal component, H, of the earth's magnetic field is given by

$$T=\pi\sqrt{\frac{K}{H\,M}} \tag{44}$$

in which K is the moment of inertia of the magnet and its stirrup and M is the magnetic moment of the magnet. The moment of inertia of the magnet and its stirrup cannot be readily calculated from the dimensions and mass; it must be determined experimentally by the kinetic method. That is, the period of the system is determined with and without a special inertia weight mounted on the magnet system. The added mass is usually in the form of a precision-ground, solid, right circular cylinder of bronze or brass, or in the form of a cylindrical brass ring. In either case the material must be homogeneous and non-magnetic.

72. The moment of inertia of a solid, right circular cylinder, oscillating about a transverse diameter through its center, is

$$K_1=W\left(\frac{l^2}{12}+\frac{d^2}{16}\right) \tag{45}$$

in which W=the mass of the cylinder in grams;
l=the length in cm;
d=the diameter in cm.

73. For a cylindrical ring oscillating about a longitudinal axis (axis through its center normal to its bounding planes), the formula is

$$K_1=\frac{W}{8}(d_1^2+d_2^2) \tag{46}$$

in which d_1 and d_2 are the inner and outer diameters.

74. It is important that the inertia weight be constructed with great precision and that the dimensions be determined for several uniformly distributed positions. The temperature at which these measurements are made should be specified. In the United States, the mass and dimensions are usually determined at the National Bureau of Standards.

75. With the inertia weight added to the system, the time of one oscillation is

$$T_1=\pi\sqrt{\frac{K+K_1}{H\,M}} \tag{47}$$

in which K_1 is the moment of inertia of the inertia weight calculated from its dimensions. Squaring equations (44) and (47) and dividing.

$$\frac{T_1^2}{T^2}=\frac{K+K_1}{K}=1+\frac{K_1}{K}$$

and

$$K=\frac{K_1}{\frac{T_1^2}{T^2}-1} \qquad (48)$$

76. Equations (44), (47), and (48) are based on the assumption that K, K_1, H, and M are constants and remain constant under all conditions and that no other forces are acting on the oscillating system. However, during a set of oscillations both H and the temperature may change. A change in temperature changes the dimensions of the magnet and the inertia weight, thus affecting both K and K_1. The magnetic moment of the magnet also changes with temperature. The directive force of torsion of the suspension filament affects the period and also changes with the load. The chronometer may not have a uniform rate during a series of observations but it is usually assumed that the rate is the same for adjacent loaded and unloaded sets, and therefore does not affect the ratio, $\frac{T_1}{T}$. Induction and damping effects may be considered as equal for loaded and unloaded sets and therefore may be neglected.

77. The moment of inertia, K_1, of the inertia weight and the observed times of oscillation, T and T_1, must be reduced to standard conditions in order to evaluate the moment of inertia, K, of the magnet at those same standard conditions. Temperatures are usually referred to 20° C and changes in H to a mean value for the elapsed time or simply to the H base-line value.

78. For the unloaded magnet, the relation between the true time of one oscillation (T) and the observed time T is,

$$(T)^2=T^2\,\frac{5400}{5400-h}\,[1+(20-t)q]\,[1+(20-t)2\alpha]\,\frac{H}{B_H} \qquad (49)$$

and for a loaded set,

$$(T_1)^2=T_1^2\,\frac{5400}{5400-h}\,[1+(20-t_1)q]\,[1+(20-t_1)2\alpha_1]\,\frac{H_1}{B_H} \qquad (50)$$

in which (T) = the corrected time of one oscillation (unloaded);
T = the observed time of one oscillation (unloaded);
(T_1) = the corrected time of one oscillation (loaded);
T_1 = the observed time of one oscillation (loaded);
t = the observed temperature of the magnet in unloaded oscillations;
t_1 = the observed temperature of the magnet and inertia weight in loaded oscillations;
q = the temperature coefficient of magnetic moment of the magnet;
α = coefficient of thermal expansion (linear) of the magnet;
α_1 = combined coefficient of thermal expansion (linear) of the inertia weight and magnet;

H and H_1 are the average values of the horizontal intensity during a set of unloaded and loaded oscillations, respectively; and B_H is the horizontal-intensity base-line value.

79. The correction factors in equations (49) and (50) are explained in appendix I, and their method of application is shown in the sample sets of observations, figures 15, 16, and 17.

UNLOADED

OSCILLATIONS FOR MOMENT OF INERTIA

Station, *Cheltenham, Md., Pier 6* — Date, *Fri., July 21, 1950*
Magnetometer No. *3054* — Observer, *RLV*
Magnet No. *RIC 6* — Inertia ring or bar No. *A* — Chron. No. *6565*
Chron. corr'n on *75* M. Time, *+0* m *10.0* s Daily rate $\left(\begin{smallmatrix}+\text{for losing}\\-\text{for gaining}\end{smallmatrix}\right)$, *+0.2* s/day

Number of oscillations		Chronometer time	Temp. t	Extreme scale readings		Remarks
		h m s	°C	div.	div.	50= 12 18 42.7
0	(1)	12 15 21.7	26.5	16.0	44.0	0= 15 21.7
5	(2)	41.8				3 21.0
10	(3)	16 02.0				
15	(4)	22.0				
20	(5)	42.2				100= 22 03.7
25	(6)	17 02.3				
30	(7)	22.3				Time of 100 oscil.
35	(8)	42.5				
40	(9)	18 02.5				
45	(10)	12 18 22.6	26.5	17.1	43.0	
100	(11)	12 22 03.6				m s
105	(12)	23 8				6 41.9
110	(13)	43.8				42.0
115	(14)	23 03.9				41.8
120	(15)	24.0				41.9
125	(16)	44.1				41.8
130	(17)	24 04.3				41.8
135	(18)	24.4				42.0
140	(19)	44.5				41.9
145	(20)	12 25 04.5	26.5	20.1	40.3	42.0
						6 41.9
		Means	26.50			6 41.90

$$\text{FORMULA: } (T)^2 = T^2 \frac{5400}{5400-h} [1+(20-t)q][1+(20-t)2\alpha] \frac{H}{B_H}$$

TORSION OBSERVATIONS

Tors. circle	Scale		Mean	Diffs.
	div.	div.	div.	div.
203	28.9	31.3	30.10	
293	25.8	27.0	26.40	3.70
113	32.7	35.1	33.90	7.50
203	29.5	30.8	30.15	3 75

Mean h = 3.74 div. = 4'.94

Magnetometer scale value, 1'.32/div.

$(20-t) = -6°.5$

Time of 1 oscil.	4.01 s 900
Log T	0.60 4118
Log T^2	1.20 824
Log $\frac{5400}{5400-h}$	+40
Log $(1+(20-t)q)$	−52
Log $(1+(20-t)2\alpha)$	−6
Log $\frac{H}{B_H}$	+339
Log $(T)^2$	1.21 145

Computed by *RRB* — Checked by *RLV* — Abstracted by *AM*

Figure 15. Observations for moment of inertia, unloaded magnet suspension system.

LOADED

Oscillations for Moment of Inertia

Station, *Cheltenham. Md., Pier 6* Date, *Fri., July 21, 1950*
Magnetometer No. *3054* Observer, *RLV*
Magnet No. *RIC-6* Inertia ring or bar No. *A* Chron. No. *6565*

Chron. corr'n on *75* M. Time, *+0* m *10.0* s Daily rate $\left(\begin{smallmatrix}+\text{for losing}\\-\text{for gaining}\end{smallmatrix}\right)$, *+0.2* s/day

Number of oscillations		Chronometer time	Temp. t	Extreme scale readings		Remarks
		h m s	°C	div.	div.	*30=12 52 23.1*
0	(1)	*12 48 56.4*	*26.8*	*10.1*	*50.0.*	*0= 48 56.4*
3	(2)	*49 17.0*				
6	(3)	*37.7*				*3 26.7*
9	(4)	*58.4*				
12	(5)	*50 19.1*				*60= 55 49.8*
15	(6)	*39.8*				
18	(7)	*51 00.5*				Time of *60* oscil.
21	(8)	*21.2*				
24	(9)	*41.8*				
27	(10)	*12 52 02.5*	*26.8*	*12.7*	*48.1*	
60	(11)	*12 55 50.0*				m s
63	(12)	*56 10.7*				*6 53.6*
66	(13)	*31.4*				*53.7*
69	(14)	*52.3*				*53.7*
72	(15)	*57 13.0*				*53.9*
						53.9
75	(16)	*33.8*				*54.0*
78	(17)	*54.2*				*53.7*
81	(18)	*58 15.1*				*53.9*
84	(19)	*35.5*				*53.7*
87	(20)	*12 58 56.4*	*26.8*	*15.3*	*46.0.*	*6 53.9*
		Means	*26.80*			*6 53.80*

$$\text{FORMULA: } (T)^2 = T^2 \frac{5400}{5400-h}[1+(20-t)q][1+(20-t)2\alpha]\frac{H}{B_H}$$

Tors. circle	Scale		Mean	Diffs.
	div.	div.	div.	div.
203	*28.5*	*31.3*	*30.40*	.....
293	*24.9*	*26.9*	*25.90*	*4.50*
113	*33.8*	*36.8*	*35.30*	*9.40*
203	*29.4*	*31.8*	*30.60*	*4.70*

Mean h=*4.65* div. =*6′.14*

Magnetometer scale value, *1′.32*/div.

$(20-t) = -6°.8$

Time of 1 oscil.	*6.89 667* s
Log T	*0.83 8639*
Log T^2	*1.67 728*
Log $\frac{5400}{5400-h}$	*+49*
Log $(1+(20-t)q)$	*−54*
Log $(1+(20-t)2\alpha)$	*−8*
Log $\frac{H}{B_H}$	*+347*
Log $(T)^2$	*1.68 062*

Computed by *RRB* Checked by *RLV* Abstracted by *AM*

Figure 16. Observations for moment of intertia, loaded magnet suspension system.

80. ***Directions for determination of moment of inertia.***—

(a) Read paragraphs 71–78, and paragraphs 447–452, appendix I.

(b) Determine the time of one oscillation of the magnet (unloaded) as in oscillations for horizontal intensity. (See fig. 15.) Mark this set *unloaded.*

(c) Observe one set of torsion observations (unloaded).

(d) Mount the inertia weight on the magnet system, *center and level it carefully*, and repeat oscillation observations as in (b) (see fig. 16). Mark this set *loaded.*

(e) Make one set of torsion observations (loaded).

(f) Observe the temperature at the beginning, middle, and end of each set, also the extreme scale readings at these times.

(g) Repeat (d), loaded.

(h) Repeat (a), 2 sets, unloaded, etc., until eight sets of loaded and eight sets of unloaded oscillations have been completed.

(i) If a magnetograph is in operation at the observatory or in the vicinity, place special time marks on the magnetogram at the beginning and end of each set of oscillations. If a magnetograph is not in operation, or if H cannot be determined independently while

COMPUTATIONS FOR MOMENT OF INERTIA

Station, *Cheltenham, Md.*
Magnet No. *RIC-6*

Observer, *RLV*
Inertia ring or bar No. *A*

Date *1950*	Set	*75* M. Time	H ordinate (h_{mm})	Log $\frac{H}{B_H}$	Log $(T_1)^2$ loaded	Log $(T)^2$ unloaded
		h m	mm			
Jul. 21	1U	*12 15/25*	*54.7*	*0.00 339*		*1.21 145*
	2L	*49/59*	*55.9*	*347*	*1.68 062*	
	3L	*13 06/16*	*57.0*	*353*	*1.68 090*	
	4U	*29/39*	*60.7*	*376*		*1.21 104*
Sept. 11	5U	*14 54/64*	*54.7*	*339*		*1.21 115*
	6L	*15 12/19*	*56.3*	*349*	*1.68 078*	
12	7L	*10 43/52*	*41.6*	*258*	*1.68 070*	
	8U	*11 01/11*	*40.3*	*250*		*1.21 115*
	9U	*16/26*	*47.9*	*297*		*1.21 109*
	10L	*12 12/22*	*43.9*	*272*	*1.68 059*	
	11L	*25/32*	*50.9*	*316*	*1.68 069*	
	12U	*50/61*	*48.8*	*303*		*1.21 101*
	13U	*13 10/20*	*52.7*	*327*		*1.21 119*
	14L	*26/36*	*51.3*	*318*	*1.68 085*	
	15L	*41/51*	*54.3*	*337*	*1.68 083*	
	16U	*14 13/23*	*53.5*	*332*		*1.21 128*

Prel. H s. v. $= S_H$	2.60γ/mm	Log $\frac{H}{B_H} = 0.434 \frac{S_H}{B_H} h_{mm}$	Sums *596*	*136*
Prel. H blv. $= B_H$	*18199γ*	$=$ *0.0000 62* h_{mm}	Means *1.68 074*	*1.21 117*

α (magnet)	*0.0000 11*	Log $\frac{[T_1]^2}{[T]^2}$	*0.46 957*
α' (inertia bar or ring)	*0.0000 18*	$\frac{[T_1]^2}{[T]^2}$	*2.94 820*
$\alpha_1 = \frac{\alpha+\alpha'}{2}$	*0.0000 14*	$\frac{[T_1]^2}{[T]^2} - 1$	*1.94 820*
For UNLOADED oscillations: Log $(1+2\alpha)$	*0.0000 10*		
For LOADED oscillations: Log $(1+2\alpha_1)$	*0.0000 12*	(a): Log $\left[\frac{[T_1]^2}{[T]^2} - 1\right]$	*0.28 965*
FORMULA: $K_{20} = \frac{K_{1.20}}{\frac{[T_1]^2}{[T]^2} - 1}$		(b): Log $K_{1.20}$	*2.47 031*
		(c): Log K_{20}: (b) − (a)	*2.18 066*
		(d): Log π^2	*0.99 430*
Remarks: *log $K_{1.20}$ from NBS, Nov. 1934 2.47 031*		(e): Log $\pi^2 K_{20}$: (c) + (d)	*3.17 496*
		K_{20} = *151.6*	$\pi^2 K_{20}$ = *1496.1*

Scaled by *RRB*
Computed by *RRB*

Checked by *RLV*

Scalings checked by *RLV*
Abstracted by *AM*

Figure 17.—Computations for moment of inertia.

oscillation observations are in progress, ignore the factor $\frac{H}{B_H}$ in equations (49) and (50).

81. ***Computation of moment of inertia of the magnet.***—(a) From the magnetogram, scale the mean ordinates, h_{mm}, in millimeters, for the elapsed times of each of the loaded and unloaded sets, fourth column on figure 17.

(b) Calculate the conversion factor, $0.434 \frac{S}{B_H}=0.434 \frac{2.60}{18199}=$ 0.000062 (see fig. 17).

(c) Calculate $\log \frac{H}{B_H}$ for each set. (For example, the first line on figure 17: $\log \frac{H}{B_H}=0.000062\ h_{mm}=0.00339$.)

(d) Transfer all values of $\log \frac{H}{B_H}$ from the fifth column on figure 17 to the appropriate boxes on the observing forms illustrated by figures 15 and 16.

(e) Calculate $\log K_1$ from the dimensions and mass of the inertia weight and reduce this value to 20° C ($=\log K_{1,20}$). (See paragraph 448, page 187.)

(f) Calculate $\log (T)^2$ and $\log (T_1)^2$ as in oscillation observations for horizontal intensity, using $\log (1+2\alpha)=0.000010$ for unloaded sets and $\log (1+2\alpha_1)=0.000012$ for the loaded.

(g) Transfer values of $\log (T)^2$ and $\log (T_1)^2$ to the proper boxes on figure 17 and complete the computation of $\log \pi^2 K_{20}$.

TEMPERATURE COEFFICIENTS

82. ***Temperature coefficient of magnetic moment.***—The magnetic moment of a magnet decreases with increase in the temperature of the magnet. If the magnet has been properly heat-treated and stabilized, the process is reversible, that is, the magnetic moment is fully restored when the magnet is brought back to its original temperature, provided the original increase in temperature is not too great. The rate of change of the magnetic moment with temperature is almost but not quite uniform over ordinary temperature ranges, say from 0° to 40° C. For all practical purposes, the relation between the magnetic moment, M_1, at a standard or reference temperature, t_1, and the magnetic moment, M_2, at a temperature, t_2, is given by

$$M_2=M_1-M_1\, q\, (t_2-t_1) \tag{51}$$

$$=M_1[1-(t_2-t_1)q] \tag{52}$$

$$=M_1[1+(t_1-t_2)q] \tag{53}$$

in which q is the mean temperature coefficient of the magnetic moment over that range of temperature. (For a more precise representation of this relation see appendix II.)

From equation (53),

$$q=\frac{M_2-M_1}{M_1\,(t_1-t_2)}=\frac{M_1-M_2}{M_1\,(t_2-t_1)} \tag{54}$$

Note that q is defined so as to have a positive value in the usual case (M diminished by a temperature rise). It is necessary to have due regard for signs in the above equations; thus, if $t_1=20°$ and $t_2=25°$, then $t_1-t_2=-5°$.

83. The temperature coefficient, q, may be determined by deflection observations, using a magnetometer. The deflecting magnet whose temperature coefficient of magnetic moment is under test is enclosed by a temperature bath, figure 18, and the deflection angle, u, is observed at several temperatures ranging from 0° C to about 40° C. The magnetic moment, M, at each temperature is calculated from the equation,

$$\frac{H}{M}=\frac{C}{\sin u} \tag{55}$$

in which H, the horizontal intensity at the time u, is observed, and

$$C=\frac{2}{r^3},$$

a constant depending primarily upon the distance r. (See p. 8.) Then

$$M=\frac{H \sin u}{C} \tag{56}$$

and

$$\log M=\log H+\log \sin u+\text{colog } C. \tag{57}$$

Since the change in the deflection angle, Δu, even for a large value of M and a large change of temperature, will be only a few minutes of arc, any changes in H and D during a set of observations must not be neglected in the calculation of q. It is necessary to observe D and H independently and simultaneously with observations of u or to scale these values from a magnetogram. After the values of log M have been derived for different temperatures, q is calculated from equation (54), or more conveniently by logarithms from equation (53). (See eq. 363, app. I.) Thus,

$$\log M_2=\log M_1+\log [1+(t_1-t_2)\,q]$$

and by equation (366)

$$\log M_2\approx\log M_1+(t_1-t_2)\log (1+q) \tag{58}$$

and

$$\log (1+q)\approx\frac{\log M_2-\log M_1}{t_1-t_2} \tag{59}$$

$$\approx\frac{\log M_1-\log M_2}{t_2-t_1}. \tag{60}$$

Suppose $\log (1+q)=0.000103$, then from tables, $1+q=1.000237$; $q=0.000237$. We may also write

$$q=2.30 \log (1+q) \text{ approximately} \tag{61}$$
$$=2.30\times0.000103=0.000237.$$

84. ***Directions for determination of temperature coefficient of magnetic moment.***—On a magnetically quiet day, set up a magnetometer as for determination of horizontal intensity, short magnet suspended, deflection bar attached.

85. Place the long magnet (or other magnet to be tested) centrally in the inner chamber of the temperature bath holder, figure 18, and mount the holder (with magnet) on the bar at such a distance, say $r=30$ cm, that a deflection of 15° to 30° is obtained. Place thermometers, fitted with cork stoppers, *C*, in each end of the magnet chamber, *B*, and see that the thermometer bulbs are in contact with the ends of the magnet so that the magnet may not creep or move during the tests. Remove the holder (with magnet) to a safe distance.

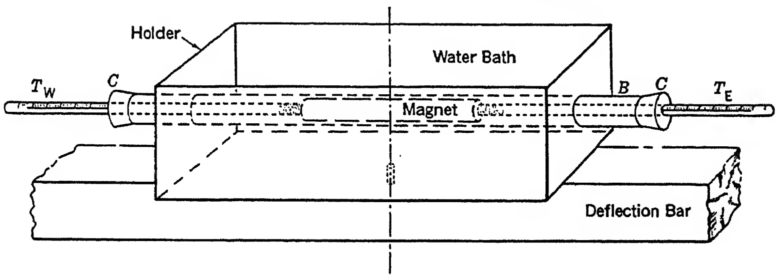

Figure 18.—Temperature bath for magnetometer magnet.

OBSERVATIONS FOR TEMPERATURE COEFFICIENT

Station, Cheltenham; Pier 7
Mgr. No. 37, Scale value, 1.38′ per div.
Deflection distance, 30 cm.
Magnet No. Colombia I Dimensions, 5.0 cm.

Date, Thursday, September 20, 1951
Observer, J. B. Townshend
Therm. Nos., 33246 (E.), 33257A (W.)
Material, Oersted steel

Column	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Line	75 M. time	Temperature observed		Circle reading		Scale reading			Middle minus mean
		East	West	A	B	Left	Right	Mean	
	h m	°C	°C	° ′ ″	′ ″	div.	div.	div.	div.
1	12 48		Away	303 30 05	30 15	49.0	51.0	50.00	0.00
2	12 54	−0.2	+0.7	287 48 20	48 20	49.3	50.7	50.00	0.00
3	13 03	+5.7	6.9			49.2	49.8	49.50	+0.50
4	09	12.2	13.0			48.5	49.4	48.95	+1.05
5	16	21.1	21.4			47.0	47.8	47.40	+2.60
6	22	28.1	28.2			46.6	47.4	47.00	+3.00
7	30	34.9	34.9			49.6	50.9	50.25	−0.25
8	39	42.3	42.2			50.2	50.9	50.55	−0.45
9	40	42.2	42.1			50.1	50.9	50.50	−0.50
10	45	34.9	34.6			50.9	51.1	51.00	−1.00
11	49	28.1	28.1			51.2	51.4	51.30	−1.30
12	53	20.8	21.1			51.8	52.0	51.90	−1.90
13	13 58	12.0	12.4			51.6	52.8	52.20	−2.20
14	14 03	5.3	6.5			51.3	52.3	51.80	−1.80
15	10	−0.5	+0.5	287 48 20	48 20	49.9	50.5	50.20	−0.20
16	14 14		Away	303 30 35	30 40	49.0	51.0	50.00	0.00
17									
18									
19									
20									

NOTES

(a) Thermometer corrections; apply to observed temperatures:
East: 0°.0, West: 0°.0;
(b) Record corrected mean temperatures in last column of figure 21;
(c) Middle division of scale: 50.00;
(d) This form is designed for use at observatories where declination is West, and where "increasing declination" means increasing West declination. If this form is used in areas where declination is East, proper corrections must be made for algebraic signs.

Figure 19.—Observations for temperature coefficient of magnetic moment.

86. Reset the horizontal circle so that the index line of the suspended magnet bisects the central division of the telescope scale. Observe and record the circle and scale readings, right and left, line 1, figure 19. This is the first *away* position. Note and record the time of the observation of the scale readings and place a special time mark on the magnetogram at this same moment.

TEMPERATURE COEFFICIENT OF MAGNETIC MOMENT
COMPUTATION OF DEFLECTION ANGLES

Station, Cheltenham; Pier 7
Magnetometer No. 37
Mgr. Scale Value, 1′.38 per div.

Date, Thursday, September 20, 1951
Magnet No., Colombia I
Middle div. of Scale, 50.00

Column	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Line	Scaled ordinates d_{mm}	Scaled ordinates h_{mm}	Mean circle reading	Scale middle minus means	D ordinate	Corrected circle reading (3+4+5)	Deflection u
			° ′	′	′	° ′	° ′
1	+26.3	+15.4	303 30.17	0.00	+26.3	303 56.47	Away
2	25.9	+9.7	287 48.33	0.00	+25.9	288 14.23	15 42.17
3	26.9	+11.8		+0.69	+26.9	15.92	40.48
4	27.5	+9.9		+1.45	+27.5	17.28	39.12
5	27.0	+9.8		+3.59	+27.0	18.92	37.48
6	28.8	+10.1		+4.14	+28.8	21.27	35.13
7	32.0	−3.8		−0.34	+32.0	19.99	36.41
8	32.9	−18.6		−0.76	+32.9	20.47	35.93
9	32.8	−17.5		−0.69	+32.8	20.44	35.96
10	32.9	−7.0		−1.38	+32.9	19.85	36.55
11	31.9	−4.5		−1.79	+31.9	18.44	37.96
12	31.8	0.0		−2.62	+31.8	17.51	38.89
13	30.8	+3.2		−3.04	+30.8	16.09	40.31
14	29.2	+7.9		−2.48	+29.2	15.05	41.35
15	26.4	+12.9	287 48.33	−0.28	+26.4	288 14.45	15 41.95
16	+25.7	+17.0	303 30.62	0.00	+25.7	303 56.32	Away
17					Mean away.	303 56.40	
18							
19							
20							

NOTES

(a) Columns (1) and (2): H and D ordinates scaled from magnetogram. Use same sign as in scaling ordinates for base-line-values.
(b) Column (3): Mean circle readings from figure 19.
(c) Column (4): Conversion of ordinates in last column of figure 19 to minutes of arc.
(d) Scale value of D-variometer: 1.00′ per mm.
(e) Column (5): D-ordinate from column (1) converted to minutes of arc; these ordinates have the same sign as the corresponding ordinates in column (1).
(f) Column (6): Corrected circle readings: Sum of columns 3, 4, and 5.
(g) Column (7): Mean circle reading, line 17, minus corrected circle readings from column (6).

Figure 20.—Computation of deflection angles, temperature coefficient of magnetic moment.

87. Replace the holder (with deflecting magnet) on the bar, adjust the circle so that the suspended magnet is centered on the scale as in paragraph 86. Fill the temperature bath with cracked ice and water. Allow 5 to 10 minutes for the magnet to reach the temperature of the bath. Re-center the suspended magnet by adjusting the horizontal circle, then observe the scale readings, temperature by both thermometers, time, and circle reading, and again place a special time mark on the magnetogram (line 2, fig. 19).

88. Raise the temperature 5° to 10° by addition of warm water to the bath. Stir continuously, avoiding mechanical disturbance of the magnetometer. After the temperature has been held constant for about 3 minutes, observe and record the scale readings, the temperature by both thermometers, and the time, and again make a

special mark on the magnetogram. Record these readings on line 3, figure 19. *Do not change the circle adjustment for this observation. All of the changes, Δu, in the angle u are to be read directly from the scale in the telescope.*

89. Continue deflection observations as in paragraph 88 as the temperature is increased by regular steps of 5° or 10° up to about 40° C, then reverse the process by reducing the temperature by uniform steps to 0° C. Observe and record the circle reading for this last test (line 15, fig. 19).

90. Remove the temperature bath holder, with deflector, and repeat the away observations as in paragraph 86 (line 16, fig. 19).

91. ***Computations.***—Scale and check the H and D ordinates for the times given in column 1, figure 19, using the same sign as in scaling ordinates for base-line values (see par. 434, p. 178).

92. Convert the D scalings in mm to minutes of arc (col. 5, fig. 20).

93. Calculate the corrected circle readings, that is, the circle readings corrected to central division of the telescope scale (col. 6, fig. 20).

94. Calculate the deflection angles, u, (Mean circle reading *away*, line 17, minus corrected circle readings, from column 6), and record the results in column 7.

95. Convert the H scalings to gammas, apply the H base-line values to get the values of H for each deflection observation (cols. 1 and 2, fig. 21).

COMPUTATIONS FOR TEMPERATURE COEFFICIENT OF MAGNETIC MOMENT

Date, Thursday, September 20, 1951 from $12^h\ 48^m$ to $14^h\ 14^m$ (75 M. T.)
Magnet, Colombia I Mgr. 37
Prel. H s. v. 2.55 γ/mm Prel. H blv. 18281 γ Prel. Mgr. log C=5.86000

Column	(1)	(2)	(3)	(4)	(5)	(6)
Line	H ord h_γ	$H\gamma$	log H_{cgs}	log sin u	log M	Mean corrected temp. t
1	+39	18320	9.26 293			
2	+25	306	259	9.43 240	2.83 499	+0.25
3	+30	311	271	165	436	6.30
4	+25	306	259	103	362	12.60
5	+25	306	259	9.43 029	288	21.25
6	+26	307	262	9.42 923	185	28.15
7	−10	271	176	981	157	34.90
8	−47	234	088	959	047	42.25
9	−44	237	095	960	055	42.15
10	−18	263	157	9.42 987	144	34.75
11	−11	270	174	9.43 051	225	28.10
12	0	281	200	093	293	20.95
13	+8	289	219	157	376	12.20
14	+20	301	247	204	451	+5.90
15	+33	314	278	9.43 231	2.83 509	0.00
16	+43	18324	9.26 302			

COMPUTATION OF LOG $(1+q)$

(Data scaled from graph of t vs log M; See fig. 22)

$t_1=0°.0$ C log M_1=2.83 504
$t_2=40°.0$ log M_2=2.83 092

$t_2-t_1=40°$ diff. .00 412

$$\log(1+q)=\frac{\Delta \log M}{\Delta t}=.000103$$

For $\Delta t=40°$, $q=.000237$
$\Delta M=6.5$ cgs.

Formulas:

$$\log M=\log H+\log \sin u+\operatorname{colog} C$$

$$\log(1+q)=\frac{\log M_1-\log M_2}{t_2-t_1}=\frac{\Delta \log M}{\Delta t}$$

$$q=2.30 \log(1+q)$$
$$\Delta M=2.30\, M_0 \Delta \log M$$

Figure 21.—Computations of temperature coefficient of magnetic moment.

96. Tabulate log H and log sin u, figure 21, and calculate log M from equation (57).

97. Plot log M against observed temperatures, figure 22, and draw a straight line through the points in such a way that it will represent the approximate mean rate of change of M with temperature over the observed range. Scale the values of t and log M from this graph for high and low values of t and calculate log $(1+q)$ and q from equation (60).

98. If the calculated points, figure 22, show a wide scattering or if the magnetic moment does not return to its original value at 0° C after the complete 40° temperature cycle, this indicates lack of care in the observing program, errors of observation (or computation), or that the magnet has not been properly heat-treated or stabilized.

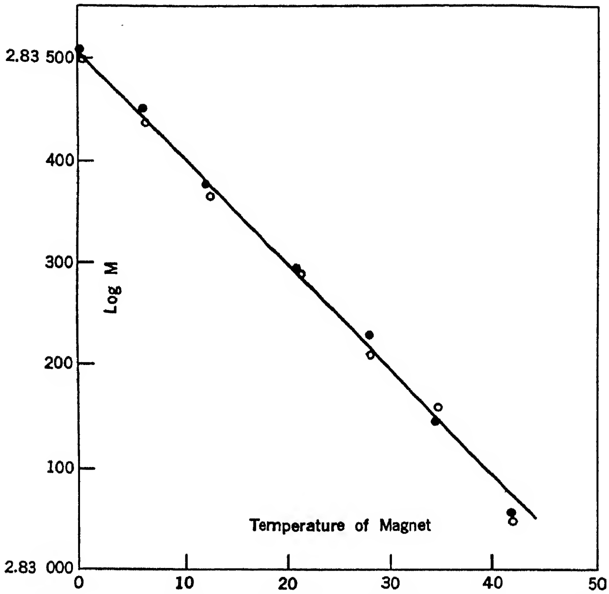

Figure 22.—Variation of magnetic moment (log M) with temperature (Cent.)

99. Note that the method described above provides for corrections for changes in declination and horizontal intensity. For example, figure 20 shows a change of 3.2 minutes in declination between lines 6 and 7. This is more than half of the change in u due to the change in magnetic moment for the full range of 40° in temperature. There was also a change of 36 γ in H during this same interval. Without taking into consideration these changes in H and D, the experimental results would have little significance, since in this case, the changes in scale readings due to changes in H and D were greater than those due to changes in magnetic moment.

100. In this method it has been assumed that the deflection distance, r, does not change during the operation, and log C is taken as a constant. Therefore the change in log H+log sin u is identical to the change in log H+log sin u+colog C so that C may be given any value and the same results will be obtained for log $(1+q)$ and q. However, it is interesting to know how the value of the magnetic

moment changes with temperature. The change in the magnetic moment, M, may be computed for the full range of t as follows: $\Delta M=2.3M\times\Delta(\log M)$. For the example given, figure 22, $\Delta M=2.3\times 684\times0.00416=6.5$ cgs. In this case the magnetic moment was reduced by 6.5 parts in 684 for a change of temperature of 40° C, or 1 part in 4200 per °C (=0.024% per °C).

101. ***Proposed alternate method for determination of*** q.—In this method the magnet, M_a, to be tested is placed in a temperature bath, north end of M_a to the south, at such a distance north or south of a properly oriented H variometer that temperature compensation of the variometer is approximately effected. The temperature bath is mounted on a separate, rigid, nonmagnetic stand, independent of the H variometer and at the proper elevation. Reduce the torsion in the H fiber until the H spot returns to its original ordinate. Insulate the variometer against temperature changes by a nonmagnetic, nonconducting shield or cover. Take M_a through several temperature cycles, say 0° to 50° to 0°, and allow the H spot to record continuously.

102. For each temperature step of 5° or 10°, observe H independently, or operate an independent magnetograph so that point scalings of H can be made.

103. Let M_a be the magnet under test; and let

M_1=the magnetic moment of M_a at a standard temperature of 0° C;

M_2=the magnetic moment of M_a at another temperature, say 50° C;

ΔM=change in moment due to a change of temperature $=M_2-M_1$

t_1=the standard temperature, 0° C;

t_2=the other temperature, 50° C;

Δt=the change in temperature$=t_2-t_1$

Δf=change in field at recording magnet due to ΔM;

r=deflection distance in cm;

q=average temperature coefficient of magnetic moment of M_a between 0° C and 50° C (that is, between t_1 and t_2).

Then, from equation (11), page 5,

$$f=\frac{2M}{r^3}$$

By differentiating,

$$df=\frac{2}{r^3}\,dM$$

or

$$\Delta f=\frac{2}{r^3}\,\Delta M$$

and

$$\Delta M=\frac{1}{2}\,r^3\Delta f. \tag{62}$$

But, from equation (53),

$$M_1-M_2=M_1q\ (t_2-t_1)$$

or

$$\Delta M=M_1q\ (-\Delta t). \tag{63}$$

Combining (62) and (63),

$$\frac{1}{2}\ r^3\Delta f=-M_1q\Delta t$$

and

$$q=-\frac{r^3\Delta f}{2M_1\Delta t}. \tag{64}$$

104. Example: Suppose:

$$M_1=100 \text{ cgs};\ r=12 \text{ cm};\ q=0.00024;$$

then $\Delta M=100\times0.00024\times(-50)$

$=-1.2$ cgs ($=$change in M due to increase of 50° in temperature);

$$\Delta f=\frac{2}{r^3}\ \Delta M=\frac{2}{12^3}\ (-1.2)$$

$=-140\gamma$ (change in F due to decrease in M).

If the H scale value of the H variometer used in the tests is 3γ per mm, then the deflection of the H spot on the magnetogram for $-\Delta t=50°$ C will be $-\frac{140}{3}=-47$ mm, a deflection of such magnitude that it may be scaled directly with sufficient accuracy.

TEMPERATURE COEFFICIENT OF MAGNETIC MOMENT

(H Variometer Method)[a]

Magnet No. 20; Solid cylinder, 5 x 25 mm; Alnico II; $M_a=100$ cgs
Scale value of standard H variometer 3.00 γ/mm; (Variometer No. 5)
Scale value of variometer used in test 3.00 γ/mm; (Variometer No. 7)

Deflection distance, r 12.6 cm

Time 75MT	Temperature			H Scalings, No. 7		H Scalings, No. 5		ΔH_7 γ	ΔH_5 γ	$\Delta H_7-\Delta H_5$ γ
	A	B	Mean	mm	γ	mm	γ			
14:50	0. 0	0. 0	0. 0	15	45	20	60	----	----	------
15:00	50. 0	50. 0	50. 0	62	186	25	75	141	15	126

$$\Delta f=\Delta H_7-\Delta H_5=126\gamma=0.00126 \text{ cgs}$$

$$q=-\frac{r^3\Delta f}{2M\Delta t}=-\frac{2000\times0.00126}{2\times100\times(-50)}=0.00025$$

[a] Observations hypothetical.

Figure 23.—Suggested form for observations of temperature coefficient of magnetic moment. H variometer method.

5. Provided the H variometer recording magnet is properly ted, it will not be necessary to take into consideration changes eclination during the tests if magnetic conditions are quiet. ever, corrections should be made for natural changes in H during ests even though these changes are small. Figure 23 gives a thetical determination of q by this method.

i. The method is especially useful for determination of q for small magnets since small deflection distances may be used, ded the dimension ratios of M_a and M_s are such that the distri- n coefficients are small.

. ***Sensitivity of temperature-coefficient observations; mag- neter method.***—Magnitude of Δu: In determining the tem- ire coefficient of the magnetic moment, it is desirable to know rance about what value of Δu may be expected from a particular et, deflection distance, temperature range, and *estimated* tem- ire coefficient. In equation (55) we have,

$$\frac{H}{M}=\frac{C}{\sin u}$$

$$\log \sin u=\log C+\log M-\log H \tag{65}$$

entiate,

$$\frac{1}{\sin u}\cos u\ du=\frac{1}{M}\,dM-\frac{1}{H}\,dH \tag{66}$$

$$du=\frac{\sin u}{\cos u}\left(\frac{dM}{M}-\frac{dH}{H}\right) \tag{67}$$

$$\Delta u=\tan u\left(\frac{\Delta M}{M}-\frac{\Delta H}{H}\right) \tag{68}$$

ince

$$\Delta M=-Mq\Delta t$$

$$\frac{\Delta M}{M}=-q\Delta t$$

uting in (68),

$$\Delta u\ (\text{radians})=-\tan u\left(q\Delta t+\frac{\Delta H}{H}\right) \tag{69}$$

$$\Delta u\ (\text{minutes})=-3438\tan u\left(q\Delta t+\frac{\Delta H}{H}\right). \tag{70}$$

If H remains constant during the test, equation (70) becomes

$$(\Delta u)'=-3438\ \tan u\ (q\Delta t) \tag{71}$$

e may expect, for example, the following results, based on n (71):

$u=15°$; $q=0.00025$; and $\Delta t=40°$, then $\Delta u=9'.7$;

$u=30°$; $q=0.00025$; and $\Delta t=40°$, then $\Delta u=20'.9$.

109. Errors of scale reductions: In this method it is assumed that the value of u as represented by the converted scale readings is identical to the angle through which the circle must be turned to bring the index of the suspended magnet to the central division of the scale. In other words, it is assumed that the suspended magnet remains fixed relative to the ground while the circle and deflector are rotated through the small angle, Δu. (Equation (55) is based on the assumption that the deflector is always at right angles to the suspended magnet.) It can be shown that the error introduced into the calculated value of Δu by following this procedure is never greater than 3 seconds of arc for values of u up to 40° provided the scale deflection does not exceed 20′. In this extreme case, the error in Δu would not be greater than 3″ in 20′ (3 parts in 1200), which is entirely negligible in this work. Likewise the effect of torsion in the fiber is negligible for an angle of 20′. For example: If 90° torsion produced a deflection of 5′ (an exceptionally large magnetometer fiber), 20′ torsion would produce a deflection of about 0.02′.

INDUCTION FACTOR

110. ***Induction and magnetic moment.***—When a permanent magnet is placed in a weak magnetic field its magnetic moment, M, is temporarily changed by induction, the amount of the change being proportional to the magnitude of the component of the applied field parallel to the magnetic axis of the magnet. The magnetic moment will be increased if the applied magnetizing field is in the same direction as the initial magnetic moment of the magnet, and it will be reduced if the magnetizing field is reversed. For a well stabilized magnet and for sufficiently small magnetizing fields, the magnitude of the change is repeatable and is the same for the field direct and reversed. The *change in the magnetic moment*, ΔM, caused by a unit magnetizing field is called the *induction factor*, μ. For any other field, X, $\Delta M=\mu X$. The ratio of the change in M to the original moment, for a unit magnetizing field is called the *induction coefficient*, h. That is $\frac{\Delta M}{MX}=h$. Combining these expressions we have $\mu=hM$.

111. The induction *factor*, μ, is approximately constant for a given magnet regardless of its magnetic moment. It depends primarily upon the volume or mass of the magnet, its dimension ratio, and its "magnetic hardness," the latter in turn being rather sensitive to temperature changes.[2] The induction *coefficient*, h, changes if M is altered, being almost inversely proportional to M for the ranges of magnetization used in geomagnetic work.

112. In this work we are concerned only with the inductive effect of the earth's field on the magnetic moments of permanent magnets which may be used in geomagnetic measurements. For this reason we shall be dealing with comparatively weak fields.

113. If a magnet, M_a, figure 24, whose induction factor is to be determined, is mounted near a suspended magnet, M_s, so that the magnetic axis of M_a is vertical and remains in a vertical plane which is perpendicular to M_s, the latter will be deflected through an angle u, since there will be a component, f_y, of the field of M_a, normal to M_s.

[2] David G. Knapp, Reversible susceptibility and the induction factor used in geomagnetism, U. S. Coast and Geodetic Survey Special Publication No. 301 (in press, 1953).

The relation between the magnetic moment of the deflector and the angle u, is given by,

$$\frac{H}{M}=\frac{C}{\sin u} \tag{72}$$

in which H is the horizontal intensity and C is a constant depending upon the relative positions of the magnets and the distribution coefficients for that position. If a uniform vertical field, X, is now

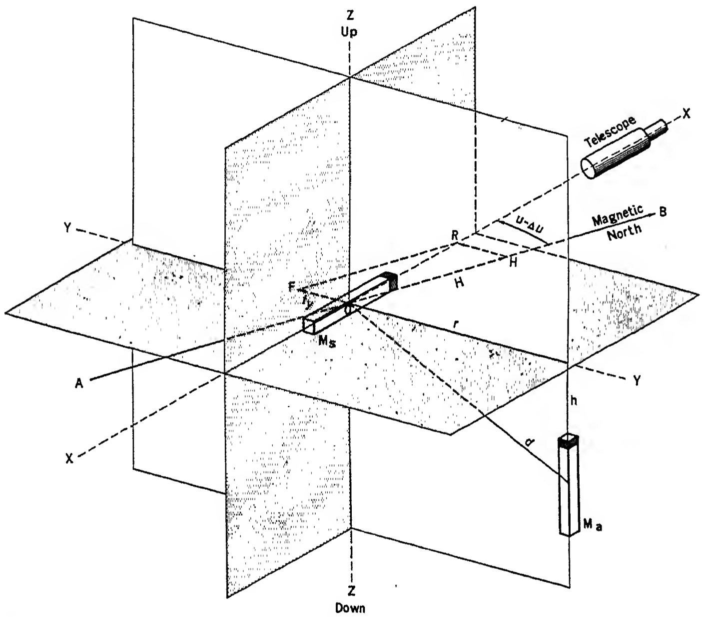

Figure 24.—Relative positions of magnets for determination of induction coefficient.

established around the deflector in the same direction as the magnetic moment of the deflector, the magnetic moment of the latter will be temporarily increased by an amount, $\Delta M = hMX = \mu X$, and the suspended magnet will be turned through an additional small angle, Δu, due to the increased magnetic moment (and field) of M_a. This uniform vertical field, X, may be either the vertical component of the earth's field or an artificially created field.

114. When the north end of the deflector is down and the magnetizing field is directed vertically downward, then,

$$\frac{H}{M+\Delta M}=\frac{C}{\sin\,(u+\Delta u)}$$

$$\frac{H}{M+hMX}=\frac{C}{\sin\,(u+\Delta u)}$$

$$\frac{H}{M\,(1+hX)}=\frac{C}{\sin\,(u+\Delta u)} \tag{73}$$

Upon reversal of the magnetizing field, X,

$$\frac{H}{M\,(1-hX)}=\frac{C}{\sin\,(u-\Delta u)} \tag{74}$$

Dividing (74) by (73), gives,

$$\frac{1+hX}{1-hX}=\frac{\sin\,(u+\Delta u)}{\sin\,(u-\Delta u)}=\frac{\tan\,u+\tan\,\Delta u}{\tan\,u-\tan\,\Delta u} \tag{75}$$

Expanding and collecting,

$$h=\frac{1}{X}\times\frac{\tan\,\Delta u}{\tan\,u}. \tag{76}$$

Equation (76) will also apply to the case where the N end of the deflector is *up* and the field is directed up, then down. Equation (76) gives the induction coefficient in terms of the measured quantities, X, u, and Δu. The magnetic moment, M, of the deflector is determined by deflecting a suspended magnetometer magnet, and the induction factor, μ, is calculated from the relation, $\mu=hM$. Two methods for the determination of μ will be described.

115. ***Lamont's method;*** [3] ***induction coefficient.***—In this method the magnet whose induction coefficient is to be determined is mounted on a special holder attached to the deflection bar of a magnetometer. The axis of the deflector is vertical and is maintained in a vertical plane at right angles to the axis of the suspended magnet and at fixed distances above and below the horizontal plane through the latter. (See fig. 25.) The deflector, M_a, may be attached to the holder with the north end of M_a up or down. (The reversal is made about a horizontal axis parallel to the magnetometer deflection bar.) Its magnetic moment is increased slightly by the inductive action of the vertical component of the earth's field, when the north end is down, and decreased by practically the same amount when the north end is up. Thus the applied magnetizing field is the vertical component of the earth's field, $+Z$ or $-Z$, relative to the direction of the magnetic moment of the deflector, and u is the angle through which M_s would be deflected if Z could be reduced to zero. Substituting Z for X in equation (76) gives

$$h=\frac{1}{Z}\,\frac{\tan\,\Delta u}{\tan\,u}. \tag{77}$$

One advantage of this method is that a fairly constant vertical field may be used throughout the test. Since Δu is a very small angle the observations must be made with extreme care in order to obtain reliable results.

116. It has been shown by Hartnell [4] and confirmed experimentally,[5] figure 26, that for a constant horizontal distance, the deflection angle, u, will be a maximum when the vertical distance between deflector

[3] J. Lamont, Handbuch des Erdmagnetismus (see item 12 of bibliography).
[4] Distribution Coefficients of Magnets, pp. 9–10 (see item 3 of bibliography).
[5] Terr. Mag., 34, 243, 1929.

and suspended magnet is just one-half the horizontal distance between them. Since the controlling error in this work lies in the determination of the small angle, Δu, the deflector should be set for a position

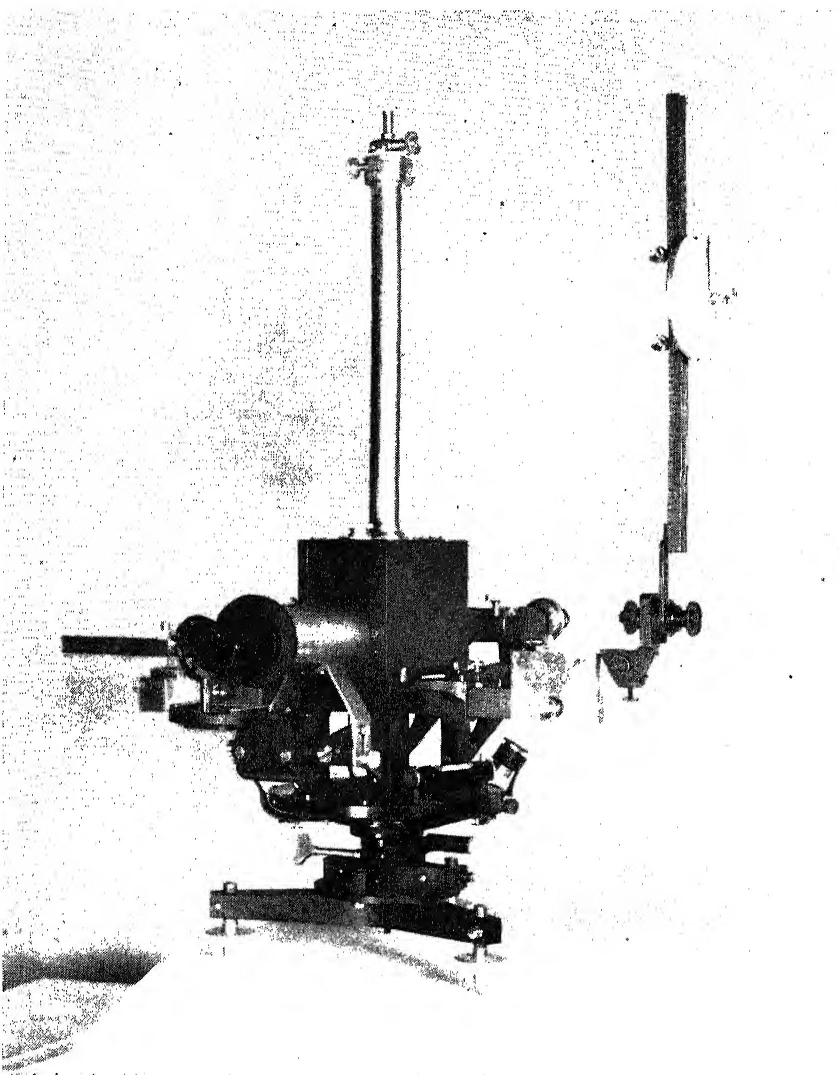

Figure 25.—D. T. M. C. I. W.[6] Induction-coefficient apparatus (Lamont's method), with modified magnet holder.

which will give a reasonably large angle for u, whereupon Δu will be correspondingly large. Figure 27 gives a typical set of induction observations made by Lamont's method.

117. *Directions for determination of induction coefficients by Lamont's method.*—Set up a magnetometer, level the instrument, and suspend

[6] Department of Terrestrial Magnetism, Carnegie Institution of Washington.

the short magnet. Adjust in azimuth until the index of the magnet is on the central division of the telescope scale. Attach the horizontal deflection bar and mount the induction apparatus on the bar, on the east side of the instrument at a horizontal deflection distance, r, of 15 to 25 cm. (Call the suspended magnet M_s and the deflector M_a.) The induction coefficient of M_a is to be determined.

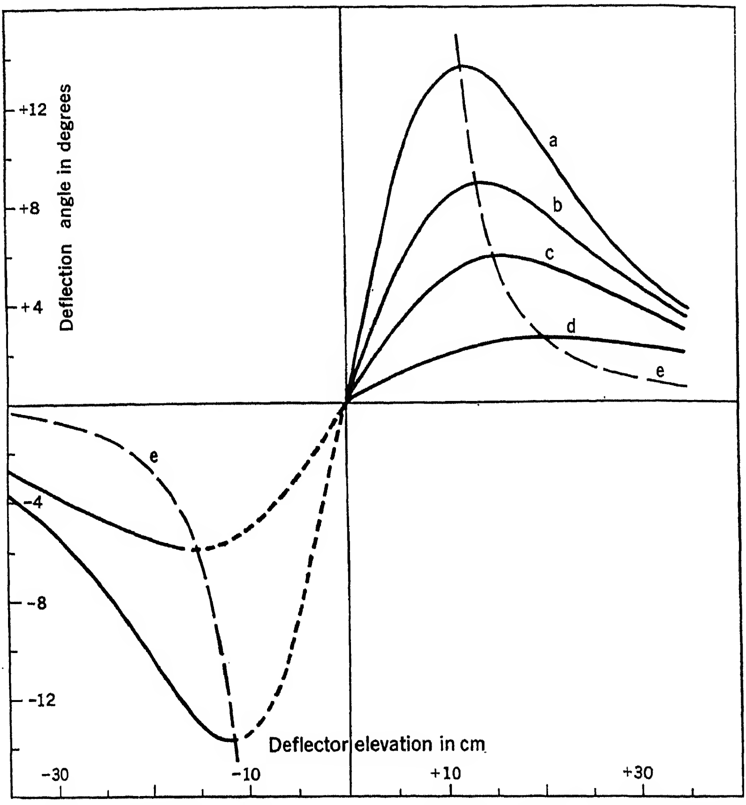

Figure 26.—Variation of deflection angle with position of deflector. Curves show changes of deflection angles for various horizontal deflection distances: a=22.5 cm, b=26.25 cm, c=30 cm, d=40 cm. Curve e is the locus of the maxima of the other curves.

118. Mount M_a, N end down, on the apparatus so that its center will be at an elevation of approximately $\frac{1}{2}r$ above the horizontal plane through M_s. In this position the f_y component of the field of M_a at M_s will deflect M_s counter-clockwise looking down.

119. Readjust the horizontal circle until the index on M_s is in the center of the telescope scale when M_s is at rest. Read the horizontal circle. This is the reading on line 1, figure 27. M_s has been deflected through the angle NOF, figure 28.

120. Reverse M_a so that its N end is up and above the bar. M_s will now be deflected clockwise, looking down. Adjust the instrument in azimuth again until the index of M_s is on the central division of the telescope scale when M_s is at rest; the deflection angle is now NOC, figure 28. Record the circle reading on line 2, figure 27.

121. Continue observations for the remaining positions on both

east and west sides as indicated in figure 27 and record all observations in their correct order on the form. Figure 24 shows the angles and positions for line 4 of figure 27.

Induction Coefficient

(Lamont's method)

Place, Cheltenham Magnetic Observatory, Pier 7. Date, March 22, 1938.
Magnetometer, 37 Observer, J. H. Nelson
Magnet, 37L (Tungsten Steel); Magnetic Moment, 874 Apparatus, Lamont's (modified).
Temperature of Magnet, 27°.8 C; Mass of Magnet, 27 gm. Length of Magnet, 9.28 cm.

OBSERVATIONS

Line	Position of deflector			Horizontal circle			
	Side	Holder	N end of M_a	A	B		Mean
				° ′ ″	′ ″		° ′ ″
1	East	Up	Down	95 58 40	59 00	(1)	95 58 50
2	East	Up	Up	117 05 30	05 40	(2)	117 05 35
3	East	Down	Down	117 12 50	13 10	(3)	117 13 00
4	East	Down	Up	96 02 30	02 40	(4)	96 02 35
5	West	Down	Up	117 11 30	11 50	(5)	117 11 40
6	West	Down	Down	95 59 10	59 20	(6)	95 59 15
7	West	Up	Up	96 06 10	06 30	(7)	96 06 20
8	West	Up	Down	117 15 20	15 40	(8)	117 15 30

COMPUTATIONS

Line	Position of M_a	Operation	Angles
			° ′ ″
9	East	(3)−(1)	21 14 10
10	West	(8)−(6)	21 16 15
11		Mean: $2u+2\Delta u=$	21 15 12
12	East	(2)−(4)	21 03 00
13	West	(5)−(7)	21 05 20
14		Mean: $2u-2\Delta u=$	21 04 10
15		$4u=(11)+(14)$	42 19 22
16		u	10 34 50
17		$4\Delta u=(11)-(14)$	11 02
18		Δu	02 46

Time, 75 Meridian.
Began, $14^h\ 30^m$.
End, 14 44.
Vertical Intensity (Z), 0.54000.

colog Z	0.2676
log sin Δu	6.9057
log cot u	0.7286
log h	7.9019
log M	2.9415
log μ	0.8434
μ	6.97
h	0.00798

Equations: $h=\frac{1}{Z}\,\frac{\tan\Delta u}{\tan u}$; $\mu=hM$

Figure 27.—Observations for Induction Coefficient (Lamont's method).

122. *Computation of h and μ.* (Refer to fig. 27.)

Line 9.—Reading 3 minus reading 1, gives one value of $2u+2\Delta u$.

Line 10.—Reading 8 minus reading 6, gives another value of $2u+2\Delta u$.

Line 11.—Take the mean. It is the largest angle, AOF, shown in figure 28 and represents the double deflection angle for the condition that M_a is used with the N end down, and its magnetic moment increased by the inductive action of the vertical component, Z, of the earth's field.

Line 14.—In like manner the angle $2u-2\Delta u$, COD, is the smaller of the large angles, in figure 28, and represents the double deflection angle for the condition that M_a is used with its N end up, and its magnetic moment decreased by induction.

Lines 11 and 14.—Solve these equations simultaneously to give values for both u and Δu.

Determine the magnetic moment of M_a by the method described in paragraph 319, page 129, or other suitable method.

Scale the vertical intensity, Z, from the magnetogram, or use the mean value for the month.

Compute h from equation (77) and μ from the relation, $\mu = h\,M$.

123. Repeat the whole operation using a slightly different value for horizontal distance or vertical distance or both.

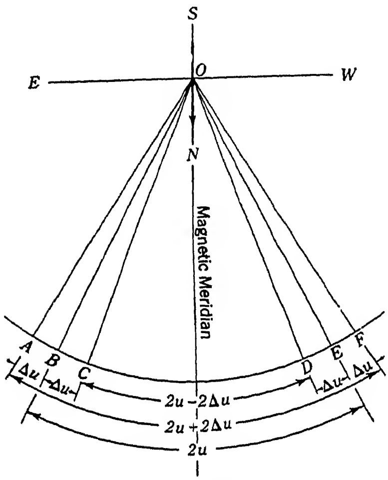

Figure 28.—Deflection angles in the measurement of the induction coefficient of a permanent magnet.

124. ***Induction coefficient, Nelson's method.***[7]—In this method the applied field is produced by a solenoid energized by a few dry cells. Figure 29 shows an observatory magnetometer upon which are mounted a special deflection bar and two solenoids. These solenoids are identical and are suspended from the deflection bar at equal distances from the suspended magnet. The coils are connected in series so that the current (and the interior field produced) in each is the same. The terminals are so connected that the fields in both coils will be directed upward or downward at the same time. By adjustment of the solenoids along the bar the components of the solenoid fields at the center of the suspended magnet of the magnetometer will be practically equal and opposite as indicated by no deflection of the suspended magnet, M_s, when the coils are energized. A precision milliammeter in the circuit indicates the current.

125. The axial magnetic field at a point on the axis of a solenoid of finite length [8] is,

$$X_s = 0.1\,n\,i\,(4\pi - \omega_1 - \omega_2) \tag{78}$$

in which ω_1 and ω_2 are the solid angles described by drawing elements from the chosen point to the last turn of the wire at each end of the solenoid.[9]

By referring to figure 30, ω_1 and ω_2 may be evaluated in terms of the coil dimensions and the position on the axis of point P,

$$X_s = 0.2\pi\,n\,i\left[\frac{\frac{l}{2}+x}{\sqrt{b^2+\left(\frac{l}{2}+x\right)^2}} + \frac{\frac{l}{2}-x}{\sqrt{b^2+\left(\frac{l}{2}-x\right)^2}}\right] \tag{79}$$

or,

$$X_s = i\,C_s$$

[7] J. H. Nelson, Terr. Mag., 43, 159, 1938.
[8] Nelson, *op. cit.*, p. 161.
[9] It may be noted that the formula, $0.4\,\pi\,n\,i$, for the axial field in an infinite solenoid may be obtained from equation (78) by making ω_1 and ω_2 equal to zero, or from equation (79) by making l infinite.

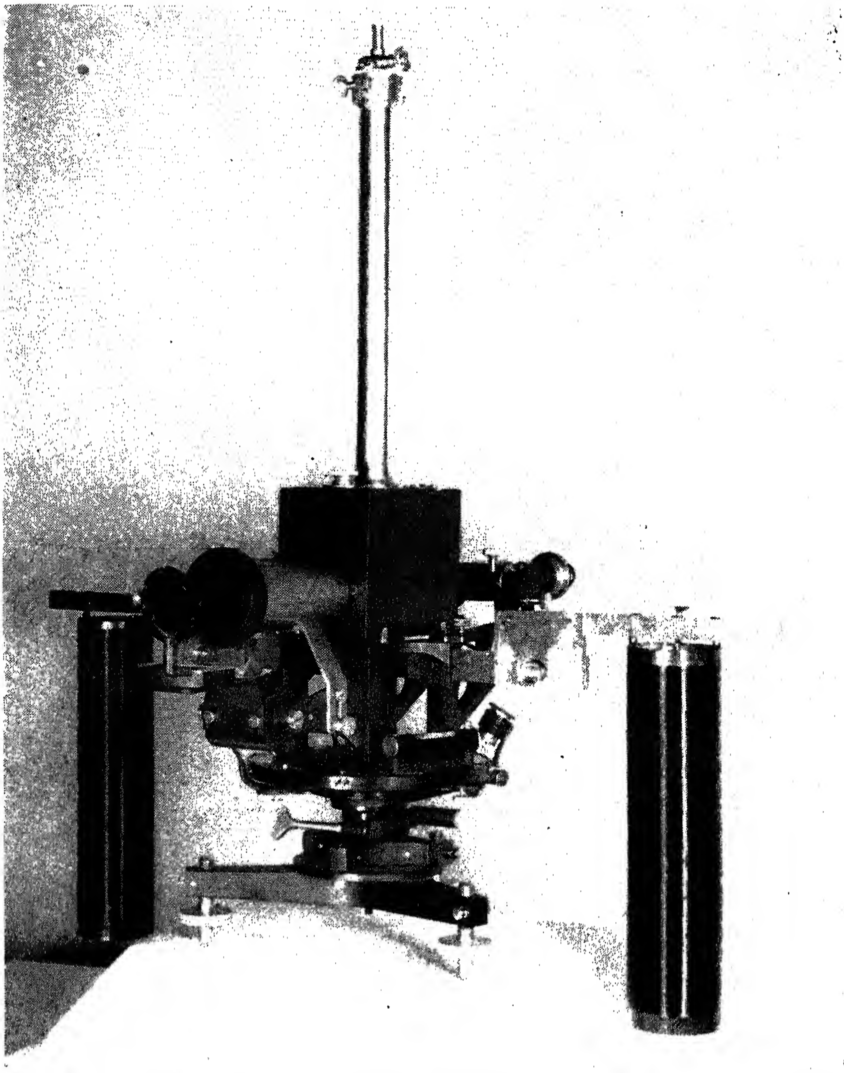

Figure 29.—Observatory magnetometer equipped with apparatus for measuring induction coefficient (Nelson's method).

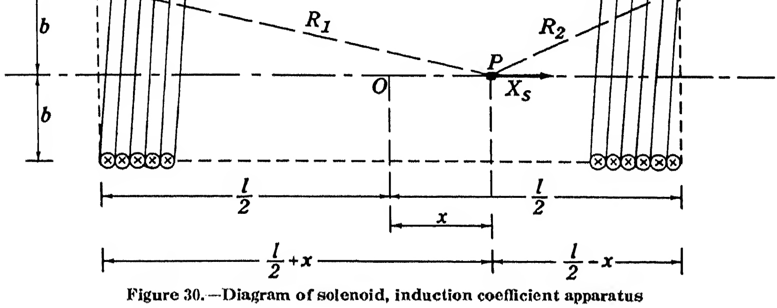

Figure 30.—Diagram of solenoid, induction coefficient apparatus

in which

X_s = axial magnetic field on axis at point P, distance x cm from coil center;
n = turns of wire per cm length of solenoid;
i = current in amperes;
l = length of solenoid in cm;
b = coil radius in cm;
x = distance from coil center O to point P on axis; x is plus in the direction of the field X_s;

$C_s = \frac{X_s}{i}$ = field per ampere at P (the coil factor).

$$C_s=0.2\pi n\left[\frac{\frac{l}{2}+x}{\sqrt{b^2+\left(\frac{l}{2}+x\right)^2}}+\frac{\frac{l}{2}-x}{\sqrt{b^2+\left(\frac{l}{2}-x\right)^2}}\right] \tag{80}$$

126. The average value of the field, $\overline{X_s}$, over the length of the magnet is related to the average of the coil factor, $\overline{C_s}$, over the same distance by

$$\overline{X_s}=i\,\overline{C_s}.$$

When working with one particular solenoid it is convenient and practical to plot the coil factor, C_s, as a function of x, as illustrated in figure 31. The solenoid is the one described in paragraph 128.

127. The average value of the coil factor, $\overline{C_s}$, over the length of the magnet to be tested, may be scaled approximately from the curve of figure 31, using the method of equal areas commonly employed in scaling averages.[10]

128. The coils used originally are still in use and have the following specifications:

Total number of turns in each coil	825
Over-all length of each coil ($=l$)	23.15 cm
Turns of wire per cm of length of coil ($=n$)	35.64
Radius of each coil ($=b$)	3.135 cm.

Figure 31 shows values of the coil factor C_s for this coil plotted as a function of x. Sufficiently accurate values of C_s or $\overline{C_s}$ may be scaled from the curve.

Example: Suppose the current is 35.0 milliamperes, the length of the magnet to be tested (deflector) is 9.28 cm, and the deflector is centered in the coil. Scale the average of the C_s curve from $x=-4.64$ to $x=+4.64$. The scaled average is 43.0 oersteds per ampere. The average field $\overline{X_s}$ over the length of the magnet is then,

$$\overline{X_s}=i\,\overline{C_s}$$

$$=0.035\times 43.0=1.50 \text{ oersteds.}$$

[10] A solenoid will produce approximately the same field as a magnet at all points at great distances, r, in any direction, θ, if (a) the distance r is very large compared to the dimensions of the solenoid and magnet, (b) the solenoid and the magnet are centered at the same point, and (c) the solenoid axis coincides with the magnetic axis of the magnet. The equivalent magnetic moment of the solenoid under such conditions is,

$$M \approx 0.1\,\pi n i l b^2$$

in the notation of figure 30.

129. *Directions for determination of induction coefficient; Nelson's method.*—Set up the apparatus as shown in figure 29, using a deflection distance of 25 to 30 cm for each coil. See that the coils hang vertically and are free from pendular motion. Suspend the short magnet, M_s in the magnetometer and bring it to rest in line with the central division of the telescope scale.

130. Connect the east coil in series with two dry cells, a milliammeter, rheostat, and a reversing switch, figure 32. Keep the lead-in wires and the ammeter well away from the magnetometer.

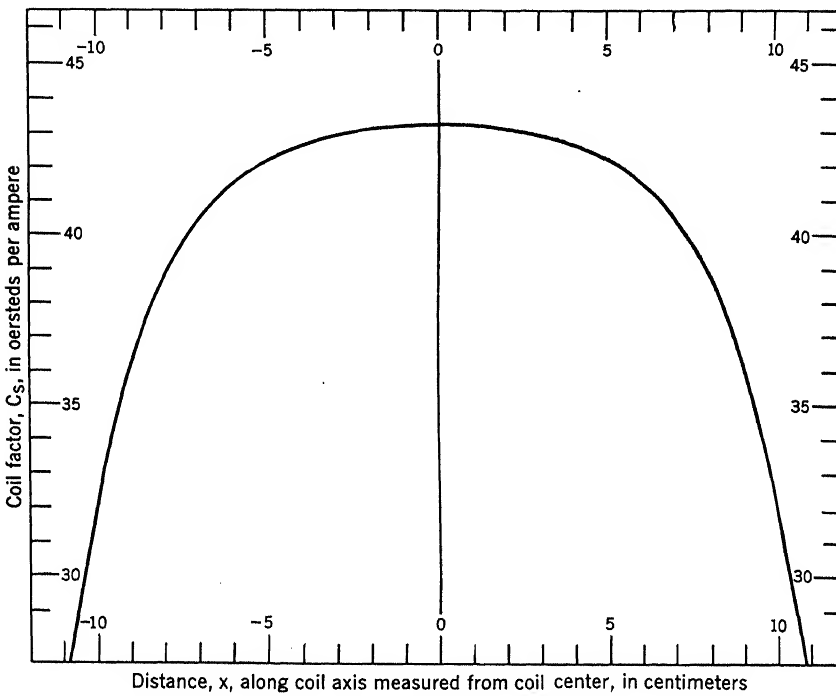

Figure 31.—Relation between coil factor and distance along axis from center of coil.

131. Close the reversing switch and note the direction of the deflection of M_s. If the scale reading *increases*, the field within the coil is directed *upward*. Mark this position of the switch *up*. If the scale reading *decreases* mark this position *down*. Repeat this operation with the west coil only in the circuit and see that it is so connected that its field will be directed as indicated by the switch. Note that when the field is directed downward in the east coil the deflection will be in the direction of decreasing scale readings but for the west coil the scale readings will increase.

132. Now connect the coils in series, leaving all other connections unchanged. Close the switch to *up*. If the suspended magnet is deflected, adjust one of the coils along the bar until the deflection is reduced to zero for both up and down positions of the switch. Adjust the horizontal circle so that the suspended magnet is in line with the center of the telescope scale. Observe and record several deflections for both up and down positions of the switch. Even though these

deflections are as small as 0.1 division they should be recorded together with ammeter and horizontal circle readings; lines 1, 2, 3, and 7 (deflector away), figure 33.

133. Without disturbing the coils or leads, mount the deflector, M_a, whose induction coefficient is to be determined, centrally within the east coil, *north end down*. With no current in the coils readjust the circle until M_s is again in line with the central division of the telescope.

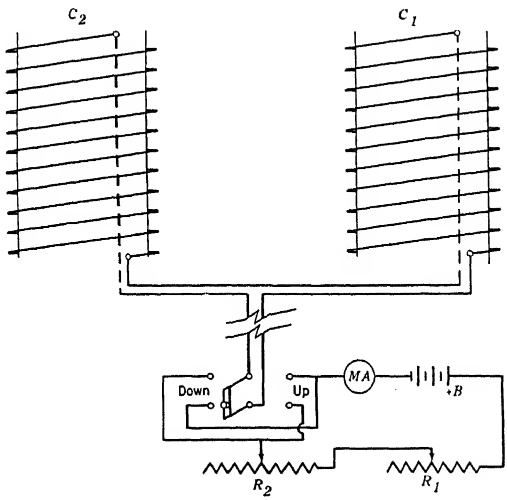

Figure 32.—Wiring diagram for induction coefficient apparatus (Nelson's method), showing solenoids C_1 and C_2; milliammeter, MA; rheostats, R_1 and R_2, for fine and coarse adjustment; reversing switch and battery.

Observe and record the circle reading; line 7, deflector in coil, and the scale readings in the telescope, line 9.

134. Energize the coils, field directed upward, using sufficient current to produce a deflection of several scale divisions. Bring the magnet to rest, observe and record the scale readings and the ammeter reading. Repeat for current reversed. Repeat these operations several times; lines 10–14.

135. Repeat the operation described in paragraph 133, and record scale readings and circle readings; line 15 and line 8 (deflector in coil).

136. Remove the deflector and repeat the operation described in paragraph 132, no current in the coil and deflector away; line 8 (deflector away) and lines 4, 5, and 6.

137. Measure the magnetic moment of the deflector by the method described in paragraph 319, page 129.

138. Calculate h and μ, using the form of figure 34. Record all pertinent data such as kind and size of deflector, magnetic moment of deflector, horizontal and vertical distances of deflector relative to M_s.

139. Repeat the whole operation, changing the current slightly for the second set.

140. *Additional notes on induction coefficients.*—Observations are taken with deflector *east*, *below* the bar, *north end down*.

141. The deflection u, caused by the magnet alone, will be in the direction of increasing circle readings, that is, clockwise, and will be taken as positive (+).

INDUCTION COEFFICIENT
(NELSON'S METHOD)

Station, *Cheltenham, Pier 7* Date, *Tuesday, August 22, 1950*
Magnetometer No. *37* Observer, *R. L. Viets*
Induction apparatus No. *1* East coil No. *1* Magnet No. *RIC-14*

DEFLECTIONS FOR BALANCE CORRECTION
Magnet away

		Direction of field	Current in coil	Scale readings Left	Right	Mean	Correction to $2\Delta u$ (U)−(D)
			amperes	div.	div.	div.	div.
(1)	Before	Up	*0.250*	*50.1*	*50.4*	*50.25*	
							0.25
(2)		Down	*0.250*	*49.8*	*50.2*	*50.00*	
							0.35
(3)		Up	*0.250*	*50.2*	*50.5*	*50.35*	
(4)	After	Up	*0.251*	*49.7*	*50.6*	*50.15*	
							0.30
(5)		Down	*0.250*	*49.6*	*50.1*	*49.85*	
							0.40
(6)		Up	*0.250*	*49.7*	*50.8*	*50.25*	

Mean current *0.250* Correction for balance +*0.32*

Description of Magnet

Material: *Alnico II*, *Chill cast*, *Density 7.1*

Dimensions: *0.62 cm diam.*, *4.9 cm long*, *Mass: 10.5 g.*

Moment: *784* c. g. s.

Solid ✓ Cylindrical ✓
Hollow Octagonal
Other

DEFLECTIONS FOR ANGLE u, NO CURRENT

		75 M. time	Magnet away A	B	Mean	Magnet in coil, east side, north end down A	B	Mean
		h m	° ′	° ′	° ′	° ′	° ′	° ′
(7)	Before	*14 07*	*47 42*	*227 42*	*47 42*	*58 33*	*238 34*	*58 34*
(8)	After	*14 15*	*47 41*	*227 41*	*47 41*	*58 33*	*238 34*	*58 34*

Mean u_1 *47° 42′*

Mean u_2 *58° 34′*
$u=u_2-u_1=$ *10° 52′*

DEFLECTIONS FOR $2\Delta u$

Magnet in coil, east side, north end down

	75 M. time	Direction of field	Current in coil	Scale readings Left	Right	Mean	$2\Delta u$ uncorrected (U)−(D)
	h m		amperes	div	div.	div.	div.
(9)	*14 10*	None	None	*49.1*	*51.0*	*50.05*	
(10)		Up	*0.250*	*52.3*	*54.0*	*53.15*	
							6.25
(11)		Down	*0.250*	*46.5*	*47.3*	*46.90*	
							6.25
(12)		Up	*0.249*	*52.3*	*54.0*	*53.15*	
							6.30
(13)		Down	*0.249*	*46.3*	*47.4*	*46.85*	
							6.30
(14)		Up	*0.248*	*52.3*	*54.0*	*53.15*	
(15)	*14 13*	None	None	*48.8*	*51.3*	*50.05*	

Mean current *0.249* Mean (U)−(D) ($2\Delta u$ uncorrected) *6.28*

Approx. horizontal deflection distance of magnet	Approx. vertical deflection distance of magnet
mm	mm
250	*170*

Remarks: *Temp. 27° C.*

Computed by *RLV* Checked by *CB* Abstracted by *WW*

Figure 33.—Observations for induction coefficient.

COMPUTATIONS FOR INDUCTION COEFFICIENT
(NELSON'S METHOD)

Station, *Cheltenham, Pier 7* Date, *August 22, 1950* Observer, *R. L. Viets*
Mgr. s. v., *1.38′* per div. Induction apparatus No. *1* East coil No. 1

(1)	Magnet No	*RIC-14*			
(2)	Magnet length (L)	4.9	cm	cm	cm
(3)	Magnetic moment (M)	784	c. g. s.	c. g. s.	c. g. s.
(4)	Coil length (l)	*23.15*	cm	cm	cm
(5)	Coil diameter ($2b$)	*6.27*	cm	cm	cm
(6)	Total turns	*825*			
(7)	Coil turns per cm (n)	*35.64*			
(8)	Average Coil factor (C_s)	*43.1*			
(9)	Current in coil (i)	*0.249*	amp.	amp.	amp.
(10)	log i	*9.396*			
(11)	log $\overline{C}_s$	*1.634*			
(12)	log $\overline{X}_s$ (10)+(11)	*1.030*			
(13)	Average field of coil over length of magnet $\overline{X}_s$.	*10.7*	c. g. s.	c. g. s.	c. g. s.
(14)	Mean $2\Delta u$ uncorrected (U)−(D)	*6 28*	div	div	div.
(15)	Correction for balance (U)−(D)	*0.32*	div.	div.	div.
(16)	$2\Delta u$ corrected (14)−(15)	*5.96*	div.	div.	div.
(17)	$2\Delta u$	*8.′22*		′	′
(18)	Δu	*4′ 07″*		′ ″	′ ″
(19)	u	*10° 52′*		° ′	° ′
(20)	log tan Δu	*7.078*			
(21)	log cot u	*0.717*			
(22)	colog $\overline{X}_s$	*8.970*			
(23)	log h =(20)+(21)+(22)	*6.765*			
(24)	log M	*2.894*			
(25)	log hM=log μ =(23)+(24)	*9.659*			
(26)	h induction coefficient	*0.000 582*			
(27)	μ induction factor	*0.456*			

$$\overline{X}_s = i\,\overline{C}_s \qquad h = \frac{1}{\overline{X}_s}\,\frac{\tan \Delta u}{\tan u} \qquad \mu = hM$$

Computed by *RLV* Checked by *CB* Abstracted by *WW*

Figure 34.—Computations for induction coefficient.

142. Under the conditions of paragraph 141, the deflection Δu, caused by the applied field must be taken as positive (+) when that field is directed *downward*, that is, in the same direction as the magnetic moment of the deflector, since the magnetic moment of M_a will be increased by a small amount ΔM.

143. Note that *decreasing* scale readings in the telescope (scale in telescope) correspond to *increasing* circle readings. Therefore when the magnetizing field is applied, decreasing scale readings indicate an increase in the magnetic moment of the deflector or an increase in the applied magnetizing field, f_v, acting on M_s.

144. If the deflection, Δu, for field *up* or field *down*, with magnet away, is in the same direction as the corresponding deflections when the deflector is within the coil, the correction for coil balance is negative.

145. *Correction for lack of balance of the coils.*—This correction is obtained while the suspended magnet is in the magnetic meridian but is applied to the deflection angle $2\Delta u$, but $2\Delta u$ is measured while M_s is deflected through an angle u. The effect of the thus unbalanced field on the deflection angle varies from a minimum when M_s is in the magnetic meridian to a maximum (infinite) when the deflection, u,

reaches 90°, at which point M_s becomes unstable. The relation between the true correction, c, for unbalance and the observed correction, x, is given by $c=\frac{x}{\cos u}$, u being the deflection angle when the deflector is within the coil. The correction to x is negligible for values of u up to 30°. Table 4 gives the factors to be applied to x to obtain c for different values of u. These factors are always positive (+).

TABLE 4.—*Correction factors*

u	Factor
°	
0	1.00
10	1.02
20	1.06
30	1.15
50	1.56
70	2.92

SUMMARY OF CONSTANTS

146. The constants of a magnetometer should be summarized in convenient form for field or observatory use as shown in figure 35. Critical tables should be prepared in all cases where their use will result in conservation of time and labor involved in routine computations, no matter how simple these computations may be. For example, in figure 35, any value of $\log \frac{H}{M}$ between $\bar{5}.79587$ and $\bar{5}.83934$ will yield the same value of $\log \left(1+\mu\frac{H}{M}\right)$ to an accuracy of ± 5 in the *sixth* decimal of logs.

ADJUSTMENT OF LOG C

147. ***Magnetometer.***—Comparison observations usually indicate that the value of H determined by a magnetometer differs consistently from that determined by the sine galvanometer or other standard instrument. This difference cannot usually be ascribed to one specific error in the determination of the constants of a magnetometer but is probably a cumulative effect of small uncertainties in measuring several of the constants. When the correction is small and consistent it may be reduced to zero (approximately) by readjustment of log C for each deflection distance. Observations for adjustment of log C consist of not less than four complete sets of the usual observations for H made at each deflection distance for which log C is to be determined. The average value of H for each whole set of observations is derived from the magnetograph by scaling average H ordinates for those time intervals during which the observations were in progress. The H base-line value is derived from measurements of H with the standard absolute instruments at the observatory. (At Cheltenham Magnetic Observatory the sine galvanometer is used as the standard for horizontal intensity.) Note also that a standard

Constants of Magnetometer 31

Effective Oct. 22, 1941

Moment of Inertia, log $\pi^2 K$ at 20° C	3.58 233
Temperature coefficient of magnetic moment, q	0.00 018
Log $(1+q)$	0.00 008
Induction factor, μ	3.5
Scale value of telescope scale, min. per div.	1.37
Middle division of telescope scale	50.00

Declination:

Reduction to middle=1.37 (50.00—mean scale reading)

Oscillations:

Correction to log T^2 for temperature; log $[1+(t-t')q]= +0.00\,008\,(t-t')$

Correction to log T^2 for induction; log $\left(1+\mu\frac{H}{M}\right)$ from table below

Correction to log π^2K for temperature; $+0.00\,001\,(t-20°)$

Deflections:

For $r=30$ cm; log $C=\bar{5}.86\,826+(0.00\,0023)(20°-t)$

For $r=40$ cm; log $C=\bar{5}.49\,406+(0.00\,0023)(20°-t)$

Log $M_{20}=\log M+0.00\,008\,(t-20°)$

$\log\frac{H}{M}$	$\log\left(1+\mu\frac{H}{M}\right)$	$\log\frac{H}{M}$	$\log\left(1+\mu\frac{H}{M}\right)$	$\log\frac{H}{M}$	$\log\left(1+\mu\frac{H}{M}\right)$
$\bar{5}.79\ 587$		$\bar{6}.00\ 848$		$\bar{6}.12\ 990$	
	0.00 010		0.00 016		0.00 021
$\bar{5}.83\ 934$		$\bar{6}.03\ 563$		$\bar{6}.15\ 059$	
	11		17		22
$\bar{5}.87\ 885$		$\bar{6}.06\ 119$		$\bar{6}.17\ 033$	
	12		18		23
$\bar{5}.91\ 506$		$\bar{6}.08\ 532$		$\bar{6}.18\ 922$	
	13		19		24
$\bar{5}.94\ 848$		$\bar{6}.10\ 818$		$\bar{6}.20\ 732$	
	14		0.00 020		0.00 025
$\bar{5}.97\ 952$		$\bar{6}.12\ 990$		$\bar{6}.22\ 469$	
	0.00 015				
$\bar{6}.00\ 848$					

NOTES

Log $(1+0.0000116\,d)^2=0.00001$ (rate of chronometer in seconds per day)

Log $\frac{5400}{5400-h}=0.00008$ (h in minutes)

Figure 35.—Constants of magnetometer.

value of H may also be determined directly by simultaneous observations with the standard instrument and the instrument being standardized.

148. We make use of the following relationships:

$$M=\frac{HM}{H}$$

$$C=\frac{H}{M}\sin u$$

$$\log M=\log HM-\log H$$

$$\log C=\log H-\log M+\log \sin u$$

From equation (448), appendix I,

$$\frac{\Delta \log C}{\Delta t} = -3(0.434)\alpha.$$

149. The H value from the magnetogram is combined with HM_t from observed oscillations to give a standard value of $\frac{H}{M_t}$. Log $\frac{H}{M_t}$ is added to log sin u for the corresponding deflection observations to obtain log C_t. Log C at 20° C is derived from log C_t by applying proper correction for temperature. (See eq. (446), app. I.) Figure 36 is a sample set of computations for adjustment of log C for Magnetometer RIC–3626. The computed values are compared with the values calculated from dimensions; lines 17 and 18, figure 36. Similar procedures may be followed in deriving certain constants for other instruments such as the Quartz Horizontal Magnetometer.

MAGNETIC STANDARDS

150. ***International standards.***—A new or reconditioned instrument is not ready for use until it has been standardized. Instruments that are apparently identical yield results that are not identical, despite the exercise of the utmost care in their construction and standardization. For this reason, there must be occasional intercomparisons to safeguard the accuracy of even the best available instruments. It is no simple matter to make correlated measurements of a shifting magnetic element in such a way that the purely instrumental discrepancies may be isolated from other effects. Programs for such comparisons are described by Hazard [11] and others. General specifications for national and international standards are recommended and published from time to time in the Bulletins of the International Association of Terrestrial Magnetism and Electricity.

151. ***Intercomparison of instruments.***—In the United States, all of the magnetic instruments of the U. S. Coast and Geodetic Survey, as well as those belonging to other agencies both foreign and domestic, are compared (standardized) directly at Cheltenham Magnetic Observatory or indirectly by comparisons first at Cheltenham and then at other observatories by use of an intermediate instrument.

152. The *Quartz Horizontal Magnetometer* (QHM)[12] has proved to be quite useful for international comparisons in horizontal intensity measurements and the *Magnetometric Zero Balance* (BMZ)[13] holds a similar position for vertical intensity. For magnetic declination comparisons the ordinary magnetometer is, of course, the most satisfactory instrument for use in indirect comparisons.

153. The *Sine Galvanometer*,[14] figure 37, requires special standardizations of its standard cells, potentiometer, and standard resistances at regular intervals of one to two years. There is no indication that the coil constant of sine galvanometer No. 1, now at Cheltenham, has changed appreciably with time.

---

[11] Dir. for Mag. Meas., pp. 37–40, (see item 4 of bibliography).
[12] D. la Cour, Le Quartz-Magnetometre QHM (see item 9 of bibliography).
[13] D. la Cour, The Magnetometic Zero Balance, the BMZ (see item 10 of bibliography).
[14] S. J. Barnett, A sine galvanometer for determining in absolute measure the horizontal intensity of the earth's field (see item 1 of bibliography).

COMPUTATION OF LOG C (H known)

Place Cheltenham, Pier 6 — H. s v: 2.53 γ/mm — Date: Oct. 31–Nov. 1, 1951
Magnetometer: RIC–3626 — Induction factor: $\mu=0.43$ — Observer: R. L. Viets
Long Magnet: 4.9 cm — $r_{25}=24.996$ $r_{30}=29.988$
Short Magnet: 4.0 cm — Deflection bar: Duralumin

Line	Terms	Step	Oct. 31	Nov. 1	Oct. 31	Nov. 1
			Begin 14:03 End 15:49	15:09 16:40	14:03 15:49	15:09 16:40
1	Distance		25 cm	25 cm	30 cm	30 cm
2	Set		1	3	1	3
3	h_{mm}		28.3	36.5		
4	h_γ		72	92		
5	B_H		18287	18288		
6	H	4+5	18359	19380		
7	log $H M_t$		1.86 547	1.86 626		
8	log H		9.26 385	9.26 435		
9	log M_t	7–8	2.60 162	2.60 191		
10	log $\frac{H}{M_t}$	8–9	6.66 223	6.66 244	6.66 223	6.66 244
11	log sin u		9.44 490	9.44 482	9.20 776	9.20 780
12	log C_t	10+11	6.10 713	6.10 726	5.86 999	5.87 024
13	t		25.20	21.90		
14	$20-t$		−5.20	−1.90		
15	Corr'n		+0.00 015	+0.00 006	+0.00 015	+0.00 006
16	log C_{20}	12+15	6.10 728	6.10 732	5.87 014	5.87 030
17	Mean			6.10 730		5.87 022
18	log C_{20} (dimensions)			6.10 733		5 87 013
19	log C_{20} (adopted)			6.10 730		5.87 022

NOTES

Line 3. Scaled from magnetogram or taken from standardization cards.
Line 4. H ordinate in gammas.
Line 5. H base line value.
Line 6. H from magnetograph (same for all distances of same set).
Line 7. Mean value from oscillations (erect and inverted).
Line 11. Log sin u; mean value from deflections (erect and inverted).
Line 13. Mean temperature of deflections (erect and inverted).
Line 15. Correction to reduce log C_t to log $C_{20}=-.0.000029\ (20-t)$ for duralumin; $0.000024\ (20-t)$ for brass.
Line 18. Log C_{20} from dimensions of magnets and deflection distances.

Figure 36.—Computation of log C.

154. ***Summary of comparisons.***—The results of comparisons of certain instruments are usually tabulated as shown in figures 38, 39, 40, and 41. For inclination and declination the final correction is shown as an additive index correction in minutes of arc. In comparisons for horizontal intensity, difference in results obtained with two magnetometers may be due primarily to errors in the adopted con-

stants, assuming of course that the instruments have no magnetic parts. All of these quantities enter factorially in the formulas from which H is derived, consequently the effect on H of an error in any one

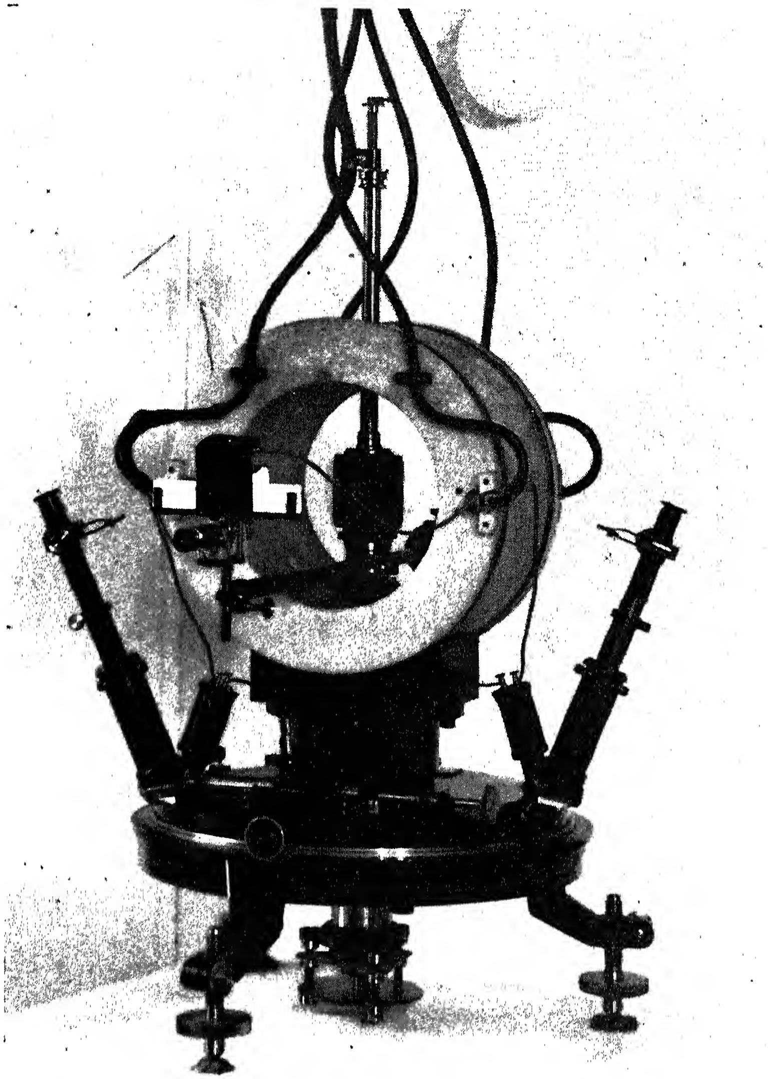

Figure 37. The Sine Galvanometer.

of them can better be expressed by a constant of multiplication rather than by an additive index correction. The errors in the constants are essentially independent of H. Hence, the correction is assumed to be proportional to H and is given as a factor $\frac{\Delta H}{H}$.

Correction to Declination Observed With Magnetometer No. 19 and Computed With Constants Dated October 10, 1941

Cheltenham Magnetic Observatory; Pier 6

	Sets			
	1	2	3	4
Observer	WEW	WEW	WEW	WEW
Date of observation 1947	Apr. 21	Apr. 21	Apr. 21	Apr. 21
Time interval (75 M. T.)	11:37–11:46	12:06–12:15	13:16–13:25	13:32–13:41
Erect−inverted in scale div	−8.6	−8.8	−8.4	−8.6
Observed declination	7°08.′6	7°10.′1	7°12.′9	7°13.′8
Variometer ordinate in mm.[a]	−13.1	−11.8	−9.0	−8.1
Scale value in ′/mm.[b]	1.00	------	------	------
Variometer ordinate in minutes	−13.1	−11.8	−9.0	−8.1
Observed base-line value	7°21.′7	7°21.′9	7°21.′9	7°21.′9
True base-line value [c]	7°22.′2	7°22.′2	7°22.′2	7°22.′2
Correction to W. declination	+0.′5	+0.′3	+0.′3	+0.′3
Mean correction	+0.′4			

[a] Ordinates corrected to 100.0 mm. shrinkage distance. Parallax allowed for.
[b] Scale value for a shrinkage distance of 100.0 mm.
[c] Includes pier correction 0.′0 added to regular base-line value to reduce to pier 6 and instrumental correction 0.′0 added to regular base-line value to reduce to International Magnetic Standard. Total correction 0′.0 added to regular base-line value.

Figure 38.—Standardization of magnetometer for declination.

Correction to Inclination Observed With Earth Inductor No. 106

Cheltenham Magnetic Observatory; Pier 5

	Sets			
	1	2	3	4
Observer	WEW	WEW	WEW	WEW
Date of observation 1946	June 18	June 18	June 18	June 18
Time interval (75 M. T.)	15:38–15:47	15:49–15:57	16:01–16:10	16:18–16:26
Observed value of inclination	71°18.′8	71°19.′2	71°19.′4	71°18.′7
Value computed from Mgph. No. 5 [a]	71°18.′6	71°19.′2	71°19.′5	71°18.′6
Index correction	−0.′2	0.′0	+0.′1	−0.′1
Mean correction	0.′0			

[a] Includes a pier correction −0.′2 added to dip to reduce to pier 5, and an instrumental correction 0.′0 added to dip to reduce to International Magnetic Standard.

Figure 39.— Standardization of earth inductor for inclination.

Correction to Horizontal Intensity Observed With Magnetometer No 19 and Computed With Constants Dated October 10, 1941

Cheltenham Magnetic Observatory; Pier 6

	Sets			
	1	2	3	4
Observer	WEW	WEW	WEW	WEW
Date of observation 1945	June 30	July 5	July 5	July 5
Total time interval (75 M. T.)	12:41–13:46	10:07–11:21	12:02–13:07	14:01–15:03
Mean $\left(\log \frac{H}{M}\right)_L - \left(\log \frac{H}{M}\right)_S$	+0. 00024	+0. 00012	+0. 00009	+0. 00010
Observed value of log M_{20}	2. 80660	2. 80671	2. 80665	2. 80666
Observed value of H in γ	18235	18195	18197	18208
Variometer ordinate in mm.*	42. 8	27. 6	29. 1	33. 4
Scale value in γ/mm.**	2. 67	----	----	----
Ordinate in γ	114	74	78	89
Observed base-line value	18121	18121	18119	18119
True base-line value***	18120	18120	18120	18120
Correction to H; (ΔH)	−1	−1	+1	+1
Mean correction	ΔH 0		$\frac{\Delta H}{H}$ 0. 00000	

*Ordinates corrected to 100.0 mm. shrinkage distance. Parallax allowed for.
**Scale value for a shrinkage distance of 100.0 mm.
***Includes pier correction $+3\gamma$ added to regular base-line value to reduce to pier 6 and instrumental correction -2γ added to regular base-line value to reduce to International Magnetic Standard. Total correction $+1\gamma$ added to regular base-line value.

Figure 40. Standardization of magnetometer for horizontal intensity.

Correction to Horizontal Intensity Observed With QHM No. 48 and Computed With Constants Dated April 23, 1942

Cheltenham Magnetic Observatory; Pier 6

	Sets			
	1	2	3	4
Observer	WEW	WEW	WEW	WEW
Date of observation 1945	Sept. 6	Sept. 7	Sept. 24	Sept. 24
Time interval (75 M. T.)	15:10–15:19	08:52–08:59	11:31–11:39	11:45–11:51
Mean temperature in °C	26. 2	24. 9	23. 6	23. 5
Observed value of H in γ	18234	18190	18199	18199
Variometer ordinate in mm.*	38. 7	21. 7	24. 9	24. 9
Scale value in γ/mm.**	2. 67	----	----	----
Ordinate in γ	103	58	66	66
Observed base-line value	18131	18132	18133	18133
True base-line value***	18120	18120	18120	18120
Correction to H; (ΔH)	−11	−12	−13	−13
Mean correction	ΔH −12		$\frac{\Delta H}{H}$ −0. 00066	

*Ordinates corrected to 100.0 mm. shrinkage distance. Parallax allowed for.
**Scale value for a shrinkage distance of 100.0 mm.
***Includes pier correction $+3\gamma$ added to regular base-line value to reduce to pier 6 and instrumental correction -2γ added to regular base-line value to reduce to International Magnetic Standard. Total correction $+1\gamma$ added to regular base-line value.

Figure 41.—Standardization of QHM for horizontal intensity.

CHAPTER 5. OPTICAL SYSTEMS AND PHOTOGRAPHIC REGISTRATION

COLLIMATION

155. Both absolute and variation instruments involve applications of simple geometrical optics which are not adequately explained in standard books on the subject. The ensuing discussion assumes a basic knowledge of the elementary principles involved.

156. ***Simple lens.***—In figure 42, let r be the radius of curvature of the convex side of the planoconvex lens, L. The optical axis is defined as the line, CO, perpendicular to both surfaces. Lenses are usually centered, that is, they are cut and edge-ground so that the optical axis passes through the geometric center of the lens itself.

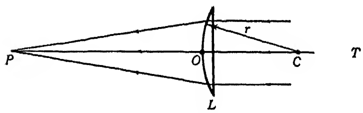

Figure 42.—Planoconvex lens.

FOR A THIN LENS, THE INDEX OF REFRACTION IS $1+r/f$

157. Parallel rays of monochromatic light falling upon the plane side of L, figure 42, at normal incidence will converge at a point, P, called the *principal focus*. The distance, OP, is called the *focal length* of this simple lens. A plane passing through P and perpendicular to the optical axis, PC, is called the *focal plane* of the lens.

158. If a point source of light be placed at the principal focus, P, figure 42, the rays will be parallel after passing through the lens. This process of producing parallel rays is called *collimation*. If an illuminated scale be placed at P and a telescope, previously focused on a distant object, be placed at T and directed parallel to OP, the image of the scale will be in sharp focus in the telescope. Many

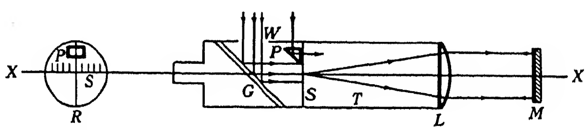

Figure 43.—Illustration of both the Gaussian ocular and the modified Gaussian ocular in an autocollimator.

magnetometer magnets are made in the form of hollow cylinders with a scale or index lines ruled on glass fixed in one end and a collimating lens fixed in the other end. Such a system is called a collimator.

159. ***Autocollimator; Gaussian ocular.***—In figure 43, a telescope, T, is provided with a scale, S, ruled on a glass reticle mounted in the principal focal plane of the objective lens, L. The eyepiece is fitted with a thin, transparent glass plate, G, set at 45° to the optical axis of the telescope. Diffuse light, after entering a small window at W, in the side of the eyepiece, is partly reflected and illuminates the scale. The telescope then serves as a collimator and if a plane mirror is interposed at M, normal to the axis of L, a real image of the scale will be found superimposed upon the scale itself. The scale and its

image may be brought into precise coincidence by rotation of the mirror or telescope in inclination or azimuth or both.

160. ***Modified Gaussian ocular.***—In this type of ocular, only the upper portion of the central line of the scale is illuminated. This is accomplished by using, instead of the inclined glass plate, an adjustable 90° prism mounted in the ocular close to the vertical line of the scale as shown at P, figure 43. Hence only the image of a portion of the central vertical line would be formed in the focal plane of the telescope. By adjustment of the telescope in azimuth and inclination the image of this central line may be made to fall upon any part of the scale and serves as an index of relative angular motion between the mirror, M, and the axis of the telescope. This type of optical system is used on some magnetometers.

MAGNETOGRAPH OPTICS

161. ***Optical scale value.***—If an optically plane mirror is interposed at any point, M, figure 44, so that its reflecting surface is normal to OP, the reflected rays will be parallel and will come to focus pre-

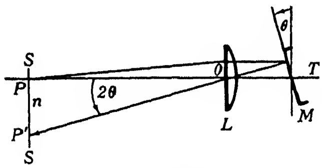

Figure 44.—Optical lever.

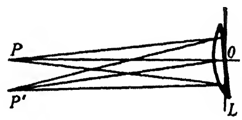

Figure 45.—Planoconvex mirror.

cisely at the position of the source, P. This will be true regardless of the distance between the mirror and the lens so long as the lens collimates the incident light. If the mirror is rotated through a small angle, θ, the reflected rays will sweep through an angle $2\,\theta$, remaining parallel until they enter the lens. After passing through the lens they will be brought to focus at some point P', in the focal plane, and the angle $P'OP$ will be practically equal to 2θ. The distance $PP'=n$ through which the image moves in the focal plane for a given small value of θ is directly proportional to the focal length of the lens. For all practical purposes this rule holds regardless of the distance between lens and mirror. If the mirror is translated in any direction without rotation, the image P' will not move. If 2θ is the angle through which the reflected ray turns when the mirror turns through an angle θ, then

$$\tan 2\,\theta=\frac{n}{OP} \tag{81}$$

and for small angles, $$\theta=\frac{n}{2OP}.$$

The ratio of the angular motion of the mirror to the linear motion of the image (in the focal plane of L) is called the *optical scale value*, designated by ϵ. That is,

$$\epsilon=\frac{\theta}{n}=\frac{1}{2OP}. \tag{82}$$

Note: See paragraphs 200–202, pages 74–76, for more exact expressions for the optical scale value and its reciprocal the optical lever.

162. The optical scale value may be halved (optical lever doubled) by allowing the recording mirror, M, to make an angle of 45° with the collimated rays whereupon the rays will be reflected at an angle of 90° to their original direction. These rays are then allowed to fall approximately at normal incidence upon a fixed plane mirror after which they will be reflected back to the mirror, M, thence through the lens, to form an image in the focal plane of the lens. Since the light is reflected twice from the recording mirror, M, the reflected ray will turn through an angle $4\,\theta$ when M is turned through an angle θ. The final result is to double the sensitivity of the system. The optical scale value is now equal to $\frac{1}{4\,OP}$ instead of $\frac{1}{2\,OP}$.

163. ***Planoconvex mirror.***—Suppose the lens is reversed as in figure 45 so that the light is incident on the convex surface of the lens. Some of the light will not pass beyond the lens but will be reflected from the inner surface of the plane side. If the lens is now inclined slightly making an angle θ with OP, the reflected rays from the plane surface, now collimated, will pass back through the lens and form a faint real image at P', such that the angle POP' is again equal approximately to $2\,\theta$. If the plane side is silvered or aluminized, practically

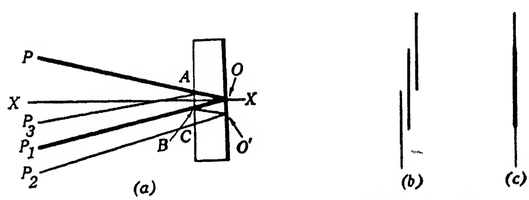

Figure 46.—(a) Origin of ghost images from front and back surfaces of plane mirror; (b) appearance o main image and ghosts on screen; (c) alignment of ghosts with main image.

all of the light will be reflected. Planoconvex mirrors of this type are frequently used as the moving mirror on some galvanometers, optical thermographs, and other laboratory instruments. Images formed by unsilvered plane surfaces of lenses or prisms are sometimes called ghost images.

164. ***Multiple reflections.***—Suppose collimated light from P, figure 46 (a), falls upon a back-surfaced mirror (front and back surfaces not quite parallel). Most of the light will be reflected at O and after again passing through the lens will form a bright image at P_1. Some of the light will be reflected at the front surface at A and since the surfaces of the mirror are not precisely parallel, a second faint image will be formed at P_3. Also some of the light reflected from O will be internally reflected at B back to O' and again back through the mirror, emerging at C, and forming another faint image at P_2. P_2 and P_3 are also called ghost images. When the light source, P, is an illuminated slit or a straight-filament incandescent lamp, the three images will usually appear as one central bright image with faint images on either side as shown at b, figure 46. Usually they may be brought into alignment as shown at c, by rotation of the mirror, M, on an axis, XX, normal to M. Ghost images of this kind cannot be formed when front-surfaced mirrors are used.[1]

[1] D. G. Knapp, Curiosities of magnetographs, Trans. Amer. Geophys. Union, p. 539, 1944.

165. ***Totally reflecting prism.***—90° prisms, figure 47, are used extensively in magnetic instruments for changing the direction of incident light approximately 90° and, as in the vertical intensity variometer, for changing the plane of motion of the reflected light beam from a vertical to a horizontal plane. Light which falls on the face AB at or near normal incidence is totally reflected by the surface AC and emerges approximately normal to the face BC.

166. ***Planoconvex prism.***—If a planoconvex lens, L, be cemented to one of the faces of a 90° prism adjacent to the 90° angle, this com-

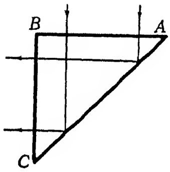

Figure 47.—90° prism.

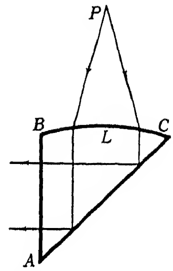

Figure 48.—Planoconvex prism.

bination will serve as a collimator for light from a source P when P is at the proper distance from the prism-lens combination. When such a combination is made from one piece of glass, figure 48, it is called a planoconvex prism. The la Cour D variometer is equipped with such a prism.

167. ***Cylindrical lens (planoconvex).***—This is equivalent to a right cylinder cut parallel to the long axis of the cylinder, figure 49.

Figure 49.—Planoconvex cylindrical lens.

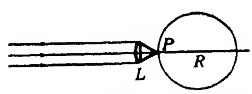

Figure 50.—Action of cylindrical lens in bringing a plane light beam to a point focus.

Parallel rays incident on either surface will converge in a line focus or line image parallel to the axis of the cylinder. If the incident light is confined to a very narrow plane at right angles to the axis of the cylindrical lens, the rays will be brought down to a point focus, figure 50. Cylindrical lenses of short focal length, 2 to 4 cm, are used on photographic recorders. Long-focus cylindrical lenses and plano cylindrical mirrors are frequently used on variometers and galvanometers. In such cases the axis of the cylinder must be adjusted so that it will be parallel to the filament of the lamp and at right angles to the axis of the cylindrical lens of the recorder in order to obtain a sharp image.

168. ***Three-faced mirrors.***—A convenient type of mirror for use on magnetic variometers to produce regular recording spots and upper and lower reserve spots is shown in figure 51. The three faces, a, b, and c are ground and polished optically flat and then aluminized. In order that the images from all three faces shall lie in the same horizontal plane, the faces must be so ground that they will all be perpendicular to a common plane. The areas of the faces are equal

and the dihedral angles are such that when the regular recording spot from face b passes off the magnetogram at the top or bottom edge, the image from a reserve face, a or c, will come on the gram at the bottom or top. The size of the desired dihedral angle is a function of the optical lever of the variometer lens and the width of the magnetogram. For example:

Let y=the width of the gram=200 mm;
x=the distance between reserve spots=180 mm;
$2OP$=the optical lever=3460 mm (for a D variometer);
α=the acute angle between adjacent faces.

Then $\tan \alpha=\frac{x}{2OP}=\frac{180}{3460}$

and $\alpha=2° 59'$

and the interior dihedral angle is 177° 01′. In the Eschenhagen arrangement of variometers the mirrors have certain specifications for the angles when the width of the drum face is 20 cm.

169. The following table gives a current set of specifications for dihedral angles, together with the approximate reserve distances which are obtained by using those angles:

Variometer	Approximate recording distance OP	Specifications for dihedral angle			Approximate reserve distance x
		Interior 180°—α	Exterior α	Tolerance	
	mm	° ′	° ′	′	mm
H	1200	175 45	4 15	±2	178
D	1730	177 00	3 00	±2	181
Z	2300	177 40	2 20	±2	188

Note: In some of the older variometers, separate plane mirrors were mounted on thin aluminum frames. With this arrangement it was necessary to bend the frames relative to each other in order to obtain proper adjustment of the spots—a tedious and time-consuming process.

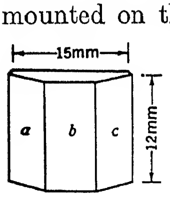

Figure 51.—Three-faced mirror.

170. ***Auxiliary spots by other means.***—La Cour devised a method of producing multiple spots from a single recording mirror, by using small, 90° prisms spaced along a rack in front of the recording lamp. The prisms were so adjusted that each one provided a beam of light for one variometer mirror—the assembly having the effect of several light sources, with the corresponding number of spots for the recording drum. Schmidt [2] used a system of inclined mirrors to reduce the sensitivity. Both the normal and the low-sensitivity traces were recorded on the same magnetogram, thus eliminating the need for reserve mirrors.

171. ***Base-line mirror.***—A plane mirror attached to an adjustable bracket within the variometer housing and back of the variometer lens reflects a beam of light to provide a fixed reference line, or base line, on the magnetogram. Each variometer usually has its own base-line mirror.

[2] Zs. f. Instrumentenkunde, 27, 137–47, 1907.

172. ***Diaphragms.***—The quality of an image may be improved in most cases by use of horizontal diaphragms (narrow horizontal windows) mounted directly in front of the variometer lens. The effect is to reduce the intensity of the light leaving the variometer and at the same time confine the reflected light or transmitted light to a small segment of the lens or mirror. Another diaphragm on the cylindrical lens of the recorder serves also as a screen to protect the photographic paper from extraneous light.

173. ***Character of image.***—The light source is usually a straight-filament incandescent electric lamp or an illuminated slit placed at either end of the recorder at approximately the same height as the cylindrical lens of the recorder. It is important that the axis of the filament or the slit be strictly vertical so that images from the variometer mirrors will be vertical as they fall upon the cylindrical lens of the recorder. The recording distances should be such that these images will be sharply focused line images on the magnetogram before the cylindrical lens is adjusted. Great care should be used in making these adjustments in order to insure satisfactory photographic registration. If the variometer lens is of the long-focus cylindrical type its axis must be vertical (parallel to the lamp filament).

PHOTOGRAPHIC REGISTRATION

174. ***Recorder.***—This is usually a clock-driven drum having a 20-cm face width and so geared as to have a peripheral (paper) speed of 20 mm per hour. To prevent loss of record and to provide space for a paper-clamp the drum makes one revolution in 25 hours. The recorder and accessories are housed in a lighttight case, figure 52. The light spots from the variometers are focused sharply on photographic paper wrapped tightly around the drum. As the drum rotates, these spots trace latent records of the base lines and variations of D, H, Z, and temperature on the photo paper.

175. ***Parallax.***—In figure 53(a) the horizontal line represents a time line across the photographic paper on the drum as seen from the variometers. When properly adjusted the D, H, and Z recording spots should be bisected on the magnetogram by this time line. In figure 53(b) the H and D spots are not properly adjusted with respect to the time line.

176. In photographic recording this maladjustment of a spot relative to a time line (or time mark) is called *parallax.* Obviously parallax of the base-line spots is of no consequence. When the centers of all variometer lenses, axis of cylindrical lens, and center of time-flash mirror are all in the same horizontal plane, little or no parallax between any recording spot and the time line would be expected. In practice, however, because of lack of perfect centering of lenses and effects of spherical aberration, the alignment of lens centers is often not sufficient to eliminate all parallax, and the final adjustments must be made by the trial-and-error method. Adjustment of any individaul spot is made by raising or lowering the whole variometer, perhaps by several millimeters, until the recording spot is on the time line. The time line itself maybe adjusted by raising, lowering, or tilting the time mirror or time lamp.

177. ***Sensitized recording paper.***—Photographic paper from several manufacturers has been used successfully for recording mag-

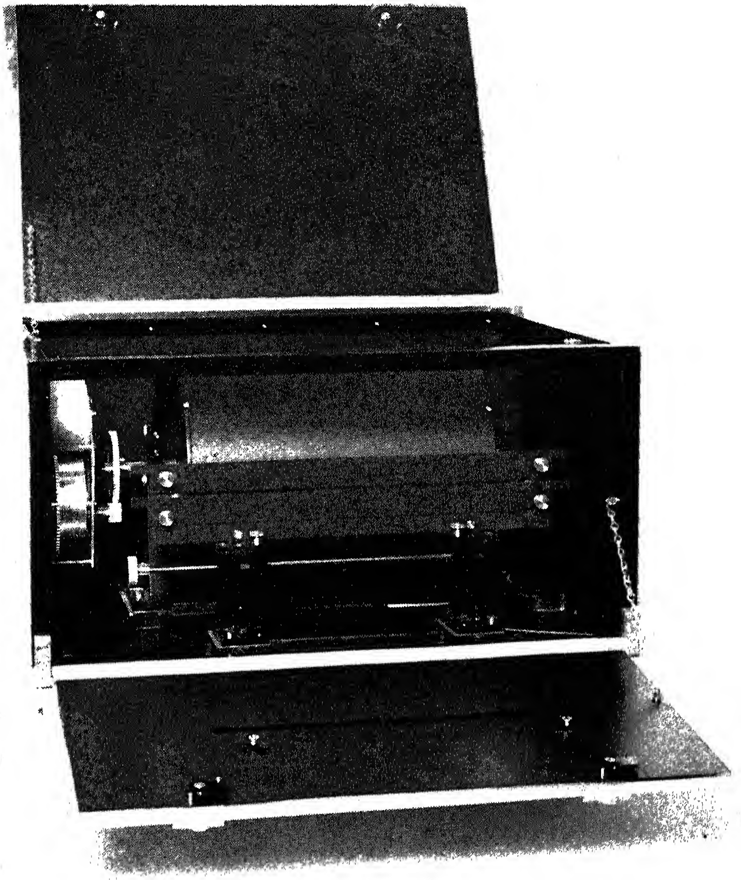

Figure 52.—Magnetograph recorder showing driving clock, gears, and cylindrical-lens diaphragm and light shield.

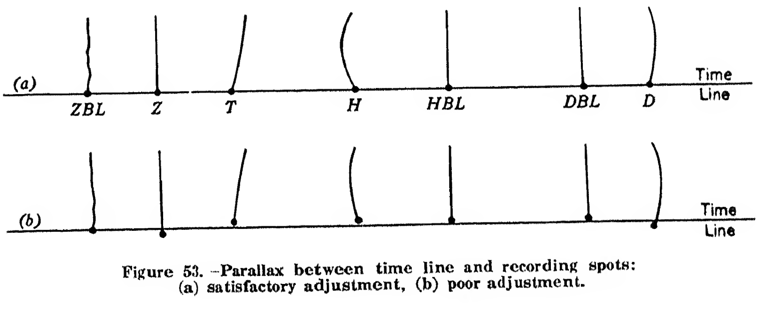

Figure 53.—Parallax between time line and recording spots: (a) satisfactory adjustment, (b) poor adjustment.

netograms. In general the paper should have high contrast qualities, producing a dense black line on a white background. Its sensitivity to red light should be as low as possible, to facilitate handling the paper by red light at the recorder and in the photographic dark room with a minimum of background fogging. Some paper that has been used appears to be quite sensitive to fingerprints, and no uniform specification as to resistance to fingerprints has been found. It has occasionally been necessary to have the observer wash his hands thoroughly with soap and water just before making the daily change of paper on the recording drum, or even to wear thin cotton gloves while handling the paper, in order to reduce the smudging effect of finger contacts.

178. Standard brands of high-contrast developing solutions for photographic prints have been used for satisfactory processing of magnetograms. It has sometimes been found that more uniform results and greater contrast can be obtained with developing solutions weaker (i. e., with more water) than the strengths recommended by the manufacturer of the chemicals. Developing is generally continued until the magnetogram has a satisfactory appearance under the red light, rather than until a specified time has elapsed. The developing, fixing, and washing baths should be warm enough to produce satisfactory speed of chemical action, but not so warm as to risk damage to the photographic emulsion by heat; temperatures of 15° to 25° C. are generally acceptable. Magnetograms will have more sharply defined traces if the intensity of the recording lamp is kept relatively low and the developing process is forced somewhat by leaving the gram in the chemical solution longer, although care must be taken that the gram is not left in the solution so long that the white background becomes fogged. Thorough washing in clean running water is, of course, necessary for magnetograms that are to be kept on file permanently.

CHAPTER 6. QUARTZ-FIBER TECHNIQUES

179. ***Procurement of fibers.***—The manufacture of quartz fibers of suitable dimensions for use in magnetic variometers and astatic galvanometers requires special equipment not usually available at magnetic observatories.[1] For this reason it is well to procure commercially a supply of fibers ranging in diameter from 10 to 75 microns (1 micron=0.001 mm) and 30 cm in length. A fiber should be reasonably uniform in diameter over its entire length, and circular in cross section. Quartz suspensions of the la Cour type, figure 54, can be manufactured with a minimum of equipment.[2]

180. ***Uses of fibers.***—Fibers ranging from 7 to 12 microns in diameter are used on astatic galvanometers and require special equipment for mounting. A frame of mounted galvanometer fibers is shown in figure 55.

181. Those ranging from 15 to 20 microns are used in D variometers and those ranging from 35 to 75 microns are for H variometers.

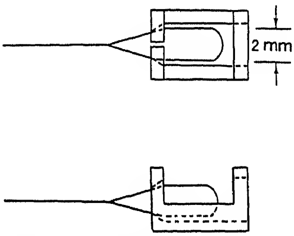

Figure 54.—Quartz fibers, la Cour type.

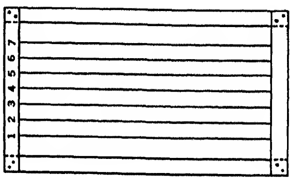

Figure 55.—Frame of mounted galvanometer fibers.

182. The sizes used in insensitive variometers and Quartz Horizontal-Intensity Magnetometers (QHM) depend upon the magnetic latitude where the instruments are to be used and upon the magnetic moment of the recording or suspended magnet (see ch. 13.)

183. ***The torsion constant of the fiber.***—When one end of a fiber is turned through an angle τ relative to the other end, the torsion couple in the fiber is

$$L=k'\tau=\left(\frac{\pi\mu d^4}{32l}\right)\tau \tag{83}$$

where k' is the torsion constant of the fiber given (par. 238, p. 88) by

$$k'=\frac{\pi\mu d^4}{32l} \tag{84}$$

and where

μ=modulus of rigidity of fiber material

$=2.83\times10^{11}\,\dfrac{\text{dyne}}{\text{cm}^2\text{-radian}}$ for quartz;

l=length of fiber (usually about 15 cm);

d=effective diameter of fiber.

[1] A. King *et al.*, Jour. Sci. Insts., 12, 249–252, 1934.

[2] Communications Magnétiques, No. 11–12, Danish Meteorological Institute, Copenhagen (see item 11 of bibliography).

184. If the fiber is used as the suspension of a torsion pendulum, the period is given by

$$T_{\sim}=2\pi\sqrt{\frac{I}{k'}} \tag{85}$$

where

$T_{\sim}$=period of a complete oscillation;

I=moment of inertia of the nonmagnetic inertia weight.

185. If the fiber is the suspension of a D variometer, the balancing couples in the torsion test are

$$k'(f-h)=HM_s \sin h \tag{86}$$

and

$$k'=\frac{HM_s \sin h}{f-h} \tag{87}$$

as shown in paragraph 203, page 76.

186. The scale value of an H variometer without sensitivity control magnet is given by

$$S_H=\epsilon\frac{k'}{M_s} \tag{88}$$

and

$$k'=\frac{M_sS_H}{\epsilon} \tag{89}$$

as shown in paragraph 242, page 89.

187. Common methods for determining the torsion constant are based on equations (84), (85), (87), and (89). The torsion-test method is described in paragraph 203, chapter 7 and the H scale-value method is described in paragraph 242, chapter 8.

188. ***Determination of the effective diameter of a fiber; classification.***—With a micrometer caliper, measure the diameter of a fiber at several uniformly spaced points, and take the 4th root of the mean of the 4th powers of the measured diameters. The accuracy of this type of measurement is of the order of ± 5 microns. After the fibers have been measured, mount them on a wooden frame with liquid shellac and label them as to diameter.

189. Continue this process of rough measurement and mounting until 50 to 100 fibers ranging from 10 to 75 microns in diameter have been mounted, classified, and labelled.

190. With a precision, 3-stage microscope having a scale in the eyepiece reading to 10 microns, remeasure the fibers as a check on the caliper readings, estimating to 1 micron. Reclassify and mount the fibers, and label them as to the more precise dimensions.

191. Calculate the torsion constant, k', for each fiber, using equation (84). Record values of k'.

192. ***Directions for determination of the torsion constant of a fiber by oscillations.***—Construct a small, nonmagnetic, solid brass cylinder, approximately 1½ to 2 mm in diameter by 50 mm in length, and drill a fine hole through its center at right angles to its long axis. Measure the diameter, length, and mass of this cylinder and calculate its moment of inertia about an axis normal to its long axis (see par. 72, p. 28). Note: The dimensions of the inertia weight used in this work by the Coast and Geodetic Survey are as follows: Length, 49.4 mm;

diameter, 1.66 mm; mass, 0.90 gram; moment of inertia, 1.83 gram-cm^2.

193. Select a fiber and with fused shellac attach one end to the inertia weight and the other end to a laboratory stand so that the length of fiber between support and inertia weight is about 15 cm. Surround the suspension by a glass cylinder to protect it against air currents. Using a stop watch or a chronometer, determine the period, $T_\sim$, (time of one cycle) of the cylinder while oscillating as a torsion pendulum through an angle not exceeding 20°, by timing 50 to 100 oscillations. Repeat the operation 4 times and take the mean as $T_\sim$. Remount the fiber in a separate wooden frame and label as to measured nominal diameter and period (say 45 microns; $T_\sim$=9.8 sec).

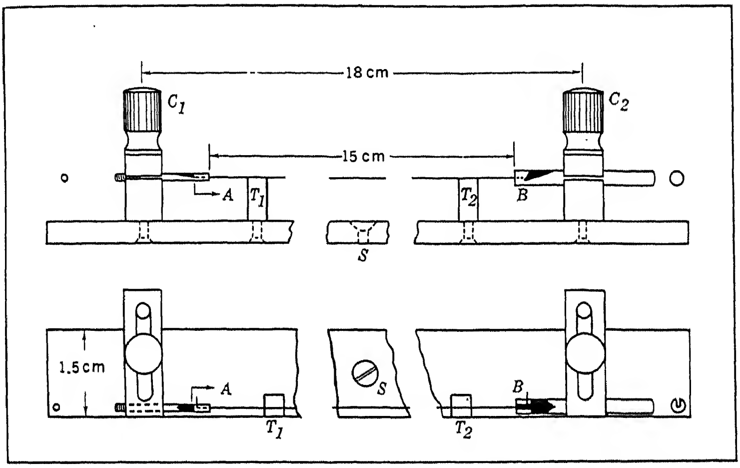

Figure 56.—Apparatus for mounting brass stems on quartz fibers.

Repeat this operation with several fibers suitable for H and D variometers. The classification of fibers by this method of oscillations is quite satisfactory and is preferred to the method of diameter measurement by microscope. Calculate from equation (85) the torsion constant, k', for each fiber, and record these values on the labels. Compare with the values obtained from direct measurement of the diameter (par. 191).

194. ***Installation of fibers in H and D variometers.***—A simple apparatus for mounting upper and lower brass stems to a fiber is shown in figure 56. It consists of a brass bar carrying two clamps, C_1 and C_2, for aligning and clamping the brass stems and two small fiber supports, T_1 and T_2. The bar is attached to a wooden block by a single screw, S. By turning the bar at right angles to the wooden base, the latter may be used as a holder when the stems are heated for melting the shellac. The stems are slotted and notched as shown in the figure.

195. To attach the stems to the fiber, proceed as follows: Using an alcohol flame and hand blowpipe (or an oxyhydrogen torch), bend the ends of the fiber at right angles, leaving about 15.5 cm of straight fiber between the bent ends. Feed the ends into the stems A and B and adjust the stems in the clamps so that the fiber is drawn taut as the bent ends engage the notches in the stems. Gently warm the

end of the bar near A in a direct flame and apply a small amount of dry shellac to the notch allowing the shellac to melt and flow around the bent end of the fiber and fill the slot and notch completely. Continue heating gently until the shellac is free of moisture and air bubbles. Avoid burning the shellac. See that the fiber is well centered in the stem at the point of emergence from the slot. In like manner attach end B. Before the shellac is entirely set, hold the bar with its long axis vertical, end B down, loosen C_2 and allow the fiber to take the load gently. After the bar has cooled and the shellac is well set, tighten C_2, leaving the fiber slightly slack. Break off the ends of the fiber which protrude laterally from the stems. In this condition the fiber may be mounted at once in a variometer or stored in the holder for future use. Experience has shown that fibers mounted in this manner give satisfactory performance and that there is little or no yielding of the shellac when the fiber is subjected to large torques over long periods of time.

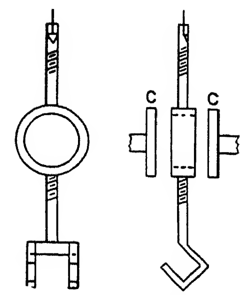

Figure 57.—Coupler for attaching quartz fiber to suspended magnet.

196. Attach the coupler, figure 57, to the stem A, figure 56, and screw it on rather firmly. Hold the apparatus with its long axis vertical, end A down, loosen C_1 and allow the fiber to take the load gently. Avoid bending the fiber near its point of attachment to a stem. Remove the fiber and attachments from the holder and while the system is still hanging vertically, attach the stem B firmly to the torsion head spindle of the variometer. Lower the whole suspension into the variometer suspension tube and adjust the torsion head and the foot screws until the suspension swings freely and the coupler disk is centered and parallel to the jaws of the fiber clamp. Then set the clamp and tighten the torsion-head set screw.

210111--53----6

CHAPTER 7. THE DECLINATION VARIOMETER

197. ***Function of a D variometer.***—This instrument should indicate visually or by photographic registration the variations in the direction of the horizontal component of the earth's magnetic field, that is, the variations in magnetic declination.

198. ***Basic requirements.***—The essentials of a *D* variometer are:

(a) A permanent magnet suspended in a nonmagnetic housing by a fine quartz (or equivalent) fiber;

(b) An optical system suitable for visual observation or photographic registration of the variations that occur in the direction of the *D* magnet as the declination changes;

(c) The recording ray and the optical axis of the *D* lens to be approximately normal to the recording drum;

(d) Magnetic damping (recording magnet surrounded by a copper or silver box);

(e) The magnetic axis of recording magnet to be parallel to the magnetic meridian in the absolute observatory; see paragraph 331, page 135 for effects of exorientation angles;

(f) A fixed base-line mirror so arranged that a straight reference line (base line) is recorded on the magnetogram near the *D* trace.

199. ***D scale value.***—The *D* scale value or minute scale value, S_D', of a *D* variometer is defined as the change in magnetic declination, ΔD, in minutes of arc, corresponding to a change of ordinate of one millimeter, Δn, of the *D* spot. Then

$$S_D'=\frac{\Delta D}{\Delta n}. \tag{90}$$

The *D* scale value is made up of three parts: (a) the *optical scale value;* (b) the *torsion factor;* and (c) the *field factor.* These component parts will be described in detail in the following paragraphs.

200. ***Terms in the expression for the D scale value.***—The various symbols used in the derivation of the general equation of the minute scale value of the *D* variometer are:

S_D' =the minute scale value; minutes per mm;

S_D^γ =the gamma scale value; gammas (*E-W* field) per mm;

k' =the torsion constant of the suspension fiber; dyne-cm per radian twist;

M_s =the magnetic moment of the recording magnet; cgs units;

k =ratio of torsion constant of of the fiber to magnetic moment of suspended magnet $=\frac{k'}{M_s}$;

H =horizontal intensity of the earth's magnetic field;

C =the algebraic sum of the north components of the fields of all other magnets of the magnetograph (Note: the C field must be the same in torsion tests as it is in normal operation of the *D* variometer);

R =the effective distance from the *D* lens to the recording drum, in mm;

$2R$ = the optical lever;

$\frac{1}{2R} = \epsilon$ = the optical scale value in radians per mm;

$\epsilon' = 3438\epsilon$ = the optical scale value in minutes per mm;
Δn = a small increment in the ordinate of the D spot, in mm;
$\Delta\theta$ = a small increment in the angular motion of the D recording magnet (or the D magnet mirror); minutes or radians;
h = the angle through which the D magnet turns when the upper end of the fiber (torsion head) is turned through an angle, f;
f = see h above;
$f-h$ = total twist in the fiber, in the torsion test;
μ = the rigidity modulus of quartz; 2.83×10^{11} dynes per cm^2 per radian;
l = the length of the fiber in centimeters;
d = the diameter of the fiber in centimeters;
I = the moment of inertia of the inertia weight suspended on the fiber and oscillating as a torsion pendulum;
$T_{\sim}$ = the period of the torsion pendulum in seconds.

201. The approximate value of R is the distance from the D lens to the recording drum. Since a small portion of the path of the reflected ray is made up of the D lens and the cylindrical lens of the recorder, certain corrections must be made to the measured distance, L, to obtain the effective recording distance, R. Assuming the glass of these lenses to have a nominal refractive index of 1.5, it can be shown that

$$R=L-\frac{c}{3}+bl \tag{91}$$

in which

R = the derived effective recording distance to be used in scale-value equations;
L = the measured distance in mm, from the front of the D lens to the point on the recording drum at which the D spot normally falls;
c = the maximum thickness of the cylindrical lens in mm;
$b=\frac{2}{3}$ for a planoconvex lens, plane side out;
$b=\frac{1}{3}$ for an equi-convex lens;
$b=0$ for a planoconvex lens, plane side in;
l = the maximum thickness of the variometer lens in mm.

202. ***The optical lever and optical scale value.***—When the recording mirror (and the recording magnet) turns through a small angle $\Delta\theta$, the D spot moves through a small ordinate, Δn. If R is the effective recording distance (*optical lever* $=2R$), the small angle, $\Delta\theta$, in radians is $\frac{\Delta n}{2R}$. Thus

$$\Delta\theta=\frac{\Delta n}{2R} \tag{92}$$

and

$$\frac{\Delta\theta}{\Delta n}=\frac{1}{2R}.$$

But the optical scale value, designated by ϵ, is

$$\epsilon=\frac{\Delta\theta}{\Delta n} \tag{93}$$

$$\epsilon=\frac{1}{2R} \tag{94}$$

and from equation (91)

$$\epsilon=\frac{1}{2\left(L-\frac{c}{3}+bl\right)}. \tag{95}$$

In radians,

$$\Delta\theta=\epsilon(\Delta n). \tag{96}$$

And in minutes,

$$\Delta\theta=\frac{3438}{2R}(\Delta n). \tag{97}$$

203. ***Torsion tests.***—If the torsion head is turned through an angle f, and the D recording magnet turns through an angle h as a result of the torsion in the fiber, the system will be in equilibrium when the opposing couples, k' $(f-h)$ and $(H+C)M_s \sin h$, are equal. That is,

$$k'\ (f-h)=(H+C)\ M_s \sin h. \tag{98}$$

For small values of h, write $\sin h \approx h$ radians. Then

$$\frac{k'}{M_s}=k=(H+C)\left(\frac{h}{f-h}\right). \tag{99}$$

Divide by H,

$$\frac{k}{H}=\left(\frac{H+C}{H}\right)\left(\frac{h}{f-h}\right) \tag{100}$$

$$=\left(1+\frac{C}{H}\right)\left(\frac{h}{f-h}\right) \tag{101}$$

and

$$\frac{h}{f-h}=\frac{k}{H+C} \tag{102}$$

$$=\frac{k'}{(H+C)M_s}. \tag{103}$$

204. ***Physical picture of the torsion factor.***—In figure 58, let O_1H_1 denote the original direction of H and M, where H is the horizontal intensity and M is the magnetic moment of the D magnet; no torsion in the fiber. Now turn the torsion head through an angle f; whereupon the magnet will turn through an angle h, and the torsion in the fiber is $f-h$. Compare figure 59 in which the original line of no

torsion remains along O_2H_2 but the declination, D, changes through a small angle ΔD and H is now along O_2H_2'. But the magnet will be restrained by torsion and its axis will lie along O_2M_2. Since the direction of the field has changed, O_2H_2' corresponds to O_1H_1 and O_2H_2 corresponds to O_1T but the torsion is oppositely directed. The change in the direction of the field, ΔD in figure 59, is equivalent to the amount the torsion head is turned in figure 58, since by keeping H constant in magnitude and direction and turning the torsion head we get the same result. Thus, when D changes, the magnet turns through the angle $\Delta\theta$, and

$$\Delta\theta = f - h \tag{104}$$

and

$$\Delta D = f. \tag{105}$$

Dividing equation (105) by equation (104)

$$\frac{\Delta D}{\Delta\theta} = \frac{f}{f-h}. \tag{106}$$

The term $\frac{f}{f-h}$ is the torsion factor.

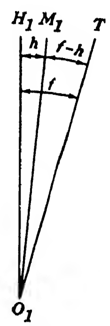

Figure 58.—Torsion factor, measured by turning torsion head.

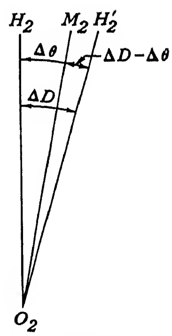

Figure 59.—Effect of fiber torsion on D variometer magnet.

205. If the preliminary optical scale value, ϵ', is near one minute per millimeter, so that h in millimeters on the drum is practically equal to h in minutes of arc, the factor, $\frac{f}{f-h}$, will not change appreciably for relatively small changes in the recording distance. If the preliminary (or final) value of ϵ' differs appreciably from unity, then h must be converted to minutes before calculating the torsion factor since the torsion-head reading will be in minutes.

206. ***Effect of the C field.***—In figure 60:

OH=the horizontal intensity, H;
$OC=HX$, the vector sum, C, of all stray fields parallel to the magnetic meridian OH;
OX=resultant of OH plus OC=original resultant.

Now suppose declination changes:

ΔD=small change in declination;
$HH'=XB$=east field corresponding to ΔD; $(=H\Delta D)$;
OH'=new horizontal intensity, approximately equal to OH in magnitude;
$CB=OH'$ and is parallel to OH';
OB=new resultant=vector sum of OC and OH'
=vector sum of OC and CB;
$\Delta\theta$=angle between OX and OB.

For a torsionless fiber, the D magnet will originally be aligned with OX; when D changes by ΔD, the D magnet will turn through the angle $\Delta\theta$ and into alignment with the new resultant OB. From the figure,

$$\Delta D \approx \frac{XX'}{OX} \tag{107}$$

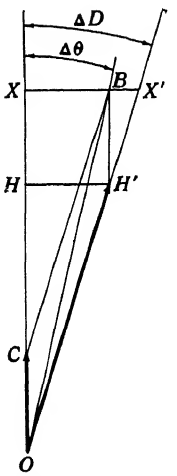

Figure 60.—Effect of the C field on D variometer magnet.

$$\Delta\theta \approx \frac{XB}{OX}. \tag{108}$$

Dividing

$$\frac{\Delta D}{\Delta\theta} = \frac{XX'}{XB} = \frac{XB+BX'}{XB} = 1 + \frac{BX'}{XB}$$

$$= 1 + \frac{BX'}{HH'}. \tag{109}$$

In the similar triangles $H'BX'$ and OHH',

$$\frac{BX'}{HH'} = \frac{H'B}{OH} = \frac{H'B}{H}. \tag{110}$$

Since $H'B=HX=OC=C$ by construction,

$$\frac{BX'}{HH'} = \frac{C}{H}$$

and

$$\frac{\Delta D}{\Delta\theta} = 1 + \frac{C}{H} = \frac{H+C}{H}. \tag{111}$$

207. ***The D scale-value equation.***—Let a small field, ΔE, act on a prefectly oriented D recording magnet, M_s, as in D scale-value deflections, or as in the normal operation of a D variometer, producing a small angular deflection, $\Delta\theta$, of the recording magnet. The couples tending to hold the recording magnet in the magnetic meridian are $(H+C)M_s \sin \Delta\theta$ and $k'\Delta\theta$. The deflecting couple is $(\Delta E)M_s \cos \Delta\theta$. When equilibrium is established these couples are equal and

$$(H+C)M_s \sin \Delta\theta + k'\Delta\theta = (\Delta E)M_s \cos \Delta\theta. \tag{112}$$

But $\sin \Delta\theta \approx \Delta\theta$ (radians) and $\cos \Delta\theta \approx 1$.

Hence, $$(H+C)M_s\Delta\theta + k'\Delta\theta = M_s\Delta E \tag{113}$$

and
$$\frac{\Delta E}{\Delta\theta}=\frac{(H+C)M_s}{M_s}+\frac{k'}{M_s}.$$

Therefore
$$\frac{\Delta E}{\Delta\theta}=H+C+k. \tag{114}$$

From equation (96), $\Delta\theta=\epsilon\,(\Delta n)$. Then also
$$\frac{\Delta E}{\Delta\theta}=\frac{\Delta E}{\epsilon\Delta n}. \tag{115}$$

Transposing, and substituting $(H+C+k)$ for $\frac{\Delta E}{\Delta\theta}$ from equation (115),
$$\frac{\Delta E}{\Delta n}=\epsilon(H+C+k) \tag{116}$$
and
$$S_D{}^{\gamma}=\epsilon(H+C+k) \tag{117}$$
since $\frac{\Delta E}{\Delta n}$ is, by definition, the gamma scale value, $S_D{}^{\gamma}$, of the D variometer. By equation (301), page 231,
$$S_D{}^{\gamma}=S_D'H\tan 1'=\frac{S_D'H}{3438}. \tag{118}$$
Then
$$S_D'=\frac{3438\,S_D{}^{\gamma}}{H}$$
$$=\left(\frac{3438}{H}\right)\epsilon(H+C+k).$$

But $\epsilon=\frac{1}{2R}$, hence,
$$S_D'=\left(\frac{3438}{2R}\right)\left(\frac{H+C+k}{H}\right)$$
and
$$S_D'=\left(\frac{3438}{2R}\right)\left(1+\frac{C}{H}+\frac{k}{H}\right). \tag{119}$$

From equation (101),
$$\frac{k}{H}=\left(\frac{h}{f-h}\right)\left(1+\frac{C}{H}\right).$$

Substituting this value of $\frac{k}{H}$ in equation (119) gives
$$S_D'=\left[\frac{3438}{2R}\right]\left[\left(1+\frac{C}{H}\right)+\left(1+\frac{C}{H}\right)\left(\frac{h}{f-h}\right)\right]. \tag{120}$$

Factoring,
$$S_D'=\left(\frac{3438}{2R}\right)\left(1+\frac{C}{H}\right)\left(1+\frac{h}{f-h}\right) \tag{121}$$
or
$$S_D'=\left(\frac{3438}{2R}\right)\left(\frac{f}{f-h}\right)\left(\frac{H+C}{H}\right). \tag{122}$$

Equation (122) is the usual expression for the minute scale value of the D variometer.

208. ***Other forms of the D scale-value equation.***—From equation (103)

$$\frac{h}{f-h}=\frac{k'}{M_s(H+C)}. \tag{123}$$

Substituting this value of $\frac{h}{f-h}$ in equation (121) gives the D scale value in terms of the torsion constant of the fiber, thus

$$S_D'=\left(\frac{3438}{2R}\right)\left(\frac{H+C}{H}\right)\left(1+\frac{k'}{M_s(H+C)}\right). \tag{124}$$

209. From equation (150), chapter 8, the torsion constant, k', of the fiber is given by,

$$k'=\frac{4\pi^2 I}{T_\sim^2}.$$

Substituting this value of k' in equation (124) gives the D scale value in terms of the period, $T_\sim$, of the fiber and an inertia weight (known value of I) when oscillating as a torsion pendulum. Thus

$$S_D'=\left(\frac{3438}{2R}\right)\left(1+\frac{4\pi^2 I}{T_\sim^2(H+C)M_s}\right)\left(\frac{H+C}{H}\right). \tag{125}$$

210. From equation (148), chapter 8, the torsion constant, k', of the fiber in terms of its dimensions is,

$$k'=\frac{\pi\mu d^4}{32l}.$$

Substituting this value of k' in equation (124) gives the D scale value in terms of the dimensions of the fiber. Thus

$$S_D'=\left(\frac{3438}{2R}\right)\left(1+\frac{\pi\mu d^4}{32l(H+C)M_s}\right)\left(\frac{H+C}{H}\right). \tag{126}$$

211. Equations (121), (122), (124), and (125) are all useful forms for the evaluation of the D scale value when the effective distance, R, is known, and the C field is negligible or small compared to H (no control magnet). Equation (126) has limited usefulness because of the difficulty in measuring the diameter of the quartz fiber to the desired accuracy, as illustrated by example (b) given in paragraph 215.

212. ***Low-sensitivity D variometer (scale value).***—Note that all of the D scale-value equations contain the factor $\frac{H+C}{H}$, indicating that the scale value might be increased indefinitely by simply increasing the field factor, C, by use of a control magnet attached to the D variometer, N end to the north, magnetic axis in the magnetic meridian

through the D recording magnet, and at such a distance as to give the required C field. Suppose a scale value of 5′ per millimeter is desired. To obtain a scale value of this magnitude by use of a control magnet attached to a sensitive variometer, for example where $S_D'=1.00$, it would be necessary to make $C=4H$. If H is 10,000γ, C would have to be around 40,000γ at the center of the D recording magnet. A control magnet having a magnetic moment of 100 cgs units properly placed 8 cm north or south of the D recording magnet would supply the approximate field required, but due to the great uncertainty of distribution at such a short distance, the effective C field would be quite uncertain even with the theoretically correct distribution factor applied. The method is not satisfactory and the use of a larger fiber without control magnet is recommended. Note also that a stronger control magnet at a greater distance, though eliminating some uncertainty in the distribution effect, would be undesirable because of the large fields it would produce at the other variometers of the magnetograph.

213. ***Alignment chart for estimating D scale values; the D nomogram.***—Figure 140, appendix VI, is a graphic solution of equations (124), (125), and (126) for the special case that $2R=3438$ mm; $l=15$ cm; $C=0$; $I=1.83$ gram-cm²; and μ is the rigidity modulus of quartz. For this special case,

$$S_D'=1+\frac{k'}{HM_s} \tag{127}$$

$$=1+\frac{4\pi^2 I}{T_\sim^2 HM_s}$$

$$=1+\frac{72.2}{T_\sim^2 HM_s} \tag{128}$$

$$=1+\frac{\pi\mu d^4}{32 l HM_s}$$

$$=1+\left[1.85\times 10^9\left(\frac{d^4}{HM_s}\right)\right]. \tag{129}$$

214. Note that equivalent values of $T_\sim$, k', and d, are read horizontally on the nomogram. In using the k' scale in conjunction with the S_D' scale and the (HM_s) scale, it is first necessary to find the equivalent diameter (on the d scale) or the equivalent period (on the $T_\sim$ scale). This is illustrated in example (a) of paragraph 215.

215. ***Use of the D nomogram.***—The following examples serve to illustrate the use of the D nomogram. (Quartz fibers only.)

(a). Given: $HM_s=1.85$ dyne cm;
$S_D'=1.01$ minutes per mm;
$2R=3438$ mm;
$C=0$;
$l=15.0$ cm;
$I=1.83$ gram-cm².

Required: d, k' and $T_\sim$.

Solution: The straight line passing through $S_D'=1.01$ and $HM_s=1.85$ intersects the d scale at 17.7 microns (0.00177 cm) and the $T_\sim$ scale at 61.4 seconds. A horizontal line through 17.7 on the

d scale intersects the k' scale at 0.0185 dyne-cm per radian. Therefore,

$d=17.7$ microns; $T_\sim=61.4$ sec.; and $k'=0.0185$.

(b). Given: $l=15$ cm;
$I=1.83$ gram-cm^2;
$d=17.7$ microns ($\pm 20\%$) measured.
Required: k' and $T_\sim$.
Solution: $17.7\pm 20\%=17.7\pm 3.5$ microns.

This means that d lies somewhere between 14.2 and 21.2 microns Opposite $d=14.2$, read $T_\sim=100$ seconds and $k'=0.007$. Opposite $d=21.2$, read $T_\sim=45$ seconds and $k'=0.038$. Thus if a torsion test were made with this fiber the value of k' might turn out to be anything between 0.007 and 0.038. If the period were determined with the inertia weight, $I=1.83$, it might have any value from about 45 seconds to 100 seconds. This example illustrates why it is impractical to rely on the measured diameter of the quartz fiber as a measure of its torsion constant or its period, $T_\sim$, as described above.

(c). Given: $2R=3438$ mm;
$C=0$;
$I=1.83$ gram-cm^2;
$l=15$ cm;
$T_\sim=4.6$ seconds;
$H=9000$ gammas ($=0.090$ cgs);
$S_D'=5.0$.
Required: Magnetic moment, M_s, of the recording magnet, to give a scale value of 5.0 minutes per millimeter.
Solution: The straight line through $S_D'=5.0$ and $T_\sim=4.6$, intersects the HM_s scale at 0.83 dyne-cm. Therefore if $H=0.090$,

$$M_s=\frac{HM_s}{H}=\frac{0.83}{0.090}=9.2 \text{ cgs, the required magnetic moment.}$$

216. ***Evaluation of the recording distance for unit scale value.***—From equation (122), for $S_D'=$one minute per mm, we have

$$R=1719\left(\frac{f}{f-h}\right)\left(\frac{H+C}{H}\right) \tag{130}$$

and from equation (91),

$$L=R+\frac{c}{3}-bl. \tag{131}$$

Then S_D' will be one minute per millimeter when

$$L=1719\left(\frac{f}{f-h}\right)\left(\frac{H+C}{H}\right)+\frac{c}{3}-bl. \tag{132}$$

For example: Let $C=+100$ gammas;

$$H\approx 20{,}000 \text{ gammas};$$

$$\frac{f}{f-h}=1.006;$$

$$b=\frac{2}{3};$$

$$l=3 \text{ mm};$$

and $c=9$ mm.

Then from equation (132),

$$\begin{aligned} L&=1719\ (1.006)\ (1.005)+3.0-2.0\\ &=1739 \text{ mm.}\end{aligned}$$

In the above arrangement, if the L distance (measured distance from the front of the D lens to the drum) is made equal to 1739 mm, the D scale value will be one minute per mm.

217. Since it is desirable to operate at a scale value near unity, the focal length of the D lens should be such that the product of the three factors on the right hand side of equation (122) shall be unity when the D spot is in sharp focus on the recording paper and when the D ordinate is about normal. It is good practice to specify a focal length of 173 to 174 cm for the focal length of the D lens. If it happens that the derived value of the distance, L, differs from the focal length by as much as one centimeter, the variometer or the recorder (preferably the recorder) may be adjusted to the correct distance for unit scale value without appreciable impairment of the character of the D spot, since the depth of focus of a well-diaphragmed D lens of this focal length may be as much as 3 or 4 cm.

CHAPTER 8. THE HORIZONTAL-INTENSITY VARIOMETER

218. ***Function of an H variometer.***—This instrument should indicate visually or by photographic registration the variations in the horizontal component of the intensity of the earth's magnetic field.

219. ***Basic requirements.***—The essentials of the instrument are:

(a) A permanent magnet supported by a bifilar or relatively coarse quartz unifilar suspension so that its magnetic axis is horizontal and is kept approximately in the magnetic prime vertical (see par. 39) by the twisted suspension;

(b) A nonmagnetic housing with suspension tube and torsion head for regulating the torsion in the suspension;

(c) An optical system for visual observations or for photographic registration;

(d) A copper or silver box surrounding the magnet to provide necessary damping;

(e) A temperature compensation device.

Note: See pages 116 and 124-125, chapter 10, for temperature compensation of the H variometer, and page 135, chapter 12, for the effects of an exorientation angle.

220. ***Operating principle.***—To place the variometer in operation, the torsion head is turned until the mechanical couple of the fiber is just sufficient to turn the magnet into a position at right angles to the magnetic meridian through the variometer. In this position the mechanical couple of the fiber is equal and opposite to the magnetic couple or restoring torque which tends to turn the magnet back into the meridian. Any slight change in H results in unbalanced couples, and the magnet turns in azimuth until the torque applied through the fiber again balances the magnetic couple. It is this motion of the magnet that is observed and recorded.

221. ***Symbols.***—The following notation will be used:

τ = the total torsion in the fiber, expressed in radians;

θ = angle between the magnetic axis of the recording magnet and the magnetic meridian;

k' = torsion constant of the fiber in dyne-centimeters per radian twist;

$k'\tau$ = total couple applied to the fiber in dyne-centimeters;

H = horizontal intensity;

M_s = magnetic moment of the recording magnet, cgs units;

ϵ = optical scale value $= \frac{1}{2R}$, in radians per mm;

R = effective distance from variometer lens to drum;

f_E = the magnetic east field at center of H variometer due to all magnets of the magnetograph, except the H recording magnet;

p =component of the earth's field along M_s when the recording magnet is not precisely in the prime vertical; plus the sum of the components of the fields of all other magnets, including a sensitivity-control magnet, parallel to M_s; p is positive when it is directed from the south to the north pole of M_s;

$\Delta\theta$ =angular displacement of the recording magnet for a change ΔH;

ΔH=small change of H;

n =ordinate in mm of the H spot measured from a fixed line (the H base line) on the gram;

Δn =small change in the ordinate, n;

k =ratio of torsion constant of the fiber to magnetic moment of suspended magnet$=\frac{k'}{M_s}$;

E_x =exorientation angle; the small angle between M_s and the magnetic prime vertical. For an H recording magnet with N end east, E_x is positive when M_s is turned clockwise, looking down, through a small angle from the prime vertical.

222. ***The H scale value.***—When there is no torsion in the fiber and the suspended magnet, M_s, is not acted upon by the fields of any other magnets, M_s will come to rest with its magnetic axis coincident with the magnetic meridian, ON, figure 61. This is called the initial line of no torsion, and the torsion-head reading establishes the direction of this line. Now turn the torsion head through the angle NOB so that M_s is precisely at right angles to the magnetic meridian, that is, so that θ is 90°. OB is the new line of no torsion, that is, the new direction the magnet would take if all fields were removed. The mechanical couple in the fiber is now $k'\tau$, τ being the difference between the angle NOB through which the upper end of the fiber turns and the angle NOE, the angle through which the lower end turns.

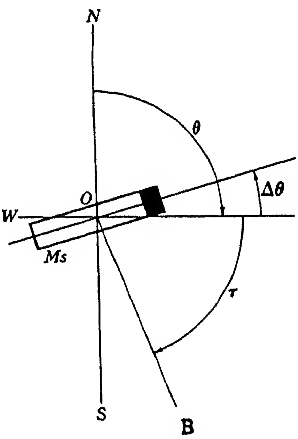

Figure 61.—Simple H variometer.

223. The mechanical couple $k'\tau$ tends to turn the magnet clockwise out of the magnetic prime vertical. The magnetic couple $HM_s \sin\theta$ tends to turn the magnet counterclockwise out of the magnetic prime vertical. Then since equilibrium has been established the couples are equal and

$$HM_s \sin\theta = k'\tau \tag{133}$$

and since $\theta=90°$,

$$HM_s = k'\tau. \tag{134}$$

224. Now let H increase by a small increment ΔH. This will cause the recording magnet to turn through a small angle $\Delta\theta$, so that,

$$(H+\Delta H)\ M_s \sin\ (90°-\Delta\theta) = k'\ (\tau+\Delta\theta). \tag{135}$$

225. If there is a fixed field, f_E, say the field of a sensitivity-control magnet, directed along the prime vertical, there will be an additional couple, $f_E M_s \sin \Delta\theta$, supplementing the couple of the fiber, and equation (135) becomes,

$$(H+\Delta H)M_s \sin (90°-\Delta\theta)=f_E M_s \sin \Delta\theta+k'(\tau+\Delta\theta). \tag{136}$$

226. When $\Delta\theta$ is small, $\sin \Delta\theta \approx \Delta\theta$; $\cos \Delta\theta \approx 1$; $\sin (90°-\Delta\theta)= \cos \Delta\theta \approx 1$; and $\cos (90°-\Delta\theta)=\sin \Delta\theta \approx \Delta\theta$; then

$$HM_s+M_s\Delta H \approx f_E M_s \Delta\theta+k'\tau+k'\Delta\theta. \tag{137}$$

Subtracting (134) from (137)

$$M_s\Delta H \approx f_E M_s \Delta\theta+k'\Delta\theta. \tag{138}$$

Transposing,

$$\frac{\Delta H}{\Delta\theta} \approx \frac{k'}{M_s}+f_E. \tag{139}$$

227. Let $\frac{k'}{M_s}=k$. (It will be shown in paragraph 232 that k may be taken as an equivalent field, expressible in gammas.) [1] Substituting in equation (139) gives

$$\frac{\Delta H}{\Delta\theta} \approx k+f_E. \tag{140}$$

228. The ratio, $\frac{\Delta H}{\Delta\theta}$, represents the rate of change of H with respect to unit angular displacement of the recording magnet and is called the *scale value* of the variometer. Equation (140) gives the approximate scale value in cgs units per radian. In practice the scale value is expressed as the change in H, in gammas, when the H spot moves one millimeter on the magnetogram. If Δn is a small change in the ordinate and R is the effective recording distance in the recording system,

$$\frac{\Delta n}{2R}=\Delta\theta=\epsilon\Delta n \tag{141}$$

and

$$\epsilon=\frac{\Delta\theta}{\Delta n} \tag{142}$$

where ϵ is the optical scale value in radians per millimeter. Multiplying the left-hand side of equation (140) by $\frac{\Delta\theta}{\Delta n}$ and the right-hand side by ϵ, we have

$$\frac{\Delta H}{\Delta\theta}\times\frac{\Delta\theta}{\Delta n}=\epsilon(k+f_E), \text{ cgs units per mm} \tag{143}$$

and the H scale value, S_H, in gammas per millimeter, is

$$S_H=\frac{\Delta H}{\Delta n}=\epsilon(k+f_E)\times10^5 \tag{144}$$

in which k and f_E are in cgs units.

[1] Ad. Schmidt, Ergebnisse der magnetischen Beobachtungen in Potsdam und Seddin im Jahre 1908, Veröff. des Preuss. Met. Inst., Berlin, p. 39.

229. A detailed analysis [2] shows that it is more accurate to use, instead of f_E, the field p which is the component of all fields along M_s. Thus

$$S_H=\epsilon(k+p)\times 10^5. \tag{145}$$

The chief difference between p and f_E is the component, along M_s, of all north or south magnetic fields. This component arises when M_s is not exactly in the magnetic prime vertical and gives rise to the a factor (change of H scale value with change of H ordinate, paragraphs 254–260), and to the b factor (change of H scale value with change of declination, paragraphs 261 and 262).

230. Equation (144) is useful in calculating the effect of k and f_E on the H scale value.

231. ***Significance of k.***—From equation (134) we have

$$\frac{H}{\tau}=\frac{k'}{M_s}=k$$

$$k=\frac{k'}{M_s}=\frac{\text{dyne-cm/radian}}{\text{dyne-cm/oersted}}=\frac{\text{oersted}}{\text{radian}} \tag{146}$$

and since angular values are dimensionless numerics, the dimensions of k are those of field intensity.

232. Also, let M_s be the recording magnet, precisely oriented in the magnetic prime vertical, north end east. In this position the mechanical couple is just balanced by the magnetic couple (equation 134). Suppose that both of these couples are reduced to zero but M_s remains suspended by a (hypothetical) torsionless fiber and free to turn in any direction in a horizontal plane. It would then take up the direction of *any* applied horizontal field of *any magnitude*. To maintain M_s in the prime vertical or to turn it into the prime vertical (if displaced) would require an *east* field directed along the prime vertical through the center of M_s. Suppose the system had a scale value of 3.00 γ per mm operating at a recording distance of $R=1180$ mm (optical scale value, $\epsilon=\frac{1}{2R}=0.000424$). Then from equation (144), since $k=0$,

$$f_E=\frac{S_H}{\epsilon\times 10^5}=S_H\times\frac{2R}{10^5}$$

and $$f_E=\left(3.00\ \frac{\text{gamma}}{\text{mm}}\right)(2360\text{ mm})\left(0.00001\ \frac{\text{oersted}}{\text{gamma}}\right)$$

$$f_E=0.07080\text{ oersted}=7080\gamma.$$

In other words, if $k'=0$ ($k=0$), f_E would have to be 7080γ to produce the same scale value as if k were acting alone without f_E. Again, if M_s is acted upon only by the field, f_E, and a field, x, of 3γ be applied normal to M_s, the latter will be deflected (take up the direction of the resultant field) through a small angle, $\Delta\theta$, such that,

$$\tan\Delta\theta=\frac{X}{f_E}=\frac{3}{7080}=0.000424. \tag{147}$$

[2] H. H. Howe, On the theory of the unifilar variometer, Terr. Mag., 42, 29–42, 1937, (see item 6 of bibliography).

But $\Delta\theta = \frac{\Delta n}{2R}$ and $\Delta n = 2R\Delta\theta$, in which Δn is the motion of the H spot on the gram when $2R = 2360$ mm. Then $\Delta n = 2360 \times 0.000424 = 1$ mm. This means that a field of 7080γ applied parallel to M_s (no other fields acting, and a torsionless fiber) produces the same scale value for M_s as we had in the original system when $f_E = 0$ and only $\frac{k'}{M_s}$ $(=k)$ determined the magnitude of the scale value. We can state then, that k is equivalent to the field which will produce the same scale value, and that k has the same dimensions as f_E, that is, k may be regarded as an equivalent field expressible in gammas.

233. ***Evaluation of k', k, and other constants.***—In the preceding paragraphs it has been assumed that M_s, k', and f_E remain constant. In practice all of these factors change with time and/or temperature. It would not be feasible to determine the values of all of these factors under changing conditions every time the H scale value is determined. (Practical methods for determining the H scale value are given in chapter 11.) However, much guesswork may be eliminated in preparing a variometer for routine operation, if one has a good understanding of the meaning of the factors which make up equation (144).

234. Since each of the terms ϵ, k', M_s, and f_E may vary continuously over wide ranges, there will be an infinite number of combinations of these terms which will give the same scale value. However, various practical considerations make it necessary to keep the ranges within fairly narrow limits.

235. The value of k' may be estimated from fiber dimensions (paragraph 238), from oscillations (paragraph 239), from torsion observations (paragraph 240), or from the H scale value (paragraph 242).

236. The magnetic moment M_s can be determined directly by deflections (par. 319, p. 129).

237. The term f_E can be estimated if the magnetic moments and relative positions of all nearby magnets are known. Usually f_E is negligible for first-order effects unless there is a sensitivity-control magnet on the variometer.

238. ***The quartz fiber and the torsion constant, k'.***—The torsion constant, k', of the fiber is the torque in dyne-centimeters required to turn one end of the fiber through an angle of one radian while the other end remains fixed. It may be calculated from Coulomb's equation,[3]

$$k' = \frac{\pi \mu d^4}{32 l} \tag{148}$$

in which EXAMPLE

μ = the rigidity modulus of quartz,	2.83×10^{11} dyne/cm²/radian
d = the diameter of the fiber,	0.00372 cm
l = the length of the fiber,	15.0 cm
k' = from equation (148),	0.354 dyne-cm/radian.

Since the diameter enters as the fourth power in equation (148), it should be measured with great precision by a suitable microscope at

[3] Max Planck, The Mechanics of Deformable Bodies. London, 1932, p. 77.

many equally spaced places along the fiber. The average of the fourth powers of d rather than the fourth power of the average should be used.

239. ***Torsion constant by oscillations.***—(Preferred method). The period, $T_\sim$, of a torsion pendulum oscillating under the directive force of the suspension is given by

$$T_\sim = 2\pi\sqrt{\frac{I}{k'}} \tag{149}$$

and

$$k' = \frac{4\pi^2 I}{T_\sim^2}, \tag{150}$$

where

	EXAMPLE
I = the moment of inertia of a suspended brass cylinder,	1.83 gram-cm^2
$T_\sim$ = the period of the system,	14.3 sec.
k' = torsion constant, by equation (150),	0.35 dyne-cm.

The moment of inertia of the cylinder is calculated from its dimensions and mass using equation (45), page 28. Length of cylinder: 4.94 cm; diameter: 0.166 cm; mass: 0.90 gram. The period is estimated by timing 50 to 100 oscillations with a stop watch.

240. ***Torsion constant by torsion observations.***—The value of k' may also be determined by torsion observations as in a D variometer. From equation (99),

$$k' = (H+C)M_s\left(\frac{h}{f-h}\right). \tag{151}$$

The values of h and f are observed by the method described in paragraph 203, page 76. Then, knowing $H+C$ and M_s, k' may be calculated. Table 5 gives some values of k' determined from torsion observations, from oscillations with known moment of inertia, and from the dimensions of the fiber, using equations (148) (150), and (151).

241. ***Evaluation of k from k' and M_s.***—Let $k'=0.35$ and $M_s=5.0$. Then from equation (146)

$$k = \frac{0.35 \text{ dyne-cm}}{5.0 \text{ dyne-cm/oersted}} = 0.07 \text{ oersted}. \tag{152}$$

242. ***Evaluation of k from scale-value observations and optical lever.***—Suppose that the H scale value has been determined in the usual manner by deflections as explained in chapter 11, and that $S_H = 2.97\gamma/\text{mm} = 0.0000297$ cgs/mm, when $\epsilon = 0.000424$ and there is no control magnet (no f_K field). Then from equation (144)

$$k = \frac{S_H}{\epsilon} \tag{153}$$

$$= \frac{0.0000297}{0.000424}$$

$$= 0.07 \text{ cgs} = 7000\gamma.$$

243. If M_s is known, then k' may be evaluated from the relation, $k=\frac{k'}{M_s}$. Suppose $M_s=5.0$, then

$$k'=kM_s=0.07\times 5=0.35 \text{ dyne-cm.} \tag{154}$$

TABLE 5.—*Torsion constant, k′, of quartz fibers by different methods.*

Place: Cheltenham Magnetic Observatory — Date: July 31, 1951
Horizontal Intensity: 0.183 — Observer: J. B. Townshend

Symbol	Quantity	Fiber No. 10	Fiber No. 36
d	Diameter .... cm..	0. 0018	0. 00655
l	Length .... cm..	15. 4	15. 6
	Oscillations:		
I	Moment of inertia of weight ...... gm-cm². .	1. 83	5. 798
$T_\sim$	Period with this inertia weight .... sec..	47. 6	5. 26
	Torsion observations:		
f	Angular motion of torsion head	1800′	100′
h	Resulting angular displacement of magnet. .	24.′9	88.′2
$f-h$	Twist in the fiber	1775.′1	11.′8
$\frac{h}{f-h}$	Displacement per unit twist	0. 014	7. 48
M_s	Magnetic moment of suspended magnet cgs-emu..	12. 0	5. 8
$(H+C)M_s$	Magnetic couple per unit displacement dyne-cm/radian..	2. 20	1. 06
k'	Torsion constant, computed:		
	From oscillations .... dyne-cm/radian..	0. 0319	9. 26
	From torsion observations..dyne-cm/radian..	0. 0308	7. 93
	From dimensions .... dyne-cm/radian..	0. 0190*	3. 28*

*Computed k' is sensitive to small error in diameter (d).
Example: If error in d is 10%, error in k' will be approximately 46%; if error in d is 20%, error in k' will be approximately 107%. For this reason, torsion constants computed from dimensions and assumed rigidity modulus are not reliable and may be used only for rough estimates in preliminary tests.

244. ***Scale value from constants.***—Let $2R=2360$ mm ($\epsilon=0.000424$), $M_s=5.0$, $k'=0.35$ ($k=0.07$), and $f_E=0$. Then from equation (144), the H scale value is given by

$$S_H=\left(\frac{0.000424}{\text{mm}}\right)(0.07 \text{ oersted})\left(\frac{10^5 \text{ gammas}}{\text{oersted}}\right) \tag{155}$$

$$=2.97\gamma/\text{mm}.$$

245. Thus, it is possible by careful selection of quartz fiber and recording magnet to assemble a system which will have a desired scale value at a given recording distance.

246. ***The sensitivity-control magnet (scale-value control).***—In figure 62 let M_s be the recording magnet of the H variometer, operating in the magnetic prime vertical with its N end east. M_a is a sensitivity-control magnet mounted on the variometer with its

axis in the magnetic prime vertical, N end east, and at such a distance from M_s that its field, f_E, at the center of M_s is equal to 5000γ (0.05 cgs units). Then the H scale value, from equation (144) will be

$$S_H=0.000424(0.07+0.05)\cdot10^5=5.09\gamma/\text{mm}. \tag{156}$$

If the control magnet is reversed, the f_E field is negative, and

$$S_H=0.000424(0.07-0.05)\cdot10^5=0.85\gamma/\text{mm}. \tag{157}$$

It is evident from the above that the H scale value may be adjusted over a wide range by simply adjusting the control magnet along its bar. The approximate field of a bar magnet, along its magnetic axis extended, at a distance r from its center is

$$f_E=\frac{2M_a}{r^3}\times10^5\gamma \tag{158}$$

in which f_E is the field, and M_a the magnetic moment, of the control magnet. If $M_a=100$ cgs and $r=10.0$ cm, then $f_E=20{,}000\gamma$ at 10 cm. The change in f_E as r is changed is given by

$$\frac{df_E}{dr}=-\frac{3f_E}{r} \tag{159}$$

and

$$df_E=-\frac{3f_E}{r}\,dr. \tag{160}$$

For $\Delta r=-0.1$ mm ($=-0.01$ cm)

$$\Delta f_E=\frac{-3\times20{,}000\times-0.01}{10}=+60\gamma. \tag{161}$$

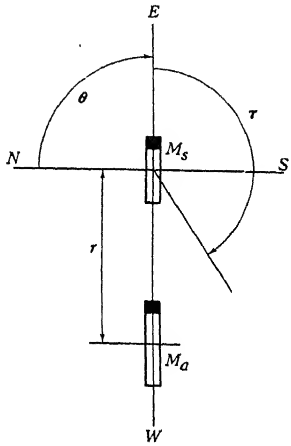

Figure 62.—H variometer with sensitivity-control magnet.

Disregarding distribution effects, this means that by reducing r by 0.1 mm the effective field at the center of the recording magnet (due to M_a) has been increased by 60γ. The nomogram of figure 138 gives the same result.

247. ***Variation of the scale value with k and f_E.***—From equation (144)

$$S_H=k\epsilon+\epsilon f_E. \tag{162}$$

The relation between changes of S_H and f_E, when k is constant, is

$$\frac{dS_H}{df_E}=\epsilon, \tag{163}$$

and when f_E is constant

$$\frac{dS_H}{dk}=\epsilon, \tag{164}$$

and when both f_E and k change,

$$dS_H=\epsilon df_E+\epsilon dk. \tag{165}$$

Equations (160) and (163) will be found quite useful for evaluating the distance at which the sensitivity magnet must be placed in order to change the scale value by a desired amount. For example: Let $S_H=5.00$; $\epsilon=0.000424$; $M_a=100$; and $r=10$ cm. How far should M_a be moved in order to change S_H from 5.00 to 5.10? In this case,

$\Delta S_H=0.10\gamma/\text{mm}$, $\Delta k=0$, and from equation (163)

$$\Delta f_E=\frac{\Delta S_H}{\epsilon}=\Delta S_H\times 2R=0.1\times 2360=236\gamma. \tag{166}$$

An additional *east* field, Δf_E, of 236γ will increase the scale value from 5.00 to 5.10γ/mm. To produce this change in the f_E field at the center of M_s, find Δr by applying equation (160),

$$\Delta r=-r\left(\frac{\Delta f_E}{3f_E}\right)=-\frac{10\times 236}{3\times 20{,}000} \tag{167}$$

$$=-0.039 \text{ cm}=-0.4 \text{ mm, approximately.}$$

248. Again, what f_E field would be required to change S_H by just one gamma per millimeter, say from 5.49 to 4.49 by means of a control magnet, and at what distance should the control magnet, $M_a=100$, be placed to accomplish this change? From equation (163)

$$df_E=\frac{dS_H}{\epsilon}=-\frac{1.0}{0.000424}=-0.02360 \text{ cgs}=-2360\gamma.$$

That is, a field of -2360γ applied at the center of M_s and directed to magnetic west would reduce S_H from 5.49 to 4.49. From equation

(158), $r^3=\frac{2M}{f_E}=\frac{200}{0.02360}=8474$ cm^3, and $r=20.39$ cm, the distance at

which M_a should be placed to produce the change of one gamma per millimeter in the scale value. If M_a is reversed it will increase the scale value from 5.49 to 6.49.

249. ***Use of two control magnets.***—The distance between the control magnet and the recording magnet may change slightly as a result of seasonal tilting of the pier (causing the suspended magnet to swing toward or away from the control magnet). This would cause a seasonal change in the scale value. The effect may be compensated in part by use of two control magnets symmetrically placed in the prime vertical, east and west of the variometer. (A second control magnet on the H variometer in the Eschenhagen type of magnetograph might have an undesirable effect on the D recording magnet. Its net effect would depend upon the direction and intensity of the other east or west fields at the center of the D magnet.)

250. Table 6 gives various values of S_H, determined experimentally for various positions of a control magnet having a magnetic moment of 50.2. The results are shown graphically in figure 63.

TABLE 6.—*Variation of H scale value with distance between control magnet and recording magnet.*

[$2R$=2360 mm; ϵ=0.000424; M_a=50.2; N-end of recording magnet east.]

Observed scale value S_H	Distance, r, between M_a and M_s	Control magnet, N-end	k	f_E	$k+f_E$
γ/mm	cm		γ	γ	γ
2.29	11	W	12950	−7540	5410
3.03	12	W	12950	−5810	7140
3.55	13	W	12950	−4570	8380
3.94	14	W	12950	−3660	9290
5.49	Away	Away	12950	0	12950
7.04	14	E	12950	+3660	16610
7.43	13	E	12950	+4570	17520
7.95	12	E	12950	+5810	18760
8.69	11	E	12950	+7540	20490

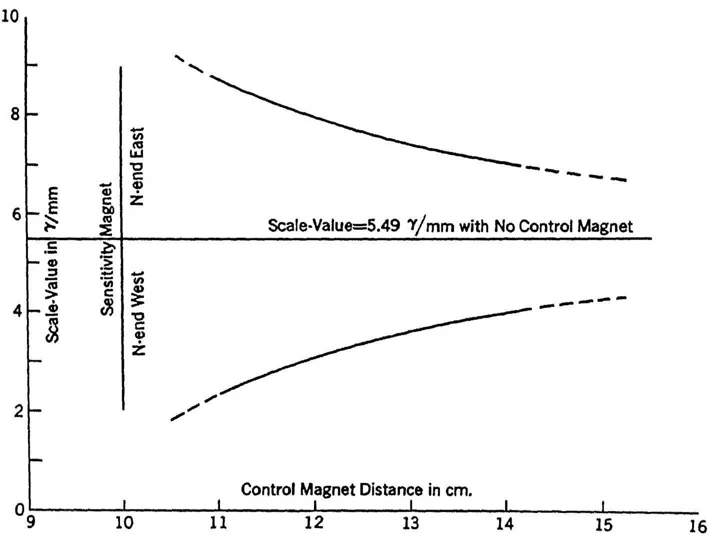

Figure 63.—Variation of scale value with control magnet distance.

251. ***Alignment chart for estimating H scale values; the H nomogram.***—Figure 141, appendix VI, may assist in attaining a particular scale value. It is based on the equation

$$S_H=\epsilon\left(\frac{k'}{M_s}+f_E\right) \tag{168}$$

with k' represented by its equivalent in terms of d (eq. 148) or of $T_\sim$ (eq. 150). When a straightedge is laid across this nomogram, the left-hand scale should be taken to represent k alone. A reading so made on the adjacent S_H scale is valid only if $f_E=0$. However, when $f_E\neq0$ the S_H scale is useful in another way. Any displacement along this scale corresponds rigorously with the identical displacement along the $(k+f_E)$ scale, permitting additive correction for f_E when necessary. Special conditions assumed in the construction of this nomogram are: $2R=2360$ mm ($\epsilon=0.000424$); $l=15$ cm; $I=1.83$ gram-cm^2; and

$\mu = 2.83 \times 10^{11}$ (rigidity modulus of quartz). For this special case, from equation (144),

$$S_H = \epsilon(k + f_E) \times 10^5$$

$$= (k + f_E)(0.000424) \times 10^5$$

$$= 42.4(k + f_E).$$

When $k + f_E = 1$ cgs unit, $S_H = 42.4\gamma$/mm. The S_H scale is placed so that 42.4 on the S_H scale is opposite 1.0 on the $(k + f_E)$ scale. For any other value of ϵ, the position of the S_H scale should be changed accordingly.

252. Note that equivalent values of $T_\sim$, k', and d are read horizontally. In using the k' scale in conjunction with the M_s and S_H scales, it is necessary first to find the equivalent diameter, d, (on the d scale) or the equivalent period, $T_\sim$ (on the $T_\sim$ scale).

253. ***Examples of the use of the H nomogram.***—The following examples serve to illustrate the use of the H nomogram:

(a) Given: $M_s = 5.0$ cgs units;

$2R = 2360$ mm;

$\epsilon = \dfrac{1}{2R} = 0.000424$;

$l = 15$ cm;

$S_H = 3.0\gamma$/mm;

$f_E = 0$.

Required: d, diameter of the quartz fiber;
$T_\sim$, period when $I = 1.83$ gram-cm^2;
k', torsion constant of fiber.

Solution: The straight line from 5.0 on the M_s scale to 3.0 on the S_H scale intersects the d scale at 37.1 microns and the period scale at 14.3 seconds. That is, a recording magnet having a magnetic moment of 5.0 when suspended on a 37.1 micron fiber of uniform diameter throughout its entire 15 cm length and operating precisely in the magnetic prime vertical would have a scale value of 3.0 γ/mm. The horizontal line through $d = 37.1$ microns ($T_\sim = 14.3$ seconds) intersects the k' scale at 0.35 dyne-cm per radian. This is the torsion constant of the fiber.

(b) Given: $T_\sim = 10.1$ sec for $I = 1.83$ gram-cm^2;

$l = 15$ cm;

$2R = 2360$ mm;

$\epsilon = \dfrac{1}{2R} = 0.000424$;

$f_E = 0$.

Required: Value of magnetic moment, M_s, of recording magnet to give a scale value, $S_H=3.0$ γ/mm.

Solution: The straight line from $S_H=3.0$ on the S_H scale, through 10.1 on the $T_\sim$ scale ($d=49$ microns) intersects the M_s scale at 10.0. That is, if $M_s=10.0$ the scale value will be 3.0 for the given conditions.

(c) Given: $S_H=6.0$ γ/mm;

$$2R=2360 \text{ mm};$$

$$\epsilon=\frac{1}{2R}=0.000424;$$

$$f_E=0.$$

Required: $S_H=3.0$ γ/mm;

f_E=sensitivity-control-magnet field (E-W field at center of M_s) to reduce S_H from 6.0 to 3.0.

Solution: On the $(k+f_E)$ scale opposite $S_H=6.0$ find $k+f_E=0.141$ (14,100γ), but $f_E=0$, so $k=0.141$; and on the same scale find $k+f_E=0.071$ (7100γ) opposite $S_H=3.0$ γ/mm. Then $f_E=(k+f_E)-k=0.071-0.141=-0.070=-7000\gamma$, the required f_E field. The negative sign means that the field must be directed toward magnetic west. This required field may be supplied by a small control magnet, M_a, having a magnetic moment of, say, 80 cgs, fixed on the variometer deflection bar at a distance $r=13.2$ cm (center of M_a to center of M_s) in the magnetic prime vertical, east or west of M_s, with N end west. In this case, from equation (144)

$$S_H=42.4(k+f_E)$$

$$=42.4\times0.071$$

$$=3.00\gamma/\text{mm}.$$

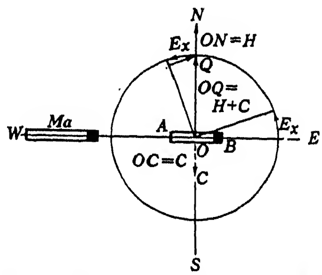

Figure 64.—The a factor in the H scale value.

254. ***Variation of H scale value with ordinate; the a factor.***—In figure 64, AOB represents the recording magnet of the H variometer. It is held in the magnetic prime vertical by the couple of the suspension, N end to the east. M_a is a control magnet, N end east, so that its field is directed along magnetic east. A magnetic field in the magnetic meridian is produced by other magnets of the magnetograph and is denoted by C. The C field is primarily a temperature-compensating field directed to magnetic south and approximately equal to half of H.

255. When H changes, the recording magnet turns through angle E_x. The field, f_E, of the control magnet along M_s is now $f_E \cos E_x$. But there is also a component of $(H+C)$ parallel to the recording magnet *when the magnet turns out of the prime vertical.* This component is $(H+C)\sin E_x$. Then the total field parallel to the recording magnet is

$$p=f_E\cos E_x+(H+C)\sin E_x \tag{169}$$

so that equation (145) becomes

$$S_H=\epsilon\,[k+f_E\cos E_x+(H+C)\sin E_x]\times10^5. \tag{170}$$

For angles of E_x up to about 3°, cos E_x may be taken as unity without introducing an appreciable error in S_H, but $(H+C)$ sin E_x is appreciable. For example: Let $H=27{,}000\gamma$; $\epsilon=0.000424$ ($2R=2360$ mm); $k=12{,}950$; $f_E=6{,}820\gamma$; $C=-13{,}500\gamma$. When $E_x=0$,

$$S_H=0.000424\ (12950+6820)=8.383\ \gamma/\text{mm}.$$

Now let H increase so that $E_x=2°$, figure 64. The term $(H+C)$ sin E_x must be added to the scale value. The scale value is now

$$\begin{aligned}S_H&=0.000424\ (12950+6820\cos 2°+13500\sin 2°)\\&=0.000424\ (12950+6816+471)\\&=8.581\gamma/\text{mm}.\end{aligned}$$

The difference in the scale value is $\Delta S_H=8.581-8.383=+0.198\gamma/\text{mm}$. But 2° angular motion of the recording magnet=4° motion of the spot, and the change in ordinate is

$$\Delta n=1180\tan 4°=82.5\text{ mm}$$

and the change in H scale value per millimeter change in ordinate is

$$\frac{\Delta S_H}{\Delta n}=+\frac{0.198}{82.5}=0.00240\gamma/\text{mm/mm}.$$

This factor, 0.00240, is called the *a factor* of the scale value. It means that the H scale value increases with ordinate; the a factor is primarily a function of $(H+C)$ and is proportional to $(H+C)$. When $(H+C)$ is reduced, from H to $H+C\ (\approx\frac{1}{2}H)$, as in magnetic temperature compensation, the a factor is reduced in the same proportion.

256. Thus it is seen that as the H ordinate, n, changes, the corresponding rotation of the recording magnet and change of E_x change the H scale value through the $(H+C)$ sin E_x term of equation (170). The rate of change, $\frac{\Delta S_H}{\Delta n}$, is the a factor, so that if S_0 is the base-line scale value (value of S_H when n is zero) then

$$S_H=S_0+an. \tag{171}$$

257. It has been shown[4] that the a factor is given approximately by

$$a\approx(H+C)\,\epsilon^2. \tag{172}$$

In magnetic temperature compensation (see par. 297, p. 115), C is approximately $-\frac{1}{2}H$, $H+C\approx\frac{1}{2}H$, and

$$a\approx\tfrac{1}{2}H\epsilon^2. \tag{173}$$

258. Table 7 gives the a factors computed from equation (173) (using $C=-\frac{1}{2}H$), and the observed a factors, for the H variometers at four Coast and Geodetic Survey magnetic observatories.

[4] H. H. Howe, *op. cit.*, p. 36 (see item 6 of bibliography).

TABLE 7.—*Computed and observed a factors at U. S. Coast and Geodetic Survey observatories*

	Observatory			
	Honolulu	San Juan	Sitka	Tucson
Recording distance, R	1126 mm	1144 mm	1127 mm	1215 mm
$\epsilon=\frac{1}{2R}$	0. 000444	0. 000437	0. 000444	0. 000412
H	28620	27470	15500	26020
$\frac{H}{2}$	14310	13735	7750	13010
$a=\frac{1}{2}H\epsilon^2$	0. 0028	0. 0026	0. 0015	0. 0022
a (observed)	0. 0028	0. 0020	0. 0020	0. 0024

259. ***Use of the a factor.***—The base-line scale value, S_0, and the point scale value, S_H, at ordinate n, are connected by the relation

$$S_H=S_0+an. \tag{174}$$

To obtain the value of the ordinate in gammas from the ordinate, n, in millimeters, multiply the ordinate in mm by the average of the scale values at zero ordinate and at the ordinate n. This average scale value is given by

$$\overline{S_H}=S_0+\tfrac{1}{2}\,an \tag{175}$$

in which

$\overline{S_H}$ = the scale value at the ordinate $\frac{n}{2}$;

S_0 = the base-line scale value.

260. To find the scale value for zero ordinate or for a particular ordinate when it is known at another ordinate, use equation (174). Note that a scale value to be used for conversion of ordinates is based on equation (175) but that the scale value applicable to small changes at a particular ordinate is based on equation (174).

261. ***Correction to H scale value for change in declination.***—For greater precision in calculating the H scale value, a correction for change in declination is sometimes applied. By taking the differential of equation (145), with k constant,

$$\Delta S_H=\epsilon\Delta p. \tag{176}$$

Suppose $\theta=90°$ and D changes but H does not change, figure 65. There will now be a component of H parallel to M_s which did not exist when M_s was normal to H. This parallel component is, from figure 65,

$$H\sin\Delta D=\Delta p. \tag{177}$$

Substituting this value of Δp in equation (176)

$$\Delta S_H=\epsilon\, H\sin\Delta D. \tag{178}$$

Let $\Delta D = 4'$ at time of H scale-value observations; $H = 18316$; and $\epsilon = 0.000424$.

Then

$$\Delta S_H = 0.000424 \times 18316 \times \sin 4'$$
$$= 0.009\ \gamma/\text{mm}.$$

262. In paragraph 261, ΔD is equal to the difference between the mean value of D, say for 1 year, and the D ordinate at the time of the

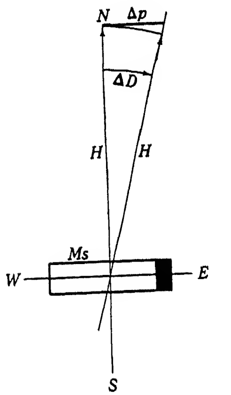

Figure 65.—Effect of change of declination on *H* scale value.

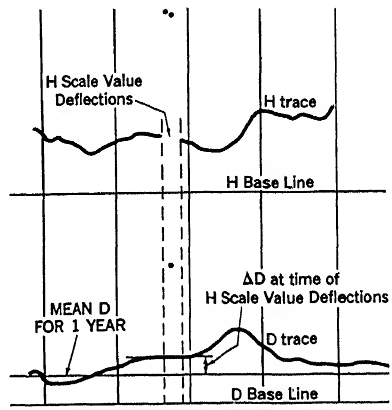

Figure 66.—Value of declination affecting *H* scale value.

scale value observations (See fig. 66.). Note: Equation (178) may be written (see eq. 381, p. 184),

$$\Delta S_H = (\Delta D)\ \epsilon\ H \sin 1' \tag{179}$$

when ΔD is a small angle. The value of $\epsilon\ H \sin 1'$ may be taken as a constant for 1 year. In this case it is equal to 0.0023. And

$$\Delta S_H = 4' \times 0.0023 = 0.009\gamma/\text{mm}.$$

To illustrate how large this correction may become even for moderate changes in declination, suppose $S_H = 3.00$; and $\Delta D = 13'$; then

$$\Delta S_H = 13' \times 0.0023 = 0.03\gamma/\text{mm}$$

or about 1 percent of the uncorrected H scale value.

CHAPTER 9. THE VERTICAL-INTENSITY VARIOMETER

263. ***Function of a Z variometer.***—This instrument should indicate visually or by photographic registration the variations in the vertical component of the intensity of the earth's magnetic field.

264. ***Basic requirements.***—The essentials of the instrument are:

(a) A permanent magnet (or pair of magnets) equipped with a quartz (or equivalent) knife edge near its center of gravity, a balancing poise for adjustment of the magnetic axis to the horizontal, and a sensitivity poise for adjustment of the scale value;

(b) A nonmagnetic housing with suitable agate or quartz supports so arranged that the magnet may oscillate in a vertical plane;

(c) An optical system suitable for visual observations or photographic registration;

(d) Copper boxes or chambers surrounding the ends of the magnet system to provide magnetic damping; and

(e) A temperature compensation device.

Note: See chapter 10 for temperature compensation of the Z variometer and paragraph 331, page 135, for the effects of an exlevel angle.

265. ***Operating principle.***—To place the instrument in operation, the magnetic system is balanced so that its magnetic axis is horizontal and (usually) in the magnetic meridian. When the vertical component, Z, of the earth's field increases, the magnetic couple increases and the south (south seeking) end rises. When Z decreases the north (north seeking) end rises. Thus any slight change in Z causes the system to become slightly unbalanced, causing the magnet to turn slightly in a vertical plane. It is this motion that is observed or recorded as a change in Z.

266. ***The equation of equilibrium.***—In figure 67, NS represents the magnetic axis of a Z recording magnet of moment M_s, in a balanced horizontal position, N end north.

H=horizontal intensity of the earth's magnetic field;
Z=vertical intensity of the earth's magnetic field;
N=north (north seeking) pole of recording magnet M_s;
S=south (south seeking) pole of recording magnet M_s;
O=position of the knife edges;
C=the center of gravity of the complete system supported on the knife edges at O;
m_1=mass of the supported system=mass of recording magnet system, hereafter called the mass of the magnet system;
$a=BC$=horizontal distance to C from the vertical, OAB; positive toward the S end of the recording magnet M_s;
$h=OB$=vertical distance to C from the horizontal, OS; positive downward.

267. When the magnet comes to rest in the horizontal position the clockwise mechanical couple, m_1ga, just balances the counterclockwise magnetic couple, $M_sZ \sin 90° = M_sZ$. That is,

$$M_sZ = m_1ga. \tag{180}$$

268. Now let Z increase so that the Z magnet turns counterclockwise through a small angle θ. The new positions are shown in figure 67, N having moved to N', S to S', B to B', and C to C'. The horizontal distance of the center of gravity from the vertical line OB is now AD, the new mechanical lever arm. With the recording magnet, M_s, in the position $N'S'$, the magnetic couples acting on the magnet system are:

$M_sZ \sin (90° - \theta) = M_sZ \cos \theta$, counterclockwise.

$M_sH \sin \theta$, clockwise.

The mechanical couple is $m_1g\ \overline{AD}$, clockwise, where g is the acceleration due to gravity. The clockwise and counterclockwise couples must

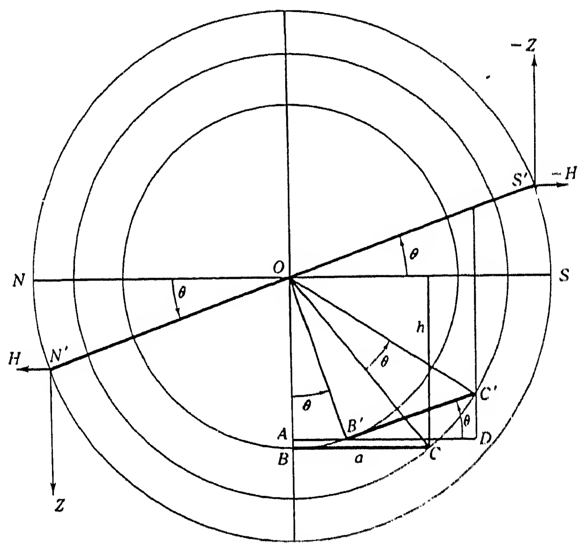

Figure 67.—Diagram for Z variometer.

be balanced for the system to be in rotational equilibrium in the new position. Therefore

$$M_sZ \cos \theta = M_sH \sin \theta + m_1g\ \overline{AD}. \tag{181}$$

From figure 67,

$$AD = AB' + B'D;$$

$$AB' = OB' \sin \theta = OB \sin \theta = h \sin \theta;$$

$$B'D = B'C' \cos \theta = BC \cos \theta = a \cos \theta;$$

and $$AD = h \sin \theta + a \cos \theta.$$

Substituting into equation (181)

$$M_sZ \cos \theta = M_sH \sin \theta + m_1g\ (h \sin \theta + a \cos \theta). \tag{182}$$

269. Equation (182) must be modified if extraneous fields exist and if M_s is not in the magnetic meridian. Let

A=Magnetic azimuth of the recording magnet=angle between magnetic north and the direction of M_s, as shown in figure 68;

f_p=horizontal component in azimuth A of all extraneous magnetic fields; the f_p field will be small unless the Z variometer has sensitivity-control magnets;

C=component of all extraneous magnetic fields parallel to Z, positive if C is downward.

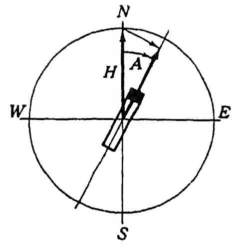

Figure 68.—View, looking down, of Z variometer magnet oriented at an angle A with the magnetic meridian.

When A, f_p, and C are taken into account, equation (182) becomes

$$M_s\,(Z+C)\cos\theta=M_s\,(H\cos A+f_p)\sin\theta+m_1g\,(h\sin\theta+a\cos\theta). \tag{183}$$

This is the more general equation of equilibrium for the Z variometer. Solving equation (183) for θ,

$$\tan\theta=\frac{M_s(Z+C)-m_1ga}{M_s(H\cos A+f_p)+m_1gh}. \tag{184}$$

Solving equation (183) for Z,

$$Z=\left[(H\cos A+f_p)+\frac{m_1gh}{M_s}\right]\tan\theta+\frac{m_1ga}{M_s}-C. \tag{185}$$

In normal operation θ is small and $\tan\theta\approx\theta$. Then

$$Z=\theta\left[(H\cos A+f_p)+\frac{m_1gh}{M_s}\right]+\frac{m_1ga}{M_s}-C. \tag{186}$$

270. ***The scale-value equation.***—In the operation of Z variometers at magnetic observatories we are concerned with the Z scale value, that is, the ratio of the change in Z to the change in the ordinate, n, of the Z spot. The rate of change of Z with respect to θ is given by differentiating equation (186) with respect to θ,

$$dZ=\left[(H\cos A+f_p)+\frac{m_1gh}{M_s}\right]d\theta. \tag{187}$$

This is the Z scale-value equation in which all quantities are in cgs units and θ is in radians. In paragraph 202, page 75, it has been shown that

$$d\theta=\frac{dn}{2R} \tag{188}$$

where dn=change of ordinate of spot, in millimeters;

and R=effective distance, Z variometer to drum, in millimeters.

Hence from equations (187) and (188)

$$\frac{dZ}{dn}=\frac{1}{2R}\left[(H\cos A+f_p)+\frac{m_1gh}{M_s}\right]. \tag{189}$$

In gammas per millimeter:

$$S_Z=\frac{1}{2R}\left[(H\cos A+f_p)+\frac{m_1gh}{M_s}\right]\times 10^5. \tag{190}$$

Equation (190) is the general equation for the scale value of the Z variometer in gammas per millimeter.

Example: $R=2244$ mm;
$H=0.1875$;
$A=0$;
$f_p=0$;
$m_1=20$ gm;
$g=980$ cm/sec/sec;
$h=0.00322$ cm$=32.2$ microns;
$M_s=314$ cgs;

by equation (190):

$$S_Z=8.66\gamma \text{ per mm.}$$

As explained later (par. 276), h may be either positive or negative.

271. ***Center of gravity and poise displacement.***—It can be seen from equation (190) that the scale value can be increased by increasing h. Before taking up this point (paragraph 274) it will be convenient to see how the position of the center of gravity is affected by displacement of each of the poises. Some Z variometers have three poises, as shown in figure 69, where

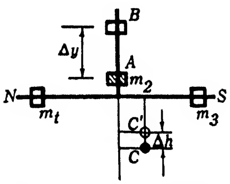

Figure 69.—The adjusting poises of the Z variometer magnet.

m_3=mass of balancing (latitude poise) on a horizontal invar spindle;
m_t=mass of temperature poise on a horizontal aluminum spindle;
and m_2=mass of sensitivity poise on a vertical spindle.

It may be shown that when the sensitivity poise, m_2, is displaced upward on its spindle a distance Δy from A to B, figure 69, the center of gravity will be displaced upward from C to C', a distance $\frac{m_2}{m_1}\Delta y$ where m_1 is the mass of the magnet system. Since h is measured positive downward and Δy is taken positive upward, then

$$dh=-\frac{m_2}{m_1}dy. \tag{191}$$

272. Similarly when m_t is displaced outward a distance dL_t, then the mechanical lever arm (a in figure 67) of the center of gravity, C, will be changed in amount da, given by

$$da=-\frac{m_t}{m_1}dL_t \tag{192}$$

and when m_3 is displaced outward a distance dL_3

$$da=+\frac{m_3}{m_1}dL_3. \tag{193}$$

When there are no temperature changes and the poises m_t and m_3 are adjusted,

$$da = -\frac{m_t}{m_1}\,dL_t + \frac{m_3}{m_1}\,dL_3. \tag{194}$$

273. When a temperature change dt occurs, it will cause displacements dL_t of the temperature poise, m_t, and dL_3 of the latitude poise, m_3. Moreover the centers of mass of the aluminum and invar spindles, and perhaps of M_s itself, will also be displaced; if these are neglected, then

$$\frac{da}{dt} \approx -\frac{m_t}{m_1}\frac{dL_t}{dt} + \frac{m_3}{m_1}\frac{dL_3}{dt}. \tag{195}$$

274. ***Effect of h on S_Z.***—From equation (190)

$$dS_Z = \frac{m_1 g \times 10^5}{2RM_s}\,dh. \tag{196}$$

The change in h may be effected by changing the position of the sensitivity poise as shown schematically in figure 69. Substitute for dh the right-hand member of equation (191), obtaining

$$dS_Z = -\frac{m_2 g \times 10^5}{2RM_s}\,dy. \tag{197}$$

Example: $m_2 = 0.0955$ gram;
$g = 980$ cm/sec/sec;
$R = 2244$ mm;
$M_s = 314$ cgs;
$dy = 0.1370$ cm $= 5$ turns;
by equation (197),

$$dS_z = 0.91\gamma/\text{mm};$$

also, if $m_1 = 20$ grams, then by equation (191),

$$dh = -0.00065 \text{ cm} = -6.5 \text{ microns for 5 turns of the poise.}$$

Table 8 shows the variation of scale value with h for two typical cases.

TABLE 8.—*Variation of Z scale value with adjustment of sensitivity poise*

SENSITIVITY POISE	Δh	$A=0°$			$A=180°$	
		SCALE VALUE	$\frac{H\cos A}{2R}\times 10^5$	$\frac{m_1gh}{2RM_s}\times 10^5$	$\frac{H\cos A}{2R}\times 10^5$	SCALE VALUE
turns	microns	γ/mm	γ/mm	γ/mm	γ/mm	γ/mm
0	0	8.66	+4.18	+4.48	−4.18	+0.30
5	−6.6	7.87	+4.18	+3.69	−4.18	Unstable
10	−13.1	6.97	+4.18	+2.79	−4.18	Unstable
15	−19.7	5.89	+4.18	+1.71	−4.18	Unstable
20	−26.2	5.12	+4.18	+0.94	−4.18	Unstable
25	−32.8	4.08	+4.18	−0.10	−4.18	Unstable
30	−39.3	3.19	+4.18	−0.99	−4.18	Unstable
35	−45.9	2.23	+4.18	−1.95	−4.18	Unstable
40	−52.4	1.33	+4.18	−2.85	−4.18	Unstable
45	−59.0	0.64	+4.18	−3.54	−4.18	Unstable
47.6	−62.4	0.00	+4.18	−4.18	−4.18	Unstable
50	−66.5	Unstable	+4.18	Unstable	−4.18	Unstable

275. Equation (190) may be solved for h:

$$h=[2RS_z\times10^{-5}-(H\cos A+f_p)]\frac{M_s}{m_1g}. \tag{198}$$

Example: $R=2244$ mm;
$S_Z=8.66\gamma$ per mm;
$H=0.1875$ cgs;
$A=0$;
$f_p=0$;
$M_s=314$ cgs;
$m_1=20$ gm;
$g=980$ cm/sec/sec;

by equation (198),

$$h=0.00322 \text{ cm}=32.2 \text{ microns.}$$

Example: Same as above, except that S_Z is now 0 instead of 8.66. By equation (198),

$h=-0.0030$ cm $=-30$ microns, that is, the center of gravity will be above the knife edge.

276. The ordinary analytical balance (nonmagnetic), is so constructed that the center of gravity is very close to the central knife edge, but always below it. Such a balance may be adjusted by means of a counterpoise so that its center of gravity will be at the same height as the knife edge or above it. In either case the balance beam would be unstable. This would be true also of the magnetic balance (Z variometer) if it were operated with no sensitivity-control magnets and with its magnetic axis strictly in the magnetic prime vertical, N end east or N end west; that is, if no component of H or f_p acts on the magnet. Suppose now the recording magnet is operated with its axis in the magnetic meridian with its N end to the north. As stated in paragraph 268, there will be a clockwise couple acting on the magnet and this couple is equal to $M_sH\sin\theta$. Thus it is possible to operate a magnetic balance with its center of gravity above the line of support (h negative) and maintain stability for long periods of time. It should be noted in passing that the distance h for a Z variometer as it is normally operated, is extremely small, usually of the order of 20 to 50 microns.

277. ***Effect of f_p on S_Z.***—If f_p is positive, that is, if it is directed the same as the Z recording magnet, S_Z will be increased, or what is the same thing, the magnet will become less sensitive to changes in Z. If f_p is negative (directed opposite to M_s) it becomes more sensitive to changes in Z, and S_Z will be decreased. Thus the scale value of a Z variometer may be adjusted over a wide range by use of control magnets properly placed and oriented with respect to the recording magnet. Control magnets have been used on la Cour Z variometers when it was not possible to obtain the desired sensitivity by other means.

278. Differentiating S_Z with respect to f_p, in equation (190),

$$dS_Z=\frac{df_p}{2R}\times10^5; \tag{199}$$

and if f_p is in gammas

$$dS_Z = \frac{df_p{}^{\gamma}}{2R}. \tag{200}$$

Example:

$$R = 2244 \text{ mm};$$
$$f_p{}^{\gamma} = 0;$$
$$S_Z = 8.66\gamma \text{ per millimeter};$$

change f_p to $+4488\gamma$; then by equation (200), $dS_Z = \frac{4488}{2 \times 2244} = +1\gamma/\text{mm}$ and S_Z becomes $9.66\gamma/\text{mm}$; or change f_p to -4488γ, dS_Z is now $-1\gamma/\text{mm}$ and S_Z becomes $7.66\gamma/\text{mm}$.

279. ***Effect of H on S_Z.***—Differentiating equation (190) with respect to H,

$$dS_Z = \frac{\cos A}{2R} \times 10^5 \, dH \tag{201}$$

$$= \frac{\cos A}{2R} \, dH_{\gamma}. \tag{202}$$

It is seen that the scale value is not affected by H when M_s is directed to magnetic east ($A = 90°$) or to magnetic west ($A = 270°$).

Example:

$$A = 0°;$$
$$R = 2244 \text{ mm};$$
$$dH_{\gamma} = 100\gamma.$$

By equation (202),

$$dS_Z = \frac{100}{4488} = 0.022\gamma/\text{mm}.$$

It is evident from the above that the change in S_Z due to a small change in H is negligible for practical purposes.

280. ***Effect of A on S_Z.***—From equation (190)

$$dS_Z = -\frac{H \sin A}{2R} \times 10^5 dA. \tag{203}$$

When dA is in degrees ($dA° = 57.3 \, dA$) and H is in gammas ($H_{\gamma} = H \times 10^5$),

$$dS_Z = -\frac{H_{\gamma} \sin A \, dA°}{114.6 \, R}. \tag{204}$$

Example:

$$H_{\gamma} = 18750\gamma = 0.1875 \text{ cgs};$$
$$A° = 90°;$$
$$R = 2244 \text{ mm};$$

change A to 95°; then $dA° = 5° = 0.0873$ radians; and by equation (203) or (204),

$$dS_Z = 0.36\gamma/\text{mm}.$$

Table 9 shows the variation of scale value, S_Z, with azimuth A of recording magnet for a typical case.

281. ***Scale values of a typical Z variometer.***—In order to give some idea of the magnitude of the various quantities entering into the Z scale value, results of various tests with a Schulze variometer are given in table 8. In these tests scale-value observations were made first with the sensitivity poise in its lowest position. The poise was then raised by steps of 5 turns (5 turns=0.1370 cm), up to the point of instability (47.6 turns=1.304 cm) keeping A constant at 0°, that is, N end north. The first column of table 8 gives the 5-turn steps. Column 2 is derived from equation (191) for $m_2=0.0955$ gm and $m_1=20$ gm. Five turns upward adjustment of the sensitivity poise decreases h and raises the center of gravity by 6.5 microns. Column 3 is the observed scale value; column 4 is the value of the constant term, $\frac{H \cos A}{2R}\times 10^5$, of equation (190); and column 5 is the difference between columns 3 and 4, that is, the variable term, $\frac{m_1 gh}{2RM_s}\times 10^5$. By equation (197),

$$\frac{dS_Z}{dy}=-\frac{m_2 g\times 10^5}{2RM_s}. \tag{205}$$

For $m_1=20$ gm, $g=980$ cm/sec/sec, $R=2244$ mm, and $M_s=314$ cgs, by equation (205),

$$\frac{dS_Z}{dy}=-6.64\ \gamma \text{ per mm, per cm of upward displacement of the poise.}$$

Since 5 turns=0.1370 cm, 1 turn=0.0274 cm; and for one upward turn of the poise,

$$dS_Z=-6.64\times 0.0274=-0.182\gamma/\text{mm}.$$

282. Suppose now that the variometer described above is operated with the N end of the recording magnet to the south and with the magnetic axis in the magnetic meridian. For this condition, $A=180°$, $\cos A=-1$, and the term

$$\frac{H \cos A}{2R}\times 10^5 \text{ becomes } -4.18\gamma/\text{mm (column 6 of table 8)}.$$

Then for the first position of the poise (0 turns), the scale value would be

$$S_Z=-4.18+4.48=0.3\gamma/\text{mm},$$

as shown in column 7. This means that the magnet would be extremely sensitive, almost to the point of instability. In fact it would require an increase of only 0.45 mm (1.64 turns) in Δy to make it unstable.

283. Table 9 gives the computed values of S_Z for all azimuths in steps of 30° from 0° to 360°.

TABLE 9.—*Variation of Z scale value with azimuth of recording magnet*

A	Cos A	$\frac{H \cos A}{2R} \times 10^5$	$\frac{m_1 gh}{2RM_s} \times 10^5$	Z Scale value
°		γ/mm	γ/mm	γ/mm
0	+1. 00	+4. 18	4. 48	8. 66
30	+0. 866	+3. 62	4. 48	8. 10
60	+0. 500	+2. 09	4. 48	6. 57
90	0	0	4. 48	4. 48
120	−0. 500	−2. 09	4. 48	2. 39
150	−0. 866	−3. 62	4. 48	0. 86
180	−1. 00	−4. 18	4. 48	0. 30
210	−0 866	−3. 62	4. 48	0. 86
240	−0. 500	−2. 09	4. 48	2. 39
270	0	0	4. 48	4. 48
300	+0. 500	+2. 09	4. 48	6. 57
330	+0. 866	+3. 62	4. 48	8. 10
360	+1. 00	+4. 18	4. 48	8. 66

284. ***Adjustment for latitude.***—The vertical intensity varies from zero at the magnetic equator to approximately $\pm 70{,}000\gamma$ at the magnetic poles. If the variometer is moved to a site where Z is considerably different, it will be necessary to rebalance the magnet by adjustment of the latitude poise, m_3, figure 69. If m_1 is the mass of the magnet system, m_3, the mass of the latitude poise, and dL_3 the distance it must be moved outward along the invar spindle in order to keep the magnetic axis horizontal, we have by equation (193)

$$da = +\frac{m_3}{m_1}\, dL_3. \tag{206}$$

By equation (186) when only Z and a change,

$$dZ = \frac{m_1 g}{M_s}\, da \tag{207}$$

and, combining equations (206) and (207),

$$dZ = \frac{m_3 g}{M_s}\, dL_3. \tag{208}$$

Example: $m_3 = 2.444$ grams;

$g = 980$ cm per sec^2;

$M_s = 1225$ cgs;

$dL_3 =$ one turn $= 0.04$ cm.

By equation (208)

$$dZ = 0.0782 \text{ cgs} = 7820\gamma.$$

That is, one turn of the poise on the latitude spindle is equivalent to 7820γ change in Z.

285. ***Adjustment of both latitude and temperature poises.***— For very large changes in Z, it would be necessary to adjust the temperature poise, m_t (see par. 292, ch. 10) and also to adjust the latitude poise to keep the magnet balanced in the horizontal plane. Combining equations (194) and (207),

$$dZ=-\frac{m_t g}{M_s}\,dL_t+\frac{m_3 g}{M_s}\,dL_3 \tag{209}$$

and

$$dL_3=\frac{M_s}{m_3 g}\,dZ+\frac{m_t}{m_3}\,dL_t. \tag{210}$$

The term $\frac{M_s}{m_3 g}\,dZ$ is the adjustment of the latitude poise needed to balance the change in Z and the term $\frac{m_t}{m_3}\,dL_t$ is the additional adjustment of the latitude poise needed to balance out the effect of the shift of the temperature poise.

Example: Z changes from 0.538 to 0.200 cgs; pitch of temperature thread 0.06 cm per turn; pitch of latitude thread 0.04 cm per turn;

$dZ=-0.338$;
$m_t=2.030$ grams;
$dL_t=-1.074$ cm to maintain temperature compensation
$=-17.9$ turns;
$m_3=2.444$ grams;
$M_s=1225$ cgs;
$g=980$ cm per sec^2.

By equation (210)

$dL_3=-0.1729$ cm $=-4.3$ turns to balance dZ;
and $dL_3=-0.8921$ cm $=-22.3$ turns to balance dL_t.

Therefore the total adjustment of the latitude poise is

$$dL_3=-0.1729\text{ cm}-0.8921\text{ cm}=-1.0650\text{ cm}$$
$$=-4.3\text{ turns}-22.3\text{ turns}=-26.6\text{ turns}.$$

286. Further details on the adjustment of the temperature poise are given in chapter 10, pages 111–112.

CHAPTER 10. TEMPERATURE COEFFICIENTS OF VARIOMETERS

287. ***Meaning of the variometer temperature coefficient.***—As stated in paragraph 267, page 99, when $\theta=0$, the magnetic axis of the recording magnet is horizontal, and

$$M_sZ=m_1ga. \tag{211}$$

Suppose that Z remains constant and the temperature of the recording magnet increases, resulting in a decrease in M_s. Then the product M_sZ will be less than m_1ga (a is assumed to be constant in this example) and the N end of the magnet will rise, indicating on the magnetogram an *apparent decrease* in Z. Now let Z increase by an amount sufficient to balance the effect due to the change in M_s. Then we may say that the change in Z is related to the change in M_s as follows:

$$\frac{dZ}{dM_s}=-\frac{m_1ga}{M_s^2} \tag{212}$$

and since $m_1ga=M_sZ$

$$\frac{dZ}{dM_s}=-\frac{M_sZ}{M_s^2}=-\frac{Z}{M_s}. \tag{213}$$

But, by definition, $-\frac{dM_s}{M_s}$ per degree centigrade change in temperature is the temperature coefficient of the magnetic moment. That is, $\frac{dM_s}{M_sdt}=-q_1$. Transposing equation (213) and dividing by dt,

$$\frac{dZ}{Zdt}=-\frac{dM_s}{M_sdt}=+q_1 \tag{214}$$

and

$$\frac{dZ}{dt}=+q_1Z=Q_Z. \tag{215}$$

The term q_1Z is the temperature coefficient, hereafter called Q_Z, of this Z variometer. In general the temperature coefficient will contain other terms in addition to q_1Z.

288. Q_Z may be defined as the *temperature coefficient of the Z spot*, always expressed in gammas per degree C. It is equal to the negative of the apparent change in Z, as recorded on the magnetogram, caused by a change in temperature of 1° C. It is also equal to the real gamma change in Z that would be required to return the Z spot to its original position after a temperature change of one degree centigrade causes the spot to be deflected. The temperature coefficient, Q_H, of the H variometer is similarly defined. The temperature coefficient, Q_D, is defined as the declination change, in minutes of arc, per degree centigrade. It may be noted that this definition of Q leads to the *correction* term $Q\ (t-t_0)$ in the derivation of values from the magnetogram (par. 424, p. 165).

289. ***Mechanical compensation of the Z variometer.***—Figure 70 is a plan view and side elevation of a Z recording magnet equipped with a device for mechanical compensation.

Let

m_1 = the mass of the recording magnet system in grams;
m_2 = the mass of the sensitivity poise;
m_3 = the mass of the balancing (latitude) poise;
m_t = the mass of the temperature poise;
L_t = effective length of the temperature compensating arm (aluminum);
L_3 = effective length of the latitude adjustment arm (invar);

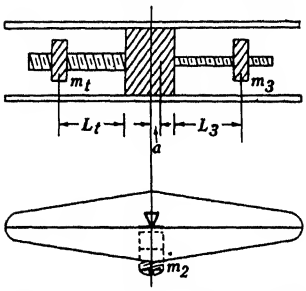

Figure 70.—Vertical-intensity recording magnet, Schmidt type.

a = the lateral distance of the center of gravity of the magnet system from the center of support when $\theta=0$;
θ = angle between the magnetic axis of M_s and the horizontal;
M_s = magnetic moment of the recording magnet;
Z = vertical intensity;
t = temperature of the magnet system;
q_1 = temperature coefficient of the magnetic moment M_s;
Q_Z = temperature coefficient of the variometer in gammas per degree C;
g = acceleration due to gravity = 980 cm/sec/sec;
α = coefficient of linear thermal expansion of aluminum = 0.000023 per degree C;
p = temperature coefficient of the distance a.

The bronze mass m_t is threaded on the aluminum spindle of effective length L_t; the bronze mass, m_3, is threaded on the invar spindle of effective length L_3. The two masses m_t and m_3, are approximately equal. The coefficient of linear thermal expansion of aluminum is taken as 0.000023 per degree C and that of invar as practically zero.

290. If the temperature of the aluminum spindle increases by an amount Δt, the mass m_t moves outward through a distance $\alpha L_t \Delta t$, and the increase in the mechanical couples on this side of the magnet system is $m_t g \alpha L_t \Delta t$. The product of m_t and its horizontal displacement due to a moderate temperature change is approximately equal to the product of the total mass of the magnet system m_1 and the horizontal displacement of the center of gravity of the system due to the same temperature change. That is

$$m_t \alpha L_t \approx -m_1 a p \tag{216}$$

or

$$ap \approx -\frac{m_t}{m_1}\alpha L_t \tag{217}$$

in which the minus sign is required because a and L_t are taken positive in opposite directions. The temperature coefficient p is a complex function of the masses, lever arms, and temperature coefficients of the various parts of the recording system.[1] If both the change of M_s

[1] C. A. Heiland and W. E. Pugh, Am. Inst. Min. and Met. Eng., Tech. Pub. 483, p. 13, 1932 (see item 5 of bibliography).

and that of a with t are taken into account, equation (215) becomes

$$Q_Z = q_1 Z + \frac{m_1 g}{M_s} ap. \tag{218}$$

Substituting for ap from equation (217),

$$Q_Z \approx q_1 Z - \frac{m_t g}{M_s} \alpha L_t \tag{219}$$

and

$$\frac{dQ_Z}{dZ} = q_1. \tag{220}$$

It should be noted in passing that the coefficient of linear thermal expansion of invar is only about 1/16 that of aluminum and even though the change in the mechanical moment on the invar side of the system (due to a temperature change) is extremely small, it *is* appreciable. This effect is almost exactly balanced by the change in the mechanical moment of the aluminum spindle itself for the same temperature change.[2]

291. Compensation for moderate temperature changes will be achieved when $Q_Z=0$, that is,

$$Q_Z = q_1 Z - \frac{m_t g \alpha L_t}{M_s} = 0 \tag{221}$$

or

$$L_t = \frac{Z M_s q_1}{\alpha m_t g}. \tag{222}$$

Example (a):
$q_1 = 0.000\ 138$ per degree C;
$Z = 0.538$ cgs;
then by equation (221),
$L_t = 0$ (uncompensated variometer);
$Q_Z = 0.000\ 074$ cgs per degree C;
$= 7.4\gamma$ per degree C.
Example (b):
$Z = 0.538$ cgs;
$M_s = 1225$ cgs;
$q_1 = 0.000\ 138$ per degree C;
$\alpha = 0.000\ 023$ per degree C;
$m_t = 2.030$ grams;
$g = 980$ cm/sec/sec;
then by equation (222),
$L_t = 1.99$ cm.

If the center of m_t is set precisely at 1.99 cm along the aluminum spindle and the latitude poise, m_3, is adjusted along the invar spindle so that the magnet is just balanced in the horizontal plane, the variometer will be compensated for temperature at a site where Z is 0.538 cgs (53,800γ). It is apparent from equation (222) that L_t must be

[2] J. W. Joyce, Manual on Geophysical Prospecting with the Magnetometer, U. S. Dept. Int., Bur. of Mines, 1937, p. 42 (see item 7 of bibliography).

increased or decreased in direct proportion to Z in order to preserve perfect temperature compensation. That is,

$$\frac{L_t}{L_t'}=\frac{Z}{Z'}. \tag{223}$$

292. If Q is of the order of $\pm 1\gamma$ per degree, it will be satisfactory for all practical purposes. What change in Z would be required to change Q_Z by 1γ per degree centigrade if $q_1=0.000\ 138$ per degree centigrade? By equation (220),

$$\Delta Q_Z=q_1\Delta Z \tag{224}$$

and

$$\Delta Z=\frac{\Delta Q_Z}{q_1}. \tag{225}$$

Then

$$\Delta Z=\frac{1}{0.000138}$$

$$=7200\gamma \text{ approximately.}$$

That is, Z may change by approximately 7200γ before Q_Z will change by more than 1γ per degree for this variometer. This variometer, compensated for temperature where $Z=53{,}800\gamma$, would be considered practically compensated for values of Z ranging from $46{,}000\gamma$ to $61{,}000\gamma$, a range of $14{,}400\gamma$. Beyond this range it would be necessary to adjust the temperature poise in accordance with equation (223) in order to preserve compensation within one gamma per degree C. For very large changes in Z, for example when a Z variometer is moved from a place where Z is 0.538 cgs to a place where Z is 0.200 cgs, both the temperature and latitude poises must be adjusted, as explained in paragraph 285. It is important *always* to make latitude adjustments with the *latitude poise only* and not to adjust the temperature poise unless one wishes to change the temperature coefficient of the instrument.

293. In south magnetic latitudes Z is negative, and the N end of the recording magnet will move downward when there is a small increase in temperature and there is no compensation. In order to achieve compensation when Z is negative, the aluminum and invar spindles should be interchanged, or what amounts to the same thing, the magnets should be removed from the central block of the system and reversed, so that the aluminum spindle is on the same end as the S end of the system. Under these conditions q_1Z is negative. For an uncompensated variometer, Q_Z is always positive in north magnetic latitudes and negative in south magnetic latitudes. Q_Z then will have the same sign as Z; and $Q_Z(t-t_0)$, the correction for temperature, is applied algebraically to observed Z in order to obtain the true Z.

294. ***Temperature coefficient and scale value.***—Heiland[3] has shown that the temperature coefficient of a variometer compensated as described above is practically independent of the scale value of the instrument. In other words, Q_Z is independent of S_Z when Q_Z is ex-

[3] Heiland and Pugh, *op. cit.*, p. 21 (seei tem 5 of bibliography).

l in gammas and not in arc or millimeters of ordinate. Heiland own also that the temperature coefficient of the scale value itself igible.[4] These conclusions are based on the assumption that s of translation of the sensitivity poise is precisely perpendicular magnetic axis of the recording magnet. (It will be shown paragraph 310, that for optical compensation the temperature ent is dependent on the scale value.)

Magnetic compensation.—Temperature compensation may omplished by use of an auxiliary magnet (or magnets) to *reduce* nponent of the vertical field acting he recording magnet. An increase perature reduces the magnetic mo- of both magnets. Reducing the tic moment of the recording magnet an apparent decrease in Z. At the ime the field of the compensating t is reduced numerically, causing *rease* in the resultant field acting recording magnet, an *apparent* e in Z. When the proper relation between the temperature coefficients two magnets and the amount of on in the field acting on the record- gnet, first-order temperature com- ion is effective. In practice the ature magnet is fixed relative to the recording magnet as in figure 71. The position of M_a is reversed in south mag- atitudes, that is, the N end of M_a is down. Equation (182) written, for $\theta=0°$,

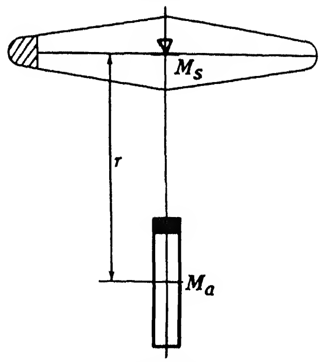

Figure 71.—Magnetic temperature compensation of Z variometer.

$$M_s(Z+C)=m_1ga \tag{226}$$

$$M_sZ+M_sC=m_1ga \tag{227}$$

ch C=the amount by which the vertical field acting on the recording magnet is altered;

M_s=the magnetic moment of the recording magnet;

m_1ga=the mechanical couple balancing the magnetic couple $M_s(Z+C)$.

be noted that Z and C have opposite signs. In north magnetic es, Z is positive and C is negative.) Let the temperature of the system rise by an infinitesimal amount, dt. This will cause a in C, M_s, and a, and the result will be an apparent change in ease or decrease). C decreases as a result of a decrease in the tic moment of the temperature magnet and because of the e in the distance r (fig. 71), due to the expansion of the material ing the magnets (usually this material is brass).

Now let Z increase by an infinitesimal amount, just sufficient the magnetic axis horizontal. Then from equation (227) we

$$\Delta M_s)(Z+\Delta Z)+(M_s+\Delta M_s)(C+\Delta C+\Delta' C)=m_1ga+m_1g\Delta a. \tag{228}$$

[4] ...d and Pugh, *op. cit.*, p. 11.

In this equation ΔC is the change in C due to a change in the magnetic moment of the temperature magnet and $\Delta' C$ is the change in C due to the change in the distance between magnets. Expanding equation (228) and neglecting second order terms,

$$M_sZ+M_s\Delta Z+Z\Delta M_s+M_sC+M_s\Delta C+M_s\Delta' C+C\Delta M_s$$
$$=m_1ga+m_1g\Delta a. \qquad (229)$$

By equation (227),

$$M_sZ+M_sC=m_1ga.$$

Subtracting equation (227) from equation (229),

$$M_s\Delta Z+Z\Delta M_s+M_s\Delta C+M_s\Delta' C+C\Delta M_s=m_1g\Delta a. \qquad (230)$$

Dividing by Δt,

$$\frac{M_s\Delta Z}{\Delta t}+\frac{Z\Delta M_s}{\Delta t}+\frac{M_s\Delta C}{\Delta t}+\frac{M_s\Delta' C}{\Delta t}+\frac{C\Delta M_s}{\Delta t}=\frac{m_1g\Delta a}{\Delta t} \qquad (231)$$

Let $\frac{\Delta Z}{\Delta t}=Q_Z=$ temperature coefficient of the variometer in gammas per degree C;

$-\frac{\Delta M_s}{M_s\Delta t}=q_1=$ temperature coefficient of the magnetic moment of the recording magnet;

$-\frac{\Delta C}{C\Delta t}=q_2=$ temperature coefficient of the compensating field at constant r;

$-\frac{\Delta' C}{C\Delta t}=q_3=$ temperature coefficient of the compensating field at constant M_a;

$\frac{\Delta a}{a\Delta t}=q_4=$ temperature coefficient of mechanical couple $=p$ (par. 289).

Substituting these values of the q's in equation (231), having regard to signs, and noting that $M_sZ+M_sC=m_1ga$,

$$M_sQ_Z-ZM_sq_1-M_sCq_2-M_sCq_3-M_sCq_1=m_1gaq_4=q_4(M_sZ+M_sC) \qquad (232)$$

from which

$$Q_Z=Z(q_1+q_4)+C(q_1+q_2+q_3+q_4). \qquad (233)$$

When compensation is effective, $Q_Z=0$, and

$$Z(q_1+q_4)=-C(q_1+q_2+q_3+q_4) \qquad (234)$$

or

$$\frac{C}{Z}=-\frac{q_1+q_4}{q_1+q_2+q_3+q_4}. \qquad (235)$$

Let

$$\sum q=q_1+q_4$$

and

$$\sum{}'q = q_1 + q_2 + q_3 + q_4. \tag{236}$$

Then

$$Q_z = (\sum q)Z + (\sum{}'q)C; \tag{237}$$

and for $Q_z = 0$,

$$\frac{C}{Z} = -\frac{\sum q}{\sum{}'q}. \tag{238}$$

From equation (235) we may compute C, the compensating field required to make $Q_z = 0$, if the values of Z and the q coefficients are known.

297. ***Evaluation of the q coefficients.***—The coefficients q_1 and q_2 are the temperature coefficients of the magnetic moments of the recording magnet and the compensating magnet, respectively, and must be determined experimentally or estimated from values previously determined for similar magnets. In the equation

$$C = -\frac{2M_a}{r^3} \tag{239}$$

we have the condition that the compensating field, C, varies with respect to the magnetic moment of the compensating magnet and with respect to r, the distance between the centers of the magnets. M_a and r both vary with temperature. Differentiating this equation with respect to M_a and r, we have

$$dC = -2r^{-3}dM_a + 2M_a \times 3r^{-4}dr \tag{240}$$

and dividing by equation (239),

$$\frac{dC}{C} = \frac{dM_a}{M_a} - \frac{3dr}{r}. \tag{241}$$

Let dt represent an infinitesimal change in temperature. Dividing equation (241) by dt,

$$\frac{dC}{Cdt} = \frac{dM_a}{M_a dt} - \frac{3dr}{rdt}. \tag{242}$$

At constant r,

$$\frac{dC}{Cdt} = \frac{dM_a}{M_a dt} \tag{243}$$

and at constant M_a,

$$\frac{dC}{Cdt} = -\frac{3dr}{rdt}. \tag{244}$$

The ratio, $\frac{dM_a}{M_a dt}$, is the change in M_a due to a change, dt, in the temperature, per unit magnetic moment. It is, by definition, the temperature coefficient of the magnetic moment of M_a, and is equal to $-q_2$. (See par. 296 where q_1 is also defined.) The ratio $\frac{3dr}{rdt}$ is simply 3

times the coefficient of linear thermal expansion of the material separating the magnets (usually brass, for which the coefficient of linear expansion is $\frac{1}{r}\frac{dr}{dt}=0.000\ 019$). The term $\frac{da}{adt}$ represents the temperature coefficient of the horizontal distance, a, from the point of support of the recording magnet to its center of gravity, and is equal to the coefficient of linear thermal expansion of the material of which the recording magnet is made. If the material is steel, this coefficient, q_4, may be taken as 0.000 01. Equation 233 then becomes,

$$Q_Z=Z(q_1+0.000\ 01)+C(q_1+q_2+0.000\ 06+0.000\ 01). \qquad (245)$$

In view of the uncertainty in q_1 and q_2, it is clear that normally the much smaller coefficients q_3 and q_4 may be neglected in computing Q_Z. In that case,

$$Q_Z\approx Zq_1+C(q_1+q_2). \qquad (246)$$

When $Q_Z=0$,

$$\frac{C}{Z}\approx-\frac{q_1}{q_1+q_2}. \qquad (247)$$

If the recording magnet and temperature magnet have the same temperature coefficients,

$$\frac{C}{Z}\approx-\frac{1}{2}. \qquad (248)$$

That is, if $q_1=q_2$ and the other coefficients are negligible, Q_Z will be practically zero when the temperature magnet is set at such a distance that the effective field at the center of the recording magnet is reduced by 50 percent.

298. ***Magnetic compensation of the H variometer.***—In the case of an H variometer, the mechanical couple is $k'\tau$ (par. 222, ch. 8) and is due to the the quartz fiber. This couple depends on the dimensions of the fiber and its rigidity modulus, both of which change with temperature. For simplicity, q_5 will be designated as the temperature coefficient, $\frac{1}{k'}\frac{dk'}{dt}$, of the torsion constant, k', of the fiber and is taken as $+0.00016$.[5] Substituting q_5 for q_4 and H for Z in equation (235) gives

$$Q_H=H(q_1+q_5)+C(q_1+q_2+q_3+q_5). \qquad (249)$$

For compensation, $Q_H=O$ and

$$\frac{C}{H}=\frac{-(q_1+q_5)}{q_1+q_2+q_3+q_5}. \qquad (250)$$

Let

$$\sum q=q_1+q_5$$

and

$$\sum{}'q=q_1+q_2+q_3+q_5. \qquad (251)$$

[5] Howe, *op. cit.*, p. 35 (see item 6 of bibliography).

Then
$$Q_H = (\Sigma q)H + (\Sigma' q)C; \tag{252}$$

and for $Q_H = O$,
$$\frac{C}{H} = -\frac{\Sigma q}{\Sigma' q}. \tag{253}$$

299. ***Values of C, r, and M_a for H and Z variometers.***—Using $q_3 = 0.000\,06$, $q_4 = 0.000\,01$, and $q_5 = 0.000\,16$, ratios $\frac{-C}{Z}$ and $\frac{-C}{H}$ for various values of q (where $q = q_1 = q_2$) are given in table 10. The product of a tabular value and the value of Z (or H) at which the instrument is operated is the amount by which Z (or H) must be *reduced* to make $Q = 0$. If q_1 is not equal to q_2, the values of $\frac{C}{Z}$ and $\frac{C}{H}$ should be computed from equations (235) and (250). Example:

Let $\frac{C}{Z} = -0.463$;

$Z = 35{,}000\gamma$ (downward);

$r = 12$ cm;

then $C = -16{,}200\gamma$, that is, $16{,}200\gamma$ *upward*.

From the nomogram, figure 137, appendix VI, $M_a \approx 140$ cgs for $16{,}200\gamma$ at $r = 12$ cm.

TABLE 10.—*Values of* $\frac{-C}{Z}$ *and* $\frac{-C}{H}$ *for various values of* q *when* $q = q_1 = q_2$

[COMPUTED FROM EQUATIONS (235) AND (250)]

Temp. coeff. of either magnet ($q_1 = q_2$) q	For Z variometers $\frac{-C}{Z}$	For H variometers $\frac{-C}{H}$
0. 000 10	0. 407	0. 619
15	0. 432	0. 596
20	0. 447	0. 581
25	0. 456	0. 569
30	0. 463	0. 561
35	0. 468	0. 554
40	0. 471	0. 549
45	0. 474	0. 545
50	0. 477	0. 541
55	0. 479	0. 538
60	0. 480	0. 535
65	0. 482	0. 533
0. 000 70	0. 483	0. 531

300. Practical limitations in the mechanical construction and installation of some Z variometers make it desirable to place the temperature magnet at 11 to 13 cm below or above the recording magnet. At such short distances, however, a distribution factor must be applied to equation (239) to obtain the *effective field*, C, for the particular pair of magnets used on the variometer. Equation (239) then becomes

$$C=\left(-\frac{2M_a}{r^3}\right)\left(1+\frac{P_A}{r^2}\right) \tag{254}$$

and

$$r^3=\frac{2M_a\left(1+\frac{P_A}{r^2}\right)}{-C} \tag{255}$$

in which the distribution coefficient P_A may be computed from the first equation on the left, table 2 (p. 13), or interpolated from table 21a. The value of P_A/r^2 may be scaled directly from the nomogram, figure 146, appendix VI. As a first approximation in determining P_A/r^2, r may be determined from equation (239). Suppose that $1+\frac{P_A}{r^2}=0.95$, then the better approximation to r is given by

$$r^3=\frac{2M_a\times0.95}{-C}$$

$$=\frac{2\times140\times0.95}{0.162}$$

$$=1642\text{ cm}^3$$

and $$r=11.8\text{ cm.}$$

301. ***Error in Q_Z due to error in C or r.***—We have from equation (237),

$$Q_Z=(\sum q)Z+(\sum{}'q)C. \tag{256}$$

Then

$$\frac{dQ_Z}{dC}=+\sum{}'q \tag{257}$$

$$\Delta Q_Z=+(\sum{}'q)\Delta C \tag{258}$$

$$\Delta C=+\frac{\Delta Q_Z}{\sum{}'q}. \tag{259}$$

From equation (244)

$$\Delta C=\frac{-3C}{r}\Delta r \tag{260}$$

$$\Delta r=-\frac{r\Delta C}{3C} \tag{261}$$

$$\Delta r=-\frac{r\Delta Q_Z}{3C\sum{}'q} \tag{262}$$

$$\Delta Q_Z=-(\sum{}'q)\frac{3C}{r}\Delta r. \tag{263}$$

Let $\Delta Q_Z = \pm 0.5\gamma/°\mathrm{C}$ (a tolerance well within the limits of accuracy of experimental determination of Q_Z) and let $\sum' q = 0.00065$. Then from equation (259)

$$\Delta C = \frac{\pm 0.5}{0.00065}$$

$$= \pm 770\gamma, \text{ approximately.}$$

This means that the compensating field (or the Z field itself) may change by as much as 770γ without affecting the value of Q_Z by more than $0.5\gamma/°\mathrm{C}$. Again suppose that $r = 11.8$ cm and $C = -16{,}200\gamma$. What tolerance may be allowed in r without seriously affecting Q_Z? From equation (261),

$$\Delta r = \frac{\pm 770 \times 11.8}{3 \times 16200}$$

$$= \pm 0.19 \text{ cm} = \text{approximately } 2 \text{ mm.}$$

302. This shows that the adjustment of the compensation magnet to a high degree of precision in the distance r is unnecessary in practice. The average Z variometer is quite sensitive to changes in Z but the temperature coefficient is relatively insensitive to changes in both C and Z, so that the temperature magnet may be adjusted over small ranges (± 2 mm in the example above) to adjust the ordinate of the Z spot without appreciably affecting the temperature coefficient of the variometer. The same reasoning applies to the adjustment of the temperature magnet of the H variometer.

303. ***Temperature coefficient of the D variometer.***—The temperature coefficient of the D variometer is given by [6]

$$Q_D = 3438\left[-(q_1 + q_5)\tan E_x + (q_1 + q_5 + q_E)\frac{f_E}{H}\right] \tag{264}$$

where

Q_D = temperature coefficient in minutes of arc of declination per degree C;

$q_1 = -\dfrac{1}{M_s}\dfrac{dM_s}{dt}$ = temperature coefficient of magnetic moment, M_s, of recording magnet;

$q_5 = \dfrac{1}{k'}\dfrac{dk'}{dt}$ = temperature coefficient of torsion constant, k' of suspension fiber;

E_x = exorientation angle of M_s;

f_E = extraneous east field at center of D variometer;

H = horizontal intensity of earth's magnetic field;

$q_E = -\dfrac{1}{f_E}\dfrac{df_E}{dt}$ = temperature coefficient of extraneous east field, f_E.

[6] Howe, *op. cit.*, p. 39 (see item 6 of bibliography).

Example:

Let $E_x = +1°.9$ (N end of M_s is $1°.9$ east of magnetic north);
$q_1 = 0.000\ 40$ per degree C;
$q_5 = 0.000\ 16$ per degree C;
$q_E = 0.000\ 55$ per degree C;
$f_E = -68\gamma$, that is, 68γ west;
$H = 18{,}500\gamma$.

By equation (264), $Q_D = -0.078$ minutes of arc of east declination per degree C, which means that a temperature *increase* of one degree C will produce an *apparent increase* of east declination equal to $0'.078$. This example shows that Q_D will usually be negligible.

304. ***Optical compensation of the Z variometer.***—In the optical system of temperature compensation, described by la Cour and Petersen,[7] the motion of the Z spot caused by a temperature change of the Z recording magnet is compensated by passing the incident and reflected light through a 90° prism attached to a nonmagnetic, bimetallic strip of silver and platinum. The strip is assumed to be at the same temperature as the Z magnet at all times. Let the prism be clamped and let Z remain constant while the temperature rises a definite amount. The magnetic moment, M_s, of the recording magnet will decrease. The Z spot will be displaced, indicating an apparent decrease in Z. Now let Z continue to remain constant but with the Z magnet clamped and the prism unclamped and let the system be subjected to the same temperature change as above. If the strip is properly faced and is adjusted to the correct sensitivity, the displacement of the Z spot caused by the bending of the strip will be just equal and opposite to that caused by the change in M_s described above. The resultant motion of the Z spot, when both magnet and prism are unclamped, will be zero. Under these conditions the variometer is optically compensated for temperature.

305. In the la Cour variometers, a portion of the light which passes through the reflecting prism is intercepted by a fixed mirror and reflected back through the prism to the drum, forming the temperature spot. The bimetallic strip then serves two purposes, to provide optical compensation for the variometer and to act as an optical thermograph for recording changes in the temperature of the instrument.

306. ***The constant, C_T, of the bimetallic strip.***—In order to adjust properly the length of the strip for a particular instrument, one should know the sensitivity of the strip to temperature changes. The strip constant, C_T, is defined as the motion of the T spot in mm on the drum, per mm length of strip, per degree change in temperature, at a recording distance of 1 cm. The constant may be furnished by the manufacturer, or determined experimentally by subjecting a known length of the strip to a known temperature change and noting the motion of the T spot at a known recording distance. If the length l mm of the strip is subjected to a temperature change $t_2 - t_1$ when the effective recording distance is R and the ordinate of the T spot changes from n_1 to n_2 millimeters, then the strip constant, C_T, is given by

$$C_T = \frac{n_2 - n_1}{t_2 - t_1}\,\frac{1}{Rl}. \tag{265}$$

Also,

$$n_2 - n_1 = C_T R l (t_2 - t_1). \tag{266}$$

[7] D. la Cour, La Balance de Godhavn, p. 17 (see item 8 of bibliography).

307. ***The sensitivity and scale value of the bimetallic strip.***—The sensitivity, T_s, of a strip is defined as the change in the millimeter ordinate of the T spot caused by a change in temperature of one degree C. Using the same notation that was used in equation (265),

$$T_s=\frac{n_2-n_1}{t_2-t_1} \tag{267}$$

and by equation (265)

$$T_s=\frac{n_2-n_1}{t_2-t_1}=C_TRl. \tag{268}$$

The reciprocal of the sensitivity, that is the change in temperature in degrees C represented by one mm change in ordinate of the T spot, is the scale value of the T spot, and is designated S_T. Thus if S_T is known, variations in temperature may be scaled directly from the temperature curve. By definition,

$$S_T=\frac{1}{T_s} \tag{269}$$

and by equation (268),

$$S_T=\frac{t_2-t_1}{n_2-n_1}=\frac{1}{C_TRl}. \tag{270}$$

Example: A platinum-silver strip was tested at Cheltenham Observatory. It was found that the angular displacement of the end of the strip was proportional to the temperature change over a range of 15° C. In these tests,

$$R=231 \text{ cm};$$

$$l=22 \text{ mm};$$

$$n_2-n_1=20 \text{ mm};$$

$$t_2-t_1=8° \text{ C}.$$

By equation (265), $C_T=0.00049$ per cm per degree.

By equation (267), $T_s=2.5$ mm/°C.

By equation (270), $S_T=0.4$°C/mm.

308. ***The temperature coefficient with optical compensation.***—For effective temperature compensation the ordinate of the Z spot should not change when Z is constant and temperature changes. Let

ΔZ_1=*apparent* change in Z due to Δt change in temperature with recording magnet free, strip clamped, and Z constant;

ΔZ_2=*apparent* change in Z due to Δt change in temperature, with recording magnet clamped, strip free, and Z constant;

$q_1=-\frac{1}{M_s}\frac{dM_s}{dt}$=temperature coefficient of magnetic moment of Z magnet;

Q_Z=temperature coefficient of the variometer, $\gamma/°$ C;
S_Z=scale value of the Z variometer, γ/mm;
R=distance from Z lens to drum;
C_T=strip constant, or motion of T spot in mm, per mm length of strip, per degree C, for R=1 cm (furnished by maker or determined from equation (265));
l=length of strip in mm.

Now let Z remain constant while the temperature of the Z recording magnet changes by Δt. This will cause the Z spot to move as a result of the change of magnetic moment of the recording magnet, and the apparent change in Z is, by equation (214) (p. 109),

$$\Delta Z_1 = -q_1 Z \Delta t. \tag{271}$$

Again, let the temperature of the strip change by Δt. With the strip facing properly, the Z spot will move in the opposite direction due to the bending of the strip and the motion of the Z spot, Δn, will be, by equation (266),

$$\Delta n = RlC_T \Delta t \tag{272}$$

and

$$\Delta Z_2 = +S_Z \Delta n = RlC_T \Delta t S_Z. \tag{273}$$

The net apparent change in Z will be

$$\begin{aligned} \Delta Z_a &= \Delta Z_1 + \Delta Z_2 \\ &= -q_1 Z \Delta t + S_Z RlC_T \Delta t; \end{aligned} \tag{274}$$

and by definition of Q_Z (par. 288),

$$Q_Z = -\frac{\Delta Z_a}{\Delta t} = q_1 Z - RlC_T S_Z. \tag{275}$$

Also

$$l = \frac{q_1 Z - Q_Z}{RC_T S_Z} \tag{276}$$

and

$$\Delta l = -\frac{\Delta Q_Z}{RC_T S_Z}. \tag{277}$$

Also, from equation (275),

$$q_1 = \frac{Q_Z + RlC_T S_Z}{Z}. \tag{278}$$

When the two *apparent* changes ΔZ_1 and ΔZ_2 just balance each other and there is no motion of the Z spot resulting from a temperature change, the variometer is compensated and Q_Z=0. When Q_Z=0,

$$q_1 Z - RlC_T S_Z = 0 \tag{279}$$

and

$$l = \frac{q_1 Z}{RC_T S_Z}. \tag{280}$$

309. ***Estimation of strip length for optical compensation.***— The following examples illustrate the application of the equations of paragraph 308:

Example (a): Let $q_1 = 0.00030$ per degree C;

$$Z = 23200\gamma;$$

$$R = 225 \text{ cm};$$

$$C_T = 0.00049 \text{ per cm per degree};$$

$$S_Z = 3.00\gamma/\text{mm}.$$

By equation (280):

$$l = 21 \text{ mm}.$$

This variometer, having a scale value of 3.00γ/mm and operating where $Z=23{,}200\gamma$, will be optically compensated for temperature when the length of the strip is 21 mm. It is seen from equation (275) that the temperature coefficient of the variometer will change if the Z scale value changes. Equation (280) shows that for $Q_Z=0$, l will vary directly as Z and inversely as R and S_Z. For a constant recording distance and scale value, l must be made longer as Z increases and shorter as R and S_Z increase. This condition is expressed in the following equation:

$$l' = l\frac{Z'}{Z}\frac{RS_Z}{R'S_Z'}. \tag{281}$$

Example (b): Let $Z = 53{,}800\gamma$;

$$R = 231 \text{ cm};$$

$$S_z = 3.60\gamma/\text{mm};$$

$$C_T = 0.00049 \text{ per cm per degree};$$

$$l = 22 \text{ mm}.$$

By changing the temperature artificially, or by comparing with another variometer, it is found that

$$Q_Z = 7.3\gamma/°\text{C}.$$

What is the strip length for $Q_Z=0$? Let $\Delta Q_Z = -7.3\gamma/°$C. By equation (277) $\Delta l = 18$ mm. Then, for $Q_Z=0$,

$$l = 22 \text{ mm} + \Delta l = 40 \text{ mm}.$$

Also, what is the value of q_1 in this case? By equation (278), $q_1 = 0.00030$ per degree C.

Example (c): Suppose it is desired to move a Z variometer from Cheltenham where it is properly compensated for temperature to San Juan. What should be the new length of the bimetallic compensating strip? At Cheltenham $l=40$ mm, $Z=53{,}800\gamma$, $R=231$ cm,

and $S_z=3.6\gamma$/mm. At San Juan Z will be 35,500γ, R will be 240 cm, and the required scale value S_z will be 2.55γ/mm. By equation (281)

$$l'=40\times\frac{35{,}500}{53{,}800}\times\frac{231}{240}\times\frac{3.6}{2.55}$$

$$=36 \text{ mm.}$$

310. ***Optical compensation of the H variometer.***—This may be accomplished in the same manner as that used for the Z instrument except that we must now take into consideration the changes in the torsion constant of the quartz fiber with changes in temperature. Let

$q_1 = -\frac{1}{M_s}\frac{dM_s}{dt}$ = temperature coefficient of the H recording magnet;

$q_5 = \frac{1}{k'}\frac{dk'}{dt}$ = temperature coefficient of the torsion constant of the quartz fiber = 0.00016 per degree C;

S_H = H scale value, in gammas per mm;

C_T = bimetallic-strip constant;

R = recording distance from drum to H lens;

l = length of the strip in mm;

Q_H = temperature coefficient of the variometer, in gammas per degree C;

ΔH_1 = apparent change in H due to change in temperature, Δt, with recording magnet free, strip clamped, and H constant;

ΔH_2 = apparent change in H due to change in temperature, Δt, with magnet clamped, strip free, and H constant.

With the prism clamped and the magnet free, and assuming that the extraneous fields at the center of H are negligible.

$$\Delta H_1 = -H(q_1+q_5)\Delta t, \tag{282}$$

and with the magnet clamped and strip free,

$$\Delta H_2 = lC_TRS_H\Delta t. \tag{283}$$

The net apparent change when both the magnet and strip are free is

$$\begin{aligned}\Delta H_a &= \Delta H_1 + \Delta H_2 \\ &= -H(q_1+q_5)\Delta t + lC_TRS_H\Delta t, \end{aligned} \tag{284}$$

and by definition of Q_H, paragraph 288,

$$Q_H = -\frac{\Delta H_a}{\Delta t} = H(q_1 + q_5) - RlC_T S_H. \tag{285}$$

For temperature compensation, $Q_H = 0$ and

$$l = \frac{(q_1 + q_5)H}{RC_T S_H}. \tag{286}$$

It is seen that in this system of temperature compensation the value of Q_H will depend upon the H scale value. The formulas developed for the optical compensation of the Z variometer, paragraph 308, will also apply to the H variometer if q_1 is changed to $q_1 + q_5$, Z to H, and S_Z to S_H.

CHAPTER 11. DETERMINATION OF SCALE VALUES

USE OF DEFLECTOR MAGNET

311. ***The H scale value.***—For small angles, the angular displacement of the recording magnet is approximately proportional to the field applied in the magnetic meridian through M_s. Then the H scale value is

$$S_H^* = \frac{f_n}{u_H} \tag{287}$$

in which S_H^* = observed H scale value in gammas per mm;

f_n = applied field in gammas;

u_H = deflection of H spot in mm on the magnetogram.

312. The field, f_n, is supplied by a small magnet mounted on the deflection bar of the variometer, or by a large deflector of high magnetic moment placed 2 or 3 meters from the variometer (in the magnetic meridian through the center of M_s and at the same elevation as M_s), or by a Helmholtz-Gaugain coil. When a small deflector is used at short distances, a correction must be applied for distribution by means of one of the formulas in table 2. When a large deflector is used at distances which are large compared with the dimensions of the magnets, corrections for distribution may be entirely negligible.

313. In practice, three deflections are made with the deflector in the A position, figure 118 (p. 204), top, first with the N end to the north and then with the N end to the south, the reversals being made in the manner shown in the sample set of observations, figure 72. For a double deflection, 2 u_H, equation (287), may be written

$$S_H^* = \frac{2f_n}{2u_H}. \tag{288}$$

314. In gammas, the field, f_n, of the deflector is $\frac{2M_a}{r^3} \times 10^5$, M_a being the magnetic moment of the deflector and r the deflection distance in centimeters (center of M_a to center of M_s). Substituting this value of f_n in equation (288), and including a distribution factor, α_H, calculated by means of the top line of table 2 (p. 13), we have

$$S_H^* = \frac{4M_a\,\alpha_H}{2u_H\,r^3} \times 10^5. \tag{289}$$

This is the observed scale value at the away ordinate, that is, at the ordinate, h_{mm}, of the undeflected H spot.

374d

DEPARTMENT OF COMMERCE

COAST AND GEODETIC SURVEY

DIV: G&G

(DEC 1949)

Observatory TUCSON

Date May 14, 1951

SCALE VALUES OF INTENSITY VARIOMETERS

Deflector Cobalt Steel 20.27 cm Deflection distance 30 6 cm Observer J B Campbell

H VARIOMETER (Deflector face up)		D VARIOMETER (Deflector face up)		Z VARIOMETER (Deflector face East)	
N end	$2u_H$	N end	$2u_D$	N end	$2u_Z$
N	mm[1]	E	mm[1]	U	mm[1]
S	67.5	W	26.9	D	29.0
N	67.3	E	26.5	U	29.0
Mean (N−S) = $2u_H$	67.4	Mean (E−W) = $2u_D$	26.7	Mean (U−D) = $2u_Z$	29.0
Away ord. Before	16.4	Away ord. Before	4.1	Away ord. Before	−6.1
After	14.7	After	5.4	After	−5.8
Mean h_{mm}	15.6 mm	Mean d_{mm}	4.8 mm	Mean z_{mm}	−6.0 mm
$\bar{S}_H$	2.97 γ/mm	S_D'	0.50 ′/mm	105	M. Time
h_γ	46 γ	d	2′	Began	18h 22m
Prel. H blv.	25932 γ	Prel. D blv.	13° 21′	Ended	18 53
H_γ	25978 γ	D	13° 23′	Mgph. Temp.	20.4°C
		$\bar{D}$	13° 24′[2]	Remarks	
		$(D-\bar{D})$	−1′		
$S_D^\gamma = S_D' H_\gamma \tan 1'$		$S_H^* = 4M(10)^5 \alpha_H / r^3\, 2u_H$		$S_Z = 2M(10)^5 \alpha_Z / r^3\, 2u_Z$	
log tan 1′	6.4637	log $4(10)^5$	5.6021	log $2(10)^5$	5.3010
log S_D'	9.6990	log α_H	0.0006	log α_Z	9.9997
log H_γ	4.4146	colog r^3	2.5429	colog r^3	2.5429
log S_D^γ	0.5773	log $4(10)^5 \alpha_H / r^3$	8.1456	log $2(10)^5 \alpha_Z / r^3$	7.8436
S_D^γ	3.78	log M	4.1603	log M	4.1603
$M = r^3\, 2u_D S_D^\gamma / 2\alpha_D (10)^5$		colog $2u_H$	8.1713	colog $2u_Z$	8.5376
log $r^3 / 2(10)^5$	2.1561	log S_H^*	0.4772	log S_Z	0.5415
colog α_D	0.0004	S_H^*	3.001	S_Z	3.48
log $2u_D$	1.4265	$-0.0039(D-\bar{D})$	+.003	$S_H = S_H^* - 0.0030\,(D-\bar{D})$	
log S_D^γ	0.5773	S_H	3.004	$S_o = S_H - 0.0024\, h_{mm}$	
log M	4.1603	$-0.0024\, h_{mm}$	−.037	$\bar{S}_H = S_o + 0.0012\, h_{mm}$	
M	14460	S_o	2.97	$h_\gamma = \bar{S}_H h_{mm}$	

[1] Scalings corrected to 100.0 mm shrinkage distance. [2] Estimated mean declination for 1951: $\bar{D}$ = 13° 24′ (to nearest minute)

Scaled by RLR Scalings checked by LRS Computed by RLR Computations checked by LRS Abstracted by LRS

M-1957-4

Figure 72.—Scale-value observations and computations.

315. The H scale value of a unifilar variometer depends on the ordinate of the H spot at the time the deflections are made and on the magnetic declination, in accordance with the equation,

$$S_H^* = S_0 + ah + b(D - \overline{D}) \tag{290}$$

where a = the a factor described in paragraph 255 (p. 95);

b = the b factor
$= \epsilon H \sin 1'$ (see par. 262);

h = the *away* ordinate of the H spot, denoted h_{mm} in figure 72;

$\overline{D}$ = standard declination value, such as the mean for one year;

D = the declination at the time of the H scale-value deflections;

S_0 = the H scale value that would be observed when $h=0$ and $D=\overline{D}$;

S_H^* = the observed H scale value when the H ordinate is h and the declination is D.

Also, if S_H is the H scale value at ordinate h and declination $\overline{D}$, then

$$S_H = S_0 + ah \tag{291}$$

and

$$S_H^* = S_H + b(D - \overline{D}).$$

In figure 72, S_H and S_0 are calculated from the observed H scale value, S_H^*, as follows:

$$S_H = S_H^* - b(D - \overline{D}) \tag{292}$$

$$S_0 = S_H - ah. \tag{293}$$

If the terms $b(D-\overline{D})$ and ah are negligible, then S_H^* and S_0 may be taken as equal to S_H. This is usually the case for low-sensitivity H variometers.

316. ***The Z scale value.***—For the Z variometer, the deflector is used in the B position, figure 118, center, that is, the center of the deflector, M_a, is placed in the magnetic meridian through the center of the Z recording magnet, M_s, axis of M_a vertical and with its magnetic center at the same elevation as the center of M_s. In this position, the field of M_a at the center of M_s is

$$f_n = \frac{M_a}{r^3} \times 10^5.$$

Substituting this value of f_n in equation (288), changing H to Z, and including a distribution factor α_Z, we have

$$S_Z = \frac{2M_a\alpha_Z}{2u_Z r^3} \times 10^5. \tag{294}$$

in which 2 u_Z is the double deflection of the Z spot in mm on the magnetogram, the deflector being reversed in making the deflection observations in accordance with the example in figure 72.

317. Equation (294) gives the Z scale value at the observed ordinate, z_{mm}, which is the *away* ordinate. The Z scale value varies slightly with ordinate, but the variation is usually negligible and will not be considered here.

318. ***The D scale value (gamma scale value).***—The deflector is used in the B position, figure 118 bottom—that is, it is placed with its center in the magnetic meridian through the center of the D recording magnet, at a distance, r, and with its magnetic axis in the same horizontal plane as the D recording magnet, at right angles to the magnetic meridian through M_s. The field, f_n, producing the deflection is $\frac{M_a}{r^3}\times 10^5$, and for a double deflection it is $\frac{2M_a}{r^3}\times 10^5$. Substituting this value of f_n in equation (288), changing H to D, and including a distribution factor α_D, we have

$$S_D^\gamma=\frac{2M_a\alpha_D}{2u_Dr^3}\times 10^5. \tag{295}$$

This is the intensity scale value (gamma scale value) of the D variometer. The relation between the gamma scale value and the minute scale value is described in paragraphs 321–23.

319. ***Computation of scale values from deflections.***—In the case of a sensitive magnetograph, the D scale value is usually known quite accurately, and is quite constant since $\frac{C+k}{H}$ is small compared with unity (see eq. 119). It is then advantageous to derive M_a from the following equation, based on equation (295):

$$M_a=\frac{2u_Dr^3S_D^\gamma}{2\alpha_D\times 10^5}. \tag{296}$$

and then to calculate S_H^* and S_Z from equations (289) and (294). This method is illustrated by the example given in figure 72. If the deflection distances are the same for all three variometers and if the same deflector is used, then we may divide equation (289) by equation (295), obtaining

$$S_H^*=\frac{4u_DS_D^\gamma\alpha_H}{2u_H\alpha_D}. \tag{297}$$

Similarly, (294) divided by (295) gives

$$S_Z=\frac{2u_DS_D^\gamma\alpha_Z}{2u_Z\alpha_D}. \tag{298}$$

Equations (297) and (298) are sometimes used to calculate S_H^* and S_Z when S_D^γ is known. This method of computation eliminates the computation of M_a.

320. In the case of a low-sensitivity magnetograph, the D scale value is sensitive to change in the magnetic moment of the D recording

BUREAU: C.&G.S.
DIVISION: G.&S.
FORM 374c

Magnetic Observatory SITKA

Date Jan 17, 1952
MO. DA. YR.

SCALE VALUES OF VARIOMETERS

(WITH MAGNETIC MOMENT OF DEFLECTOR KNOWN)

Deflector Cobalt Steel 1 x 1 x 20 cm Deflection dist.(r) 25.2 cm. Moment of defl.(M) 12160

135 Meridian Time began 1041 ended 1048 Observer TL Skillman

Z VARIOMETER (DEFLECTOR FACE EAST)		D VARIOMETER (DEFLECTOR FACE UP)		H VARIOMETER (DEFLECTOR FACE UP)	
N end	$2u_Z$	N end	$2u$	N end	$2u_H$
U	4.8 mm.	E	8.5 mm.	N	14.1 mm.
D	4.8	W	8.6	S	14.1
U		E		N	
Mean (U-D)	4.8	Mean (E-W)	8.6	Mean (N-S)	14.1
Away Ordinate	Before 14.0 After 14.0	Away Ordinate	Before 13.6 After 13.2	Away Ordinate	Before 30.3 After 30.4
All scalings corrected for shrinkage to 100.0 mm.		H from sensitive magnetograph 15520 γ		Magnetograph Temperature 15 °C	
$S_Z = 2M \cdot 10^5 \alpha_Z / r^3 2u_Z$		$S_D^\gamma = 2M \cdot 10^5 \alpha_D / r^3 2u$ $S'_D = S_D^\gamma \cot 1' / H$		$S_H = 4M \cdot 10^5 \alpha_H / r^3 2u_H$	
$\log 2 \cdot 10^5$	5.3010	$\log 2 \cdot 10^5$	5.3010	$\log 4 \cdot 10^5$	5.6021
$\log M$	4.0849	$\log M$	4.0849	$\log M$	4.0849
$\log \alpha_Z$	0.0000	$\log \alpha_D$	9.9993	$\log \alpha_H$	0.0009
colog r^3	2.7958	colog r^3	2.7958	colog r^3	2.7958
$\log C_Z$	2.1817	$\log C_D$	2.1810	$\log C_H$	2.4837
$\log 2u_Z$	0.6812	$\log 2u$	0.9345	$\log 2u_H$	1.1492
$\log S_Z$	1.5005	$\log S_D^\gamma$	1.2465	$\log S_H$	1.3345
		$\log \cot 1'$	3.5363		
		colog H	5.8091		
		$\log S'_D$	0.5919		
$S_Z =$	31.7 γ/mm.	$S_D^\gamma =$	17.6 γ/mm.	$S_H =$	21.6 γ/mm.
		$S'_D =$	3.91 ′/mm.		

Scaled by T L Skillman Checked by K Cravens.

Computed by TLS Checked by KC Abstracted by KC

Figure 73.—Scale-value observations and computations; magnetic moment of deflector known.

magnet, and likewise to change of the C field (sensitivity-control field) if there is a control magnet. That is because $\frac{C+k}{H}$ is large compared with unity (see eq. 119). For this reason it is better to derive M_a by deflections on a sensitive D variometer of known scale value as explained above, or by magnetometer deflections. Then S_D^γ, S_H, and S_Z may be determined by equations (289), (294), and (295) as shown by the sample observations in figure 73. Note that for low-sensitivity H variometers the ah and $b(D-\overline{D})$ terms of equation (290) are negligible, so that S_H^*, S_H, and S_0 are practically equal and are denoted by S_H.

321. ***Relation between the minute and gamma D scale values.***—In figure 74, NS is the magnetic meridian through the center of the D recording magnet, M_s. Let the horizontal intensity be denoted by

$$H=ON.$$

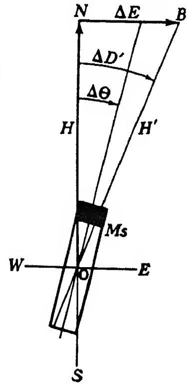

Figure 74.—Relation between the minute and gamma D scale values.

322. Suppose an east field applied at O, of magnitude $\Delta E=NB$. The direction of the resultant of H and ΔE will be OB and its magnitude will be H', practically equal to H. The change in declination will be the angle $\Delta D'$ (the D' meaning D expressed in minutes of arc). (The magnet turns through the angle $\Delta\theta$, being restrained by the suspension fiber and by any existing C field—see par. 200, p. 74.)

We may write

$$\frac{\Delta E}{H}=\tan\,\Delta D'$$

$$=\Delta D'\,\tan\,1'\ \text{(when } \Delta D' \text{ is small, by eq. 380)}$$

and $$\Delta E=\Delta D'\,H\,\tan\,1'. \qquad (299)$$

323. We are concerned here with the ratio of the applied field, ΔE, to the change of the ordinate of the D spot, Δn (Δn in mm). Dividing both sides of (299) by Δn,

$$\frac{\Delta E}{\Delta n}=\frac{\Delta D'}{\Delta n}\,H\tan 1'. \qquad (300)$$

But $\frac{\Delta D'}{\Delta n}$ is defined as the *minute* scale value, SD', and $\frac{\Delta E}{\Delta n}$ as the *gamma* scale value, S_D^γ. Hence,

$$S_D^\gamma=S_D'H\tan 1'=\frac{H}{3438}\,S_D' \qquad (301)$$

and $$S_D'=\frac{S_D^\gamma}{H\tan 1'}=\frac{3438}{H}\,S_D^\gamma. \qquad (302)$$

Conversions based on this relation may be approximated with the aid of a nomogram (app. VI, fig. 142).

ELECTROMAGNETIC METHODS

324. ***Helmholtz-Gaugain coil.***—A simple Helmholtz-Gaugain coil consists of two similar coils on a common axis, connected in series and separated by a distance, b, equal to the radius of either coil.[1] A direct current flowing in the system produces a magnetic field of high uniformity along the axis at the center of the coil system. The intensity of the field, f, at the geometric center of the system is, in oersteds,

$$f=\frac{32\pi}{5\sqrt{5}}\cdot\frac{ni_{(emu)}}{b} \tag{303}$$

$$=\frac{8.99\,ni_{(emu)}}{b} \tag{304}$$

in which

n = the number of turns of wire on each coil;
b = the radius of either coil (also their separation);
$i_{(emu)}$ = the current in absolute units (abamperes), 1 abampere being equivalent to 10 amperes (practical units).

If the current is expressed in milliamperes, i, and the field is expressed in gammas, f_γ, equation (304) becomes,

$$f_\gamma=\frac{8.99\,ni}{10\times 1000b}\times 10^5 \tag{305}$$

$$=\frac{89.9\,ni}{b}. \tag{306}$$

If C, called the coil constant, represents the gamma field per milliampere, then

$$C=89.9\,\frac{n}{b} \tag{307}$$

and

$$f_\gamma=iC. \tag{308}$$

Example: Given, $n=1$; $b=12$ cm; and $i=1$ milliampere; then, by equation (306),

$$f_\gamma=\frac{89.9}{12}=7.49\gamma.$$

325. ***Scale-value observations with coil.***—The field, f, may be reversed by reversing the current. Special precautions are taken in the details of construction of the coils to insure a close approximation to a circular current in each coil. Also, the lead-in wires must be twisted together for some distance away from the coils. The coils are so constructed that they may be placed around any one of the three variometers of a magnetograph, with the axis of the coil on the line along which the field is to be applied and so that the recording magnet is well centered within the coil system in each case (see fig. 75). For the D variometer, the coil axis is in the magnetic prime vertical; for the

[1] S. G. Starling, Electricity and magnetism for degree students, 7th edition, 1941, p. 53.

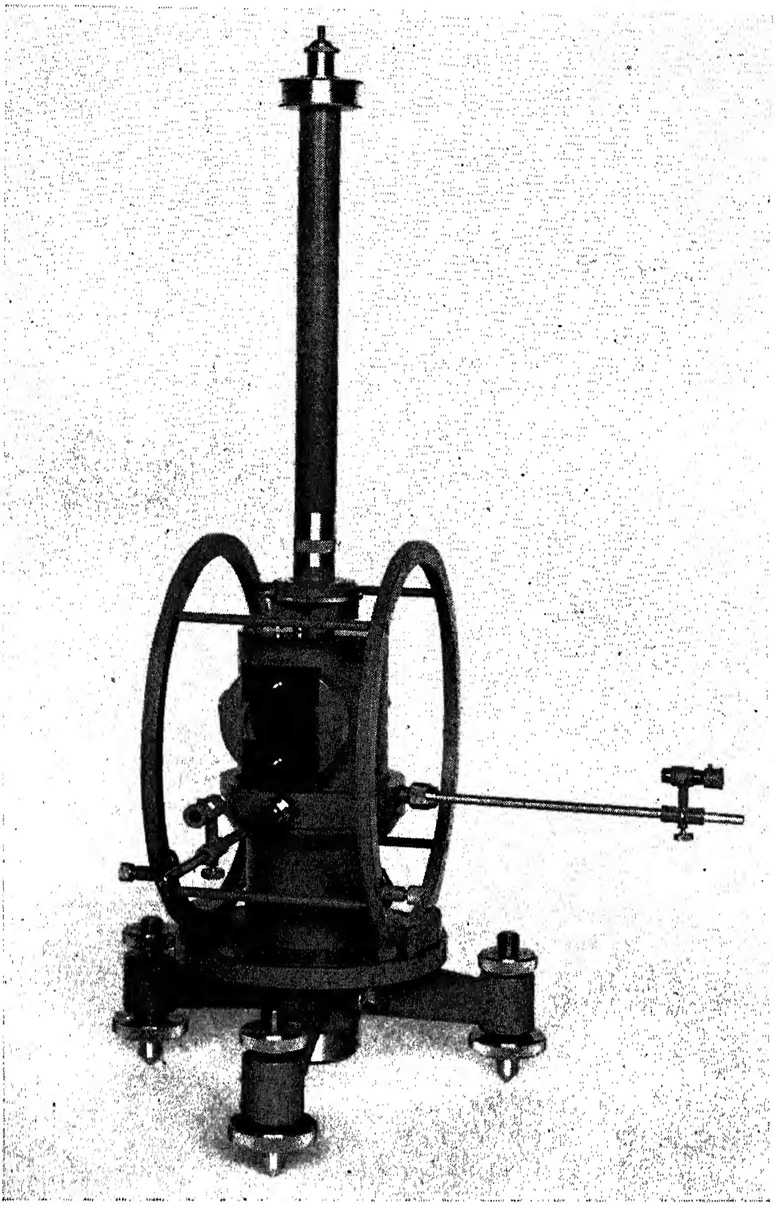

Figure 75.—Horizontal-intensity variometer equipped with Helmholtz-Gaugain coil, temperature compensating magnet, and sensitivity-control magnet.

H variometer, it is in the magnetic meridian; and for the *Z* variometer, it is vertical.

326. To determine scale values, apply the current necessary to produce the desired deflection, first direct and then reversed, repeating the process 2 or 3 times, the last deflection to be made with the current flowing in the same direction as in the first, say, 3 direct and 2 reversed.

The gamma scale value, S, of the variometer is simply the field, f_γ, divided by the deflection, u, in mm; that is,

$$S=\frac{f_\gamma}{u}.$$

327. ***External field.***—The approximate external field of a Helmholtz-Gaugain coil at a distance, r, from its center may be calculated from equations (23), (25), and (26) by substituting for M_a the equivalent magnetic moment of the energized coil, provided the distance, r, is great compared with the coil radius, b. The equivalent magnetic moment, M_a, is given by

$$M_a=2\pi b^2 i_{\text{(emu)}} \tag{310}$$

or

$$=2\pi b^2 i\times 10^{-4}. \tag{311}$$

In this case, the angle θ of equations (23), (25), and (26) is the angle between the direction of r and the direction of the field along the coil axis.

SCALE VALUE AND GEOMAGNETIC LATITUDE

328. ***Selection of scale value.***—In general, the scale values shown in table 11 would seem to meet most requirements.

329. The D variometer of the U. S. Coast and Geodetic Survey Magnetic Observatory at Tucson, Arizona, is operated at a scale value of 0.5 minute per mm. It has an optical lever of double the normal value, obtained by using a double reflection from the moving mirror, as explained in paragraph 162, page 64.

TABLE 11.—***Recommended scale values for observatories in different latitudes*** [a]

GEOMAGNETIC LATITUDE OF OBSERVATORY	SCALE VALUE WHEN OBSERVATORY HAS VARIOMETERS OF NORMAL SENSITIVITY ONLY			SCALE VALUE WHEN OBSERVATORY HAS VARIOMETERS OF BOTH HIGH AND LOW SENSITIVITY					
				High sensitivity			Low sensitivity		
	D	H	Z	D	H	Z	D	H	Z
°	′/mm	γ/mm	γ/mm	′/mm	γ/mm	γ/mm	′/mm	γ/mm	γ/mm
0–30	1	4	4	1	3	4	1	15	4
30–50	1	5	6	1	3	4	4	25	15
50–60	3	15	15	1	5	5	5	30	25
60–90	5	25	25	1	7	7	9	45	45

[a] For the committee report in which this table first appeared, see Intl. Assn. Terr. Mag. Electr., Bul. 13, 1950, pp. 335-338.

CHAPTER 12. ORIENTATION OF VARIOMETER RECORDING MAGNETS

NECESSITY FOR TESTING ORIENTATION

330. ***Standard orientation.***—For satisfactory performance, the recording magnets of variometers should be oriented as follows:

D variometer: magnetic axis in or very close to the *magnetic meridian* through its center.

H variometer: magnetic axis in or very close to the *magnetic prime vertical* through its center; N end east or N end west.

Z variometer: magnetic axis *horizontal*, usually N end to the north or south, but may be oriented in any azimuth if the optical system is suitably designed.

331. Maladjustment of any variometer will cause it to respond not alone to the element for which it is intended but to another element as well. Thus, a misoriented D or H magnet registers a composite of D and H fluctuations, while an out-of-level Z magnet is spuriously affected by H (assuming it to be operating in the magnetic meridian). For the magnitude of these spurious effects, see the nomograms shown in figures 143–145.[1] It is no easy matter to achieve, or to maintain, the high accuracy of adjustment needed to render these spurious effects wholly negligible. Either gradual relaxation of the H suspension or secular change of declination, for example, will result in a slow change in the effective orientation of the H magnet. The orientation of each recording magnet should be determined at the time of installation of the variometers, at regular intervals of one to three years thereafter, and at any time there is reason to suspect maladjustment.

332. ***Maladjustments.***—If there is appreciable torsion in the D fiber or if the D magnet is acted upon by the E–W component of any stray field (such as the field of another magnet of the magnetograph) the axis of the D magnet will be deflected out of the magnetic meridian. The angle between the magnetic axis of the D magnet and the magnetic meridian is called the *exmeridian angle*. If the torsion in the H fiber is not properly adjusted, the magnetic axis of the H magnet will not lie precisely in the magnetic prime vertical. The angle between the prime vertical and the magnetic axis of the H magnet is called the *ex-prime-vertical angle*. If the balancing poises of the Z magnet are not properly adjusted the magnetic axis of the Z magnet will not be horizontal. The angle between the magnetic axis of the Z magnet and the horizontal is called the *exlevel angle*. All of these angles of maladjustment are exorientation angles and are usually designated as E_x angles.

333. The established method of testing for misorientation is based on the application of a known field to the recording magnet along the direction of its *standard* orientation. For a D variometer, for example, this field is established in the magnetic meridian. If the recording magnet is properly oriented it will not be deflected, whereas

[1] The formulas used for two of the nomograms are derived by Howe, *op. cit.*, p. 41 (see item 6 of bibliography).

if it is maladjusted there will be a component of the field normal to the recording magnet and it will be deflected. For small exmeridian angles, the angular deflection will be proportional to the field applied, and will be practically proportional to the exmeridian angle itself; therefore the angular deflection is a measure of the exmeridian angle. This deflecting field may be supplied by a large magnet, M_a, having a magnetic moment of 10,000 to 15,000 cgs units, properly oriented and placed with respect to the D magnet. It is usually placed at one to two meters from the recording magnet so that its field at the center of the variometer magnet will be from 200γ to 1000γ, depending upon the sensitivity of the variometer. In general, the larger the gamma scale value of the variometer, the larger the field required for the tests.

334. In like manner the E_x angles of the H and Z recording magnets may be determined. Detailed directions for determination of these angles are given in paragraphs 341 to 362.

ORIENTATION FORMULAS

335. ***D variometer***.—In figure 76, let y be the field, or its equivalent, that is causing the D recording magnet, M_s, to have a small exmeridian angle, α, and let H be the horizontal component of the earth's field. Then $\frac{y}{H}=\tan\alpha$. When a deflector, M_a, figure 77, is placed with its

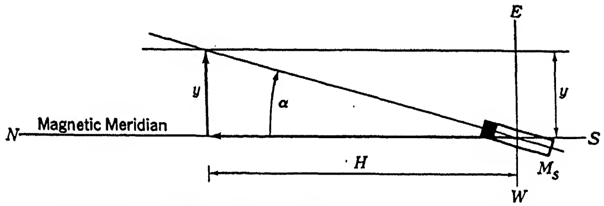

Figure 76.—*D*-variometer magnet not in magnetic meridian.

magnetic axis horizontal and in the magnetic meridian through M_s, at a distance r from M_s, at the same elevation as M_s, and with its N end south, it will have a field $-f_a=\frac{-2M_a}{r^3}$ at M_s and the field will be directed south (opposite to H). M_s will now be deflected through a small angle β_1, due to a small component of the field of M_a normal to M_s. Evidently M_s will now lie along the resultant of $(H-f_a)$ and y, consequently also along the resultant of H and $(y+\Delta y_1)$. That is, Δy_1 is the equivalent east field that would produce the deflection actually produced by f_a. Let u_1 be the deflection in mm of the D spot on the magnetogram corresponding to the deflection angle β_1. If S_D^γ is the gamma scale value of the variometer, then

$$\Delta y_1=S_D^\gamma u_1. \tag{312}$$

Also

$$\tan(\alpha+\beta_1)=\frac{y}{H-f_a}. \tag{313}$$

By similar triangles,

$$\frac{y}{H-f_a}=\frac{\Delta y_1}{f_a} \tag{314}$$

and

$$\Delta y_1=f_a\tan(\alpha+\beta_1)=S_D^\gamma u_1. \tag{315}$$

336. Reversing the deflector, figure 78, reverses the field f_a at M_s so that M_s is now deflected through a small angle β_2, by a small field normal to M_s, with corresponding deflection, u_2, of the D spot.

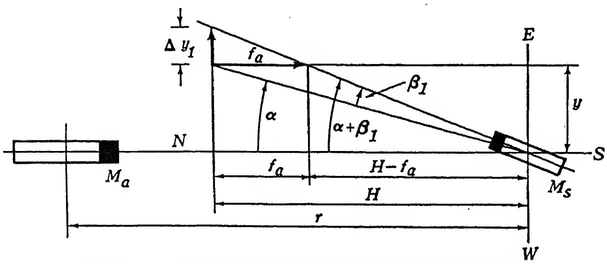

Figure 77.—Test of orientation of D-variometer magnet; deflector placed N end south

The equivalent west field that would produce this deflection if f_a were zero is Δy_2. Then,

$$\Delta y_2 = S_D^y u_2 \tag{316}$$

and

$$\tan\,(\alpha-\beta_2)=\frac{y}{H+f_a}. \tag{317}$$

By similar triangles

$$\frac{y}{H+f_a}=\frac{\Delta y_2}{f_a}=\tan\,(\alpha-\beta_2) \tag{318}$$

and

$$\Delta y_2 = f_a \tan\,(\alpha-\beta_2)=S_D^y u_2. \tag{319}$$

For small angles

$$\tan\,(\alpha+\beta_1)\approx\tan\alpha+\tan\beta_1 \tag{320}$$

and

$$\tan\,(\alpha-\beta_2)\approx\tan\alpha-\tan\beta_2. \tag{321}$$

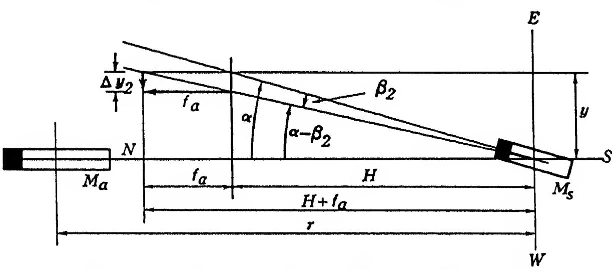

Figure 78.—Test of orientation of D-variometer magnet; deflector placed N end north.

Then from equation (315)

$$\frac{S_D^y u_1}{f_a}=\tan\alpha+\tan\beta_1 \tag{322}$$

and from equation (319)

$$\frac{S_D^y u_2}{f_a}=\tan\alpha-\tan\beta_2. \tag{323}$$

Adding equations (322) and (323),

$$\frac{S_D^\gamma(u_1+u_2)}{f_a}=2\tan\alpha+\tan\beta_1-\tan\beta_2 \tag{324}$$

and
$$\tan\alpha=\frac{S_D^\gamma(u_1+u_2)}{2f_a}-\tfrac{1}{2}(\tan\beta_1-\tan\beta_2). \tag{325}$$

Let $u_1+u_2=2u$; $2f_a=\frac{4M_a\times10^5}{r^3}$; $\tan\beta_1=\beta_1$; and $\tan\beta_2=\beta_2$; then

$$\tan\alpha=\frac{2ur^3S_D^\gamma}{4M_a\times10^5}-\tfrac{1}{2}(\beta_1-\beta_2). \tag{326}$$

337. In normal operation of a D variometer, it is evident from an examination of figures 77 and 78 that larger values of β_1 and β_2 will be obtained when f_a increases with respect to H. An extreme case might be when $H=8000\gamma$ and $f_a=1000\gamma$. From equations (313) and (317),

$$\tan(\alpha+\beta_1)=\frac{y}{H-f_a} \tag{327}$$

and
$$\tan(\alpha-\beta_2)=\frac{y}{H+f_a}. \tag{328}$$

Applying equations (320) and (321) and assuming that $\tan\alpha=\alpha$; $\tan\beta_1=\beta_1$; and $\tan\beta_2=\beta_2$,

$$\alpha+\beta_1=\frac{y}{H-f_a} \tag{329}$$

and
$$\alpha-\beta_2=\frac{y}{H+f_a}. \tag{330}$$

Adding equations (329) and (330),

$$2\alpha+\beta_1-\beta_2=\frac{y}{H-f_a}+\frac{y}{H+f_a}.$$

Since $\tan\alpha=\frac{y}{H}$,

$$\beta_1-\beta_2=\frac{y}{H-f_a}+\frac{y}{H+f_a}-\frac{2y}{H}.$$

Divide both sides by 2α,

$$\frac{\beta_1-\beta_2}{2\alpha}=\frac{\frac{y}{H-f_a}+\frac{y}{H+f_a}}{\frac{2y}{H}}-\frac{\frac{2y}{H}}{\frac{2y}{H}} \tag{331}$$

$$=\tfrac{1}{2}H\left(\frac{1}{H-f_a}+\frac{1}{H+f_a}\right)-1 \tag{332}$$

which simplifies to

$$\frac{\frac{1}{2}(\beta_1-\beta_2)}{\alpha}=\frac{f_a^2}{H^2-f_a^2}. \tag{333}$$

Thus if $f_a=1000$ and $H=8000$, $\frac{f_a}{H}=\frac{1}{8}$ and the ratio $\frac{f_a^2}{H^2-f_a^2}=\frac{1}{64-1}=$ 0.016. This shows that in equation (326) the term $\frac{1}{2}(\beta_1-\beta_2)$ may be safely neglected in computing tan α and tan E_x. Then

$$\tan\alpha=\tan E_x=\frac{2ur^3S_D^\gamma}{4M_a\times10^5} \tag{334}$$

in which $E_x=\alpha$ and $2u$ is the double deflection of the D spot in mm. Note: The scale value of the D variometer will be changed as a result of the large component of the deflector field parallel to M_s. This component enters as the term C in the general equation of the gamma scale value. That is,

$$S_D^\gamma=\left(\frac{\cot 1'}{2R}\times\frac{f}{f-h}\times\frac{H+C}{H}\right)H\tan 1'. \tag{335}$$

In orientation tests the C field, practically equal to f_a, may be of the order of 500γ. Suppose $H=20{,}000\gamma$. Then the factor, $\frac{H+C}{H}=$ $1+\frac{C}{H}=1+\frac{500}{20{,}000}=1.025$ for the N end of the deflector north and $1-\frac{C}{H}=0.975$ for the N end south. As a result of this change in scale value, the deflections will be unequal by a small amount; in this case, by 5 percent. Through the use of the mean gamma scale value in the computation of E_x by equation (334), these effects will cancel each other for all practical purposes. The foregoing proof disregards torsion in the fiber, but even for a stiff fiber the important results (eqs. 326 and 334) still hold.

338. ***Z variometer.***—In making orientation tests of the Z recording magnet the deflector is placed in the same relative position as for the D variometer. Figures 76, 77, and 78 apply except that they now represent side elevations as seen from the west side of the Z variometer, and the line marked *magnetic meridian* is now the horizontal plane through the center of the deflector and the center of the Z magnet. The development of the orientation equation follows the same steps as for D, and

$$\tan E_x=\frac{2ur^3S_Z}{4M_a\times10^5} \tag{336}$$

in which S_Z is the mean Z scale value and $2u$ is the double deflection of the Z spot in mm.

339. ***H variometer.***—In figure 79, let x be a small field or its equivalent that is deflecting the recording magnet, M_s, out of the magnetic prime vertical by a small angle α. Then since k (par. 232, p. 87) may be treated as an equivalent field,

$$\tan\alpha=\frac{x}{k} \tag{337}$$

and if there is a sensitivity magnet having a field, p, at M_s, parallel to and directed the same as M_s, then

$$\tan\,\alpha = \frac{x}{k+p}. \tag{338}$$

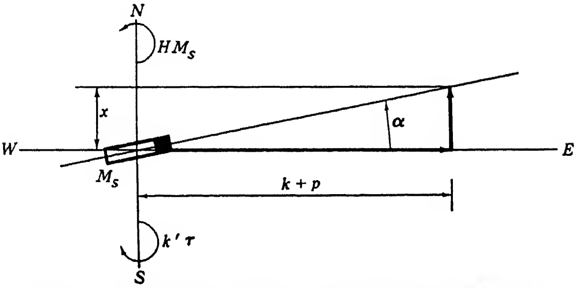

Figure 79.—*H*-variometer magnet not in magnetic prime vertical.

340. With the deflector in position for orientation tests, M_s will be deflected through a small angle β_1, figure 80, such that

$$\tan\,(\alpha-\beta_1) = \frac{x}{k+p+f_a} = \frac{\Delta x_1}{f_a} \tag{339}$$

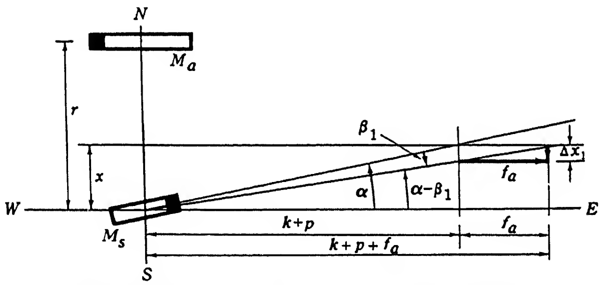

Figure 80.—Test of orientation of *H*-variometer magnet; deflector placed *N* end west.

in which f_a is the field of M_a at the center of M_s, and Δx_1 is the component of f_a practically normal to M_s and causing the small deflection β_1. When the deflector is reversed, figure 81,

$$\tan\,(\alpha+\beta_2) = \frac{x}{k+p-f_a} = \frac{\Delta x_2}{f_a}. \tag{340}$$

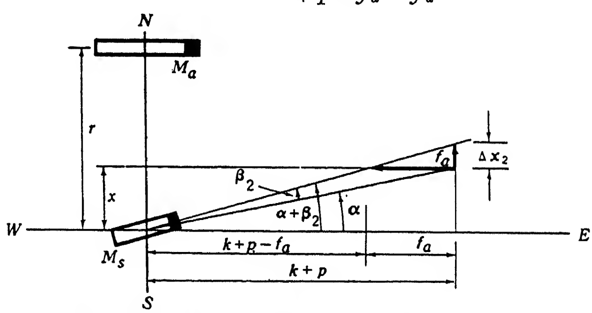

Figure 81.—Test of orientation of *H*-variometer magnet; deflector placed *N* end east.

For small angles,

$$\tan(\alpha-\beta_1)\approx\tan\alpha-\tan\beta_1,$$

and

$$\tan(\alpha+\beta_2)\approx\tan\alpha+\tan\beta_2.$$

Then

$$\tan\alpha-\tan\beta_1=\frac{\Delta x_1}{f_a}=\frac{S_H u_1}{f_a} \tag{341}$$

and

$$\tan\alpha+\tan\beta_2=\frac{\Delta x_2}{f_a}=\frac{S_H u_2}{f_a} \tag{342}$$

in which S_H is the H scale value, and u_1 and u_2 are the deflections in mm of the H spot corresponding to β_1 and β_2. Putting $u_1+u_2=2u$ and adding equations (341) and (342),

$$2\tan\alpha=\frac{2S_H u}{f_a}+(\tan\beta_1-\tan\beta_2). \tag{343}$$

Neglecting the term $(\tan\beta_1-\tan\beta_2)$, which will always be small compared to α,

$$\tan\alpha=\frac{2uS_H}{2f_a}$$

$$\tan\alpha=\tan E_x=\frac{2ur^3S_H}{2M_a\times10^5}, \tag{344}$$

since $2f_a=\frac{2M_a}{r^3}\times10^5$ (deflector in B position).

DIRECTIONS FOR ORIENTATION TESTS

341. ***Critical adjustment.***—Read carefully appendix III, giving special attention to those parts dealing with critical adjustments of the deflector. Table 18 of appendix III gives a summary of critical adjustments.

342. ***Orientation bench.***—If space in the variation room permits, construct a permanent bench or table similar to that shown in figure 82. Use nonmagnetic materials throughout. Adjust the bench plates (wood, brass, or aluminum plates upon which the deflector rests) in elevation so that when the deflector is in proper position for deflections, its geometric axis will be horizontal and at the same elevation as the recording magnet to be tested.

343. ***Adjustment of the deflector to correct elevation.***—Place a short piece of glass tubing in each end of a long piece of flexible rubber tubing and fill the tube with water, free of air bubbles. Hold one tube vertical near the recording magnet to be tested and the other end near the bench plate opposite the variometer. When the water surface, bottom of the meniscus, is at the precise elevation of the center of the recording magnet and the water is at rest, mark on the bench-plate support the height of the water meniscus in the adjacent glass tube. Adjust the bench-plate supports in thickness until the deflector is at the correct height when resting in proper position on the plate. Check carefully. (Note: The recording magnets may not

all be at the same elevation; the three bench plates must be adjusted separately.)

344. If it is not feasible to construct a permanent bench as described in paragraph 342, use a small nonmagnetic plane table for supporting the deflector.

345. ***Establishing magnetic meridians through the variometers.***—If magnetic meridians have been established previously, proceed as outlined in paragraph 348. Otherwise establish these magnetic meridians as outlined below.

346. Using a transit or theodolite, run a traverse from the declination pier in the absolute observatory to the variation room. This line, preferably about parallel to the magnetic meridian, should be

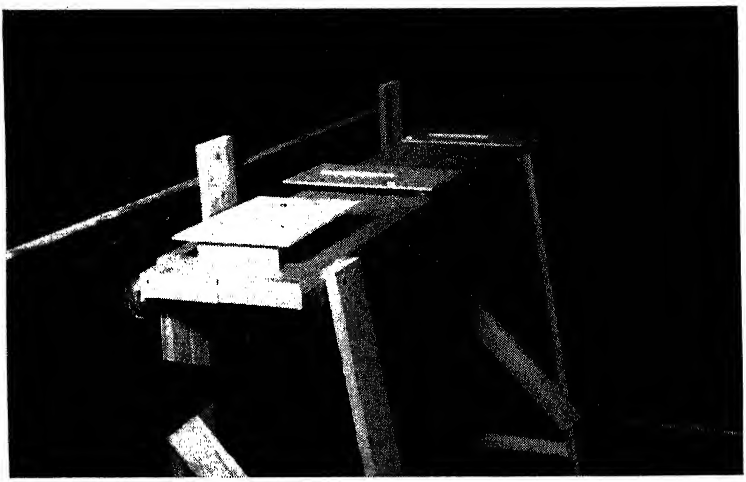

Figure 82.—Orientation bench.

marked with permanent vertical lines inscribed on brass plates on the north and south (inside) walls of the variation room. The plates should bear the inscription: *True Bearing,* ------. (For example: *N 14° E.*) A simple computation will then, at any time, furnish data for establishment of the magnetic meridian by offsetting one end of the line established by the two permanent marks. An illustration of this is given in figure 83. The line AB, established by traverse, has a true bearing of N 14° E. If the mean magnetic declination in the absolute observatory is 13°48′ East, then the desired magnetic meridian in the variation room will be AB', making an angle of 12′ with AB. If the length of the line AB=300 cm, the offset distance, BB' will be 3000 tan 12′ or 10.5 mm to the west of the point B.

347. Make a sketch of the traverse, showing all turning points, courses, and angles on the sketch. Furnish also a record of all circle readings. Check carefully.

348. At approximately the same elevation as the marks described in paragraph 346, mount two wooden strips horizontally on the north and south walls of the variation room. A convenient size for the strips is 2 by 10 cm in section by 3 meters in length, and they should be held away from the walls by spacing blocks 2 cm thick.

349. Stretch a white linen or cotton thread between the wooden strips and attach a small weight to each end of the thread to keep it taut as it passes over the top edges of the strips. Attach a light-weight plumb bob to the thread in such a way that the point of the bob can be precisely centered over the top of the *D* variometer. By moving the thread along the wall strips so the two ends of the thread are the same distance (within ½ mm) from the points *A* and *B′*, figure 83, the thread can be made to lie in the magnetic meridian passing through the center of the *D* variometer. Now file small grooves in the wooden strips directly beneath the ends of the threads and mark these grooves with a label; such as, *D, Jan. 1, 1951.*

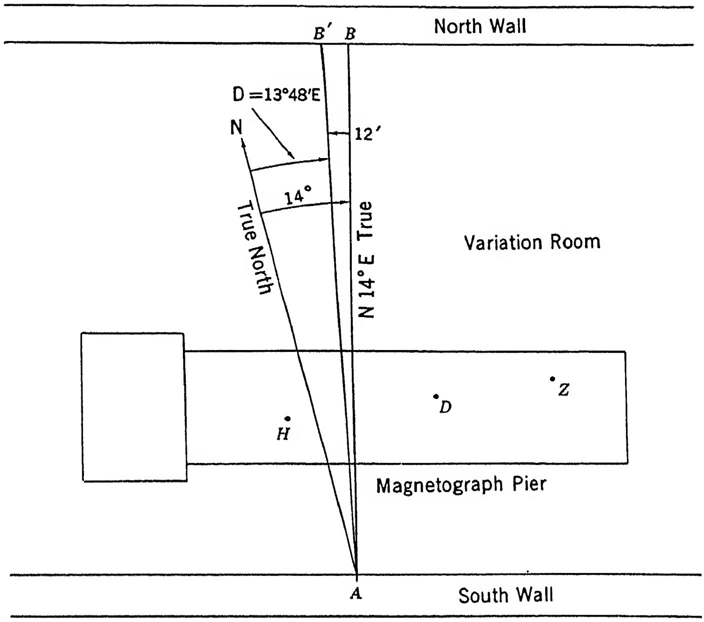

Figure 83.—Establishing the magnetic meridians through the variometers.
AB=established true bearing;
AN=true north;
AB′=magnetic meridian (for magnetic declination of 13°48′ East).

350. In like manner establish magnetic meridians through the *H* and *Z* variometers. (Note: Center of variometer means center of recording magnet.) Label and date each mark.

351. Suspend a light-weight plumb bob (perhaps a suitably sharpened copper nail or brass screw so tied to a piece of thread that it will hang straight with the vertical) from the meridian thread of the *D* variometer and carefully mark a point on the bench plate. Move the bob along the meridian thread and mark a second point, making the two marked points as far apart as possible. These two points then mark the centerline of the orientation deflector. Prepare a wooden strip and fasten it to the bench plate with brass screws (preferably in a way that will permit final adjustment of the guide strip after the screws are started) making due allowances for the size of the deflector magnet or its case so that the deflector will be centered on the meridian and parallel to the meridian. Fasten a

second wooden strip to the bench plate to form a stop that will determine the distance between the center of the deflector magnet and the center of the *D* recording magent. Both wooden strips (guides) should be well made, with straight sides, so that the deflector will fit firmly and positively in one position only when placed against them on the bench plate. All markings of points and lines should be done as precisely as the materials at hand will permit.

352. Mark a meridian line on the bench plate that will be used for the *H* variometer in the same manner as that described for the *D* plate. With a draftsman's triangle having a true 90° angle, mark a line on the bench plate perpendicular to the meridian line at the desired distance from the center of the *H* recording magnet. This prime vertical line will be the guide line for determining the position of the guide strip that should now be fastened to the bench plate as was done for the *D* position. Fix an end position stop, also, to the bench plate so that the center of the deflector magnet will be precisely in the magnetic meridian through the *H* variometer. Check the work carefully. It is important that the work be done with great precision.

353. Establish the meridian line on the *Z* bench plate in the manner described for *H* and *D*. Make a second check on the *Z* bench plate to see that the center of the deflector magnet will be at the *same elevation* as the *Z* recording magnet, and to see that the deflector will be *level.* (Test the level of the deflector with the sensitive stride level of the earth inductor.) Fasten wooden guide strips to the bench plates to serve as guides or stops against which the deflector may be placed in making orientation tests.

354. The deflection distances, *r*, need not be the same for all variometers. Measure and record these distances to the nearest millimeter.

355. It cannot be emphasized too strongly that every effort should be made to obtain precisely fixed positions for the deflector magnet for orientation tests, particularly to insure that the deflector shall be accurately parallel to the established magnetic meridian for *D*, accurately parallel to the established prime vertical for *H*, and accurately level for *Z*.

356. The procedures described above have been outlined in detail. There are, of course, other ways of accomplishing the results with equal if not greater precision, and the observer may adopt whatever methods seem to be most desirable. Experience has shown the author, however, that in any event a complete, detailed record of the methods and procedures followed will inevitably be highly useful at some later time.

357. ***Orientation deflections.***—On a magnetically quiet day, make orientation deflections precisely as outlined in figure 84, making certain that the deflector is always against the guides. Begin with the *D* variometer and see that the first deflection is made with the deflector *face up, N end to the north.*

358. Complete all of the deflections indicated on the form, in precisely the order indicated and with the deflector oriented properly for the indicated deflections.

359. Measure and record, as in figure 84, the deflection distances for each variometer. Record also the magnetic moment of the deflector (in terms of colog *M*, line 19), and the gamma scale value of all variometers (line 15).

360. From the magnetogram, scale and record all deflections and the *away* ordinates, and complete the computations for the numerical values of the orientation angles.

ORIENTATION TESTS

(DEFLECTOR NORTH OR SOUTH OF VARIOMETER)

Magnetograph: {*Eschenhagen D & H.* / *la Cour Z* — Magnetic Observatory: *Cheltenham*
Observer: *S. G. Townshend*

1. Variometer	*D*	*H*	*Z*
2. Date	*June 24 1937*	*June 24 1937*	*June 24 1937*
3. Approximate time	*15 20*	*15 40*	*16 10*
4.	Ord. mm	Ord. mm	Ord. mm
5. Away	*−3.0*	*4.6*	*34.8*
6. Face up; N end of deflector	N *+2.8*	W *6.1*	N *36.6*
7. Face up; N end of deflector	S *−9.5*	E *4.3*	S *32.6*
8. Face down; N end of deflector	S *−9.5*	E *8.7*	S *32.9*
9. Face down; N end of deflector	N *+2.8*	W *7.0*	N *36.7*
10. Away	*−3.0*	*10.0*	*34.4*
11. Mean, N end (A)	N *+2.8*	W *+6.6*	N *36.6*
12. Mean, N end (B)	S *−9.5*	E *+6.5*	S *32.8*
13. $2u$ (A)−(B)	*+12.3*	*+0.1*	*+3.8*
14. r	*129 cm*	*129 cm*	*134 cm*
15. $S\gamma$	*5.30 γ/mm*	*3.30 γ/mm*	*3.54 γ/mm*
16. Log $2u$	*1.090*	*9.000*	*0.580*
17. 3 log r	*6.333*	*6.333*	*6.381*
18. Log $S\gamma$	*0.724*	*0.519*	*0.549*
19. Colog M	*6.129*	*6.129*	*6.129*
20. Colog 2×10^5	----------	*4.699*	----------
21. Colog 4×10^5	*4.398*	----------	*4.398*
22. Log tan E_x	*8.674*	*6.680*	*8.037*
23. E_x (to nearest minute of arc)	*2° 42′*	*0° 02′*	*0° 37′*
24. N end of recording magnet	*E of N*	*S of E*	*Up*
25. Formula tan $E_x=$	$\frac{2\,u\,r^3\,S_D\gamma}{4M\times10^5}$	$\frac{2\,u\,r^3\,S_H\gamma}{2M\times10^5}$	$\frac{2\,u\,r^3\,S_Z\gamma}{4M\times10^5}$

Record all ordinates to nearest 0.1 mm with same sign as in scaling an ordinate for a base line value.
Colog M derived from: *Regular scale-value observations.*

Figure 84.—Observations for orientation tests.

361. Prepare a tabulation similar to that shown in table 12.

362. By means of table 13, identify the quadrant in which the *N* end of a recording magnet lies for the away position of the deflector. Record the results on line 24, figure 84.

363. ***Adjustment of recording magnets.***—If a calculated E_x angle for any recording magnet is in excess of 1°, the recording magnet should be adjusted for correct orientation and the orientation tests repeated. Adjusting and testing should be continued until satisfactory orientation is indicated by the recorded deflections.

TABLE 12.—*Miscellaneous data*

OBSERVATORY: Cheltenham DATE: Dec. 31, 1951

D variometer	(a) Declination is West. (b) *D* spot moves up on the magnetogram for numerical increase in declination.
H variometer	(c) *N* end of *H* recording magnet is toward the East. (d) *H* spot moves up on the magnetogram for numerical increase in horizontal intensity.
Z variometer	(e) *N* end of *Z* magnet is toward the North. (f) Absolute value of vertical intensity is positive (+). (g) *Z* spot moves up on the magnetogram for numerical increase in vertical intensity.

TABLE 13.—*Orientation of variometer magnets*

LINE	*D* Variometer	West Declination		East Declination	
1	When N end of deflector is toward	Magnetic north		Magnetic north	
2	The field of the deflector at the *D* variometer is directed	Magnetic north		Magnetic north	
		(a)	(b)	(c)	(d)
3	If the *D* spot moves in the direction of	Increasing West *D*	Decreasing West *D*	Increasing East *D*	Decreasing East *D*
4	The N end of the recording magnet lies in	NE quadrant	NW quadrant	NW quadrant	NE quadrant
5	*H* Variometer	N end of recording magnet east		N end of recording magnet west	
6	When N end of deflector is toward	Magnetic west		Magnetic west	
7	The field of the deflector at the *H* variometer is directed	Magnetic east		Magnetic east	
8	If the *H* spot moves in the direction of	Increasing *H*	Decreasing *H*	Increasing *H*	Decreasing *H*
9	The N end of the recording magnet lies in	SE quadrant	NE quadrant	NW quadrant	SW quadrant
10	*Z* Variometer	N end of recording magnet north		N end of recording magnet south	
11	When N end of deflector is toward	Magnetic north		Magnetic north	
12	The field of the deflector at the *Z* variometer is directed	Magnetic north		Magnetic north	
13	If the *Z* spot moves in the direction of numerically	Increasing *Z*	Decreasing *Z*	Increasing *Z*	Decreasing *Z*
14	In North magnetic latitudes, the N end of the recording magnet is	Too high	Too low	Too low	Too high
15	In South magnetic latitudes, the N end of the recording magnet is	Too low	Too high	Too high	Too low

364. The *D* recording magnet can be brought into correct orientation by slight adjustment of its corrector magnet (see par. 381, p. 152). If a corrector magnet is used the E_x angle should not exceed 20′; if a corrector magnet is *not* used, an E_x angle up to 1° may be permitted. In any event, when acceptable orientation of the *D* magnet is achieved, if the *D* ordinate on the magnetogram is not correct (too large, too small, or even negative in sign), it will be necessary to adjust the *D* magnet relative to the mirror frame, in the manner described in paragraph 377, page 150. Recording of orientation deflections must, of course, be repeated after any mechanical adjustments are made.

365. In like manner the *H* and *Z* recording magnets may be adjusted to correct orientation by slight adjustments of the temperature-compensating magnets. In the case of *H*, for example: Let $E_x=1°$, *L* (recording distance)$=1200$ mm, and Δn=change in ordinate corresponding to 1° change in orientation. Then

$$\tan E_x=\frac{\Delta n}{2L}$$

and $$\Delta n=0.0175\times 2400=42 \text{ mm}.$$

If the H scale value is 5.0γ/mm, this would be equivalent to a change of 210γ in the field of the temperature-compensating magnet, an amount which would not appreciably affect the temperature coefficient of the variometer. In like manner the change of ordinate of the Z trace corresponding to an angular motion of the Z magnet may be computed. After proper orientation of the recording magnets is obtained, if the H or Z ordinates on the magnetogram are too large, or are negative, it will be necessary to adjust the recording mirror without changing the orientation of the magnet—for H, by turning the magnet relative to the mirror; for Z, by rotating the whole variometer and then readjusting the base line mirror, or by adjusting the prisms if the variometer provides for such adjustment.

366. In ordinary routine operation, readjustments of a recording magnet should not be made without the prior approval of the administrative authority responsible for the processing of the results.

CHAPTER 13. DIRECTIONS FOR INSTALLING A MAGNETOGRAPH

PRELIMINARY STEPS

367. ***The variation room.***—These directions apply to the installation of a complete magnetograph, in which the instruments are arranged as shown in figure 13 and schematically in figure 85. It is assumed that the absolute values of D, H, and Z on the observing piers in the absolute observatory are practically identical with the respective values on the variation-room pier; that the magnetograph pier and scale-value and orientation shelves and guides have been constructed in accordance with the general specifications in chapter 3; and that a line with known bearing has been established and permanently marked on the walls of the variation room.

368. *Comparison observations.*—If there is any doubt about magnetic materials having been introduced into the piers or the structural parts of the buildings, compare the values of D and H on the variometer pier and the piers in the absolute observatory by making one or two sets of horizontal intensity with a magnetometer or a Quartz Horizontal Magnetometer and at least two sets of declination with a magnetometer or a compass declinometer at both sites. Differences of more than a few minutes in declination or 50γ in horizontal intensity should not be tolerated.

369. ***Constants of component parts.***—Prepare or have on hand several D and H quartz fibers, calibrated and mounted as explained in chapter 6. Determine the dimensions and magnetic moments of all of the magnets to be used with the variometers. The magnetic moments may be determined by deflections, using a magnetometer, at a place where H is known. Label these magnets by number and by magnetic moments for future use. Determine the temperature coefficients of magnetic moment of the H and Z recording magnets and of the temperature compensating magnets for the H and Z variometers for use in the computations which follow, or use the temperature coefficients furnished with the magnets.

370. Record all available data on the appropriate lines in tables 14, 15, 16, and 17, as the work progresses.

371. Calculate the approximate distances at which the temperature compensating magnets for the H and Z variometers should be placed on these variometers for effective temperature compensation, and mount the magnets at those distances on their respective variometers. The H temperature magnet should be mounted with its N end to the south and the Z temperature magnet with its N end *up* in north magnetic latitudes and with its N end *down* in south magnetic latitudes.

372. ***Test of the optical system.***—Set up the three variometers, the recorder, time flasher, and thermograph at the approximate positions they will occupy in routine operation of the magnetograph. See that the recording magnets are in the instruments and that the N end of the Z recording magnet is to the north; also that the temperature magnets and sensitivity magnets are attached to their deflection bars

or holders in approximately the same positions and orientations in which they will be operated later. Details regarding the methods of deriving this information may be found in the chapter on temperature coefficients and in paragraphs 387, 390, 393, 394, and 401 of this chapter. Adjust the D recording distance to 174 cm (D lens to recording drum) and adjust the D base-line spot to the desired ordinate on

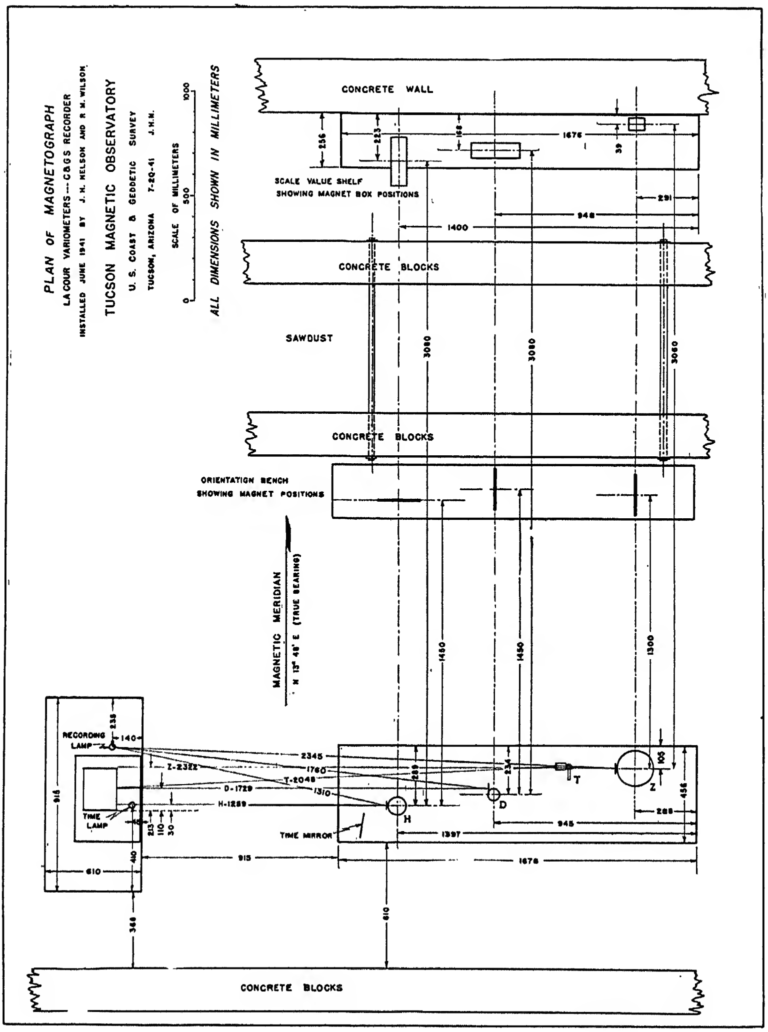

Figure 85.—Plan of magnetograph installation at Tucson Magnetic Observatory.

the drum. In like manner adjust the H and Z recording distances so that the H base-line spot and the Z base-line spot are in good focus on the drum, axes of all variometer lenses approximately perpendicular to the face of the recorder. See that the Z base line is not eclipsed by the D or H variometers when the Z variometer is rotated on its vertical axis for the full aperture of the recorder window. In like manner see that the D base line is not eclipsed by the H variometer. Using marble

blocks or nonmagnetic metal spacers, adjust the heights of all of the variometers so that the centers of all lenses are in the same horizontal plane as the axis of the recorder when all instruments are resting on their footplates and when each foot screw is approximately at the middle of its run. Also adjust the axis of the cylindrical lens to this same elevation. Mark on the pier top the positions of the three variometers and then take the H and Z instruments several meters away from the pier.

INSTALLING THE D VARIOMETER

373. ***Removal of torsion.***—Consult the nomogram, figure 140, appendix VI, and select a D fiber and D magnet combination that will give a D scale value close to one minute per millimeter at a recording distance of approximately 174 cm. Make preliminary visual tests for the torsion factor. Level the instrument and install the D fiber and the mirror frame *without the D magnet* but with a small torsion weight attached. Adjust the torsion head in azimuth and the mirror in inclination until the *regular D* spot (from the central mirror if a triple-faced mirror is used) falls at the desired ordinate when the suspended system is centered in the magnet chamber and is at rest. Clamp the torsion head in this position and set the fiber clamp. Remove the mirror frame from the hooks, then replace it, release the fiber clamp, and allow the system to come to rest. If the D spot comes to rest at approximately the same ordinate as before, the mirror frame is in good adjustment and probably hanging evenly on both hooks. *Always set the fiber clamp before removing or replacing the mirror frame.*

374. Record the torsion-head reading, R_0, when the torsion-weight axis is approximately in the magnetic meridian and the *regular D* spot is at the desired ordinate. Record also the position of the D spot on the recording-box scale, figure 86. Call this original scale reading n_0. Clamp the fiber.

375. Replace the torsion weight with the D magnet, N end to the north. Release the fiber clamp; carefully raise the damping chamber around the magnet. See that the magnet is well centered within the damping chamber by slight adjustment of the level of the instrument. Test the damping by deflecting the magnet and allowing it to come to rest. The system should be so adjusted that it will be somewhat underdamped. If the regular D spot comes to rest within 10 mm of the original scale reading, n_0, the D magnet is practically in the magnetic meridian and there is little or no torsion in the fiber.

376. If the D spot does not come to rest within 10 mm of the original scale reading, n_0, the magnet is not properly oriented with respect to the mirror, the mirror frame is not properly suspended on the hooks, magnetic conditions are not quiet, or there may be some magnetic material near the instrument. Observe the spot for several minutes. If magnetic conditions are not quiet, postpone operations. If magnetic conditions are quiet, remove the mirror frame and readjust the magnet in azimuth relative to the frame and test again. Repeat this operation until satisfactory adjustment has been obtained. If available, use the mirror-adjusting apparatus described below for adjusting the magnet relative to the frame.

377. ***Mirror-adjusting apparatus.***—This apparatus, figure 87, is used for turning the mirror frame and the attached mirror through

a small angle relative to the magnetic axis of the recording magnet. The lenses have focal lengths equal, respectively, to those on the *H* and *D* variometers. The whole apparatus is mounted on a non-magnetic laboratory stand. To turn the mirror through a small angle relative to the magnet, proceed as follows: Place the stand on the variometer pier so that the longer focus lens is parallel to and at the same height as the *D* lens, also at the same distance from the recorder. Clamp the magnet so that the frame stands vertically and the triple-

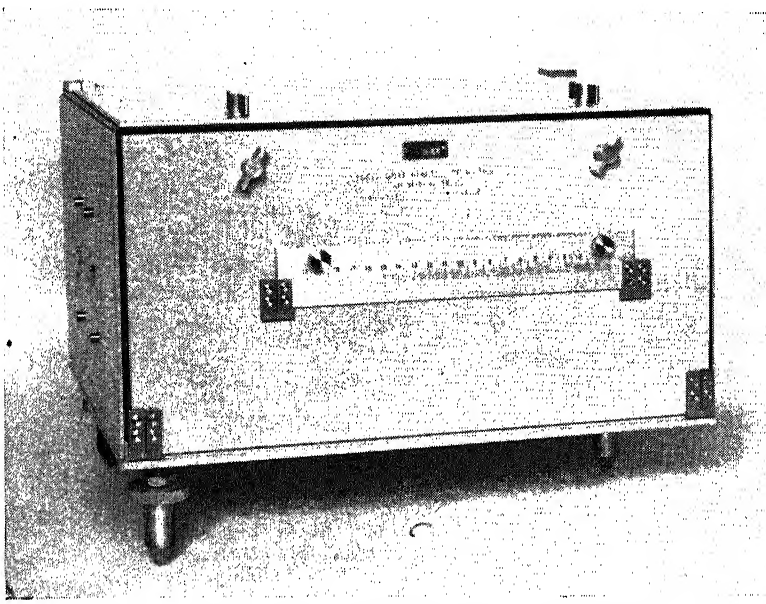

Figure 86.—Magnetograph recorder showing scale on front of cylindrical-lens window.

face mirror faces the lens. Find the image of the lamp filament as reflected from the *regular* mirror and adjust the stand in azimuth and the apparatus in elevation until the image of the *regular* *D* spot falls on the recorder scale. Then by means of a long adjusting pin, turn the mirror frame in azimuth until the image has moved through the desired change of ordinate.

378. Adjust the *D* base-line mirror to correct position in azimuth and elevation so that the *D* spot has an ordinate of about 20 mm and so that numerically increasing ordinate corresponds to increasing magnetic declination. Note that in some cases it may be necessary to have the *D* base line *above* the *D* trace in order to accomplish this.

379. Establish the magnetic meridian through the *D* variometer, adjust the guides of the orientation bench for *D*, and make orientation tests as described in chapter 12. Record the observations as in figure 84.

380. ***Measurement of stray east fields.***—When magnetic conditions are reasonably quiet, record *D* and the *D* base line photographically for one hour. Close the recorder window and quickly replace the *H* and *Z* variometers in their correct positions on the pier with temperature magnets, sensitivity magnets, and recording magnets all in place, as described in paragraph 371. Cover the *H* and *Z* lenses

with paper screens. The D spot may be deflected away from its original position, n_0, because of the resultant east (or west) component, f_E, of the fields of the magnets of the H and Z variometers. Record D and the D base line photographically for 30 minutes. Close the recorder window. Develop the magnetogram and scale the change in ordinate, Δn, of the D spot due to the resultant field, f_E, at D. Then

$$f_E = (\Delta n) S_D^{\gamma}. \tag{345}$$

Compute f_E, noting the sign. It may be as large as $\pm 150\gamma$.

381. ***The D corrector.***—If the deflection, Δn, is large, attach a short deflection bar to the D variometer on the side nearer the Z variometer, axis of the bar in the magnetic prime vertical through D. Attach the D corrector to this bar and adjust the deflection distance until the D spot returns to its original position, n_0, on the scale of the recorder; that is, make $\Delta n = 0$. A D corrector having a magnetic moment of 5 cgs units at a distance of 20 to 25 cm would furnish the required compensating field at D. If the field, f_E, deflects the D spot only a few mm, the D corrector is unnecessary, since an exmeridian angle this small would introduce no appreciable error in the recorded changes in declination.

382. Determine the torsion factor, $\frac{f}{f-h}$, by turning the torsion head 30° clockwise, then 60° counterclockwise (that is, 30° counterclockwise from its original position), then 30° clockwise. Record the torsion-head readings and the corresponding ordinates of the D spot. Record photographically or observe the ordinates of the D spot on a paper millimeter scale *attached to the drum.*

TABLE 14.—*D variometer; miscellaneous data*

OBSERVATORY: Cheltenham, Md. DATE: December 12, 1948
VARIOMETER: Schulze No. XI OBSERVER: J. Q. P.

Line	Symbol	Items	Examples
1		SECTION I. THE OPTICAL SCALE VALUE	
2	c	Maximum thickness of cylindrical lens	9 mm
3	b	Factor in equation (91); plane side of D lens out	$\frac{2}{3}$
4	l	Thickness of D lens at center	3 mm
5	$\frac{f}{f-h}$	Estimated torsion factor (or derived from preliminary visual tests)	1.006
6	H	Horizontal intensity	18300γ
7	C	Estimated north field or caculated (see appendix V)	+100γ
8	$\frac{H+C}{H}$	Field factor; equation (111)	1.005
9	L	Estimated distance, D lens to drum, for unit scale value, equation (132).	1739 mm
10	$\frac{f}{f-h}$	Torsion factor, measured; see Section IV below	1.011
11	L	Calculated from equation (132) using new torsion factor	1748 mm
12	R	Effective recording distance calculated from equation (91)	1747 mm
13	ϵ	Optical scale value, $\frac{1}{2R}$, in radians per mm	0.000 2862
14		SECTION II. VARIOMETER MAGNETS	
15	M_r	Magnetic moment of D recording magnet (Alnico II; cylindrical; 2 by 10 mm).	8.8 cgs
16	M_a	Magnetic moment of D corrector (Alnico II; 2 by 10 mm)	5.0 cgs
17	r	Deflection distance 25 cm toward Z variometer; N end	West

TABLE 14.—*D variometer; miscellaneous data*—Continued

Line	Symbol	Items	Examples
18		SECTION III. QUARTZ FIBER	
19	l	Effective length of fiber	15 cm
20	d	Diameter of fiber; several measurements uniformly spaced; calculated diameter as described in paragraph 188.	0.0017 cm
21	T	Period of system with torsion weight, I=1.83, attached	63 sec
22	k'	Torsion constant from dimensions; from equation (148)	0.0155
23	k'	Torsion constant from period; from equation (150)	0.0182
24	k'	Torsion constant from torsion tests; from equation (151)	0.0181
25		SECTION IV. TORSION OBSERVATIONS; SENSITIVE VARIOMETER	
26		Torsion head readings 0 30° right 30° left 0	
27	f	Angular motion of torsion head 1800′ 3600′ 1800′	
28	n	Ordinate of the D spot 42.2 62.1 22.1 42.2	
29	h	Displacement of the D spot 19.9′ 40.0′ 20.1′	
30	h	Mean h for f=1800′ 20.0′	
31	$f-h$	Twist in the fiber 1780.0′	
32	$\frac{f}{f-h}$	Torsion factor 1.011	
33		SECTION V. FIELD FACTOR	
34	C	Resultant N or S field at D magnet due to stray fields	+100γ
35	H	Mean value of H at the variometer pier	18300γ
36	$\frac{H+C}{H}$	Field factor; equation (111)	1.005
37		SECTION VI. EAST FIELD AT D VARIOMETER	
38	n_0	D ordinate on magnetogram, *H and Z variometers away*	47.5
39	n_1	D ordinate on magnetogram, H and Z variometers in place	60.0
40	Δn	Change in D ordinate due to resultant east field, f_E	12.5
41	f_E	East field from equation (345)	66γ
42	f_E	East field, calculated from equations in appendix V (Item 42 is optional).	70γ
43			
44		SECTION VII. COMPUTATION OF D SCALE VALUE	
45	S_D'	From equation (122); minute scale value	1.000
46	S_D^{γ}	From equation (301); gamma scale value	5.32
47		SECTION VIII. PARALLAX TESTS	
48	D	Correction in mm between time line and D spot	0.1 mm
49	H	Correction in mm between time line and H spot	0.0
50	Z	Correction in mm between time line and Z spot	0.0
51		SECTION IX. RESERVE DISTANCES	
52	D^R-D	Distance in mm on magnetogram, regular spot to upper reserve	187.1 mm
53	$D-D_R$	Distance in mm on magnetogram, regular spot to lower reserve	186.2 mm
54		SECTION X. LOW-SENSITIVITY VARIOMETER; TORSION TESTS	
55	ϵ	Optical scale value, D variometer (from line 13)	0.000 2862
56	L_1	Recording distance, auxiliary scale	1000 mm
57	ϵ_1	Optical scale value, auxiliary scale	0.000 500
58	n_0	D ordinate, no torsion in fiber 46.0 46.0	
59	n	D ordinate, magnet deflected by torsion 116.6 188.2	
60	$n-n_0$	Change of D ordinate 70.6 142.2	
61	s_0	Auxiliary scale reading, no torsion 1.5 1.5	
62	s	Auxiliary scale reading, magnet deflected by torsion. 52.1 103.3	
63	$s-s_0$	Change in scale reading 50.6 101.8	
64	h	Angular motion in radians of the D magnet= $\epsilon(n-n_0)$. 0.0202 0.0407	
65	f	Angular motion in radians of torsion head= $\epsilon_1(s-s_0)$. 0.0253 0.0509	
66	$f-h$	Torsion in the fiber 0.0051 0.0102	
67	$\frac{f}{f-h}$	Torsion factor 4.96 4.99	
68		Mean torsion factor 4.98	
69	$\frac{h}{f-h}$	3.96 3.99	

383. Adjust the time flasher apparatus in elevation until there is no parallax between the D spot and the time line. The parallax may be examined visually while the D spot and the time flasher lamp are activated simultaneously.

384. ***Computation of the effective D scale value.***—Compute the D scale value (minute scale value) from equation (122) having regard for the direction (sign) and magnitude of the C field (see app. V) in the factor $\frac{H+C}{H}$. If C is directed to magnetic north it is positive (+); if directed to magnetic south it is negative (−). If the calculated D scale value is (1.000 ± 0.002) minutes per mm, the installation of the D variometer may be considered satisfactory in this respect. If the scale value differs from unity by more than 2 parts in 1000, adjust the D recording distance by moving the recorder directly away from, or toward, the D variometer by the required amount. For example: Suppose the effective recording distance is 1720 mm, and the D scale value, calculated from equation (122), is 1.005. Then the distance should be increased approximately $0.005 \times 1720 = 9$ mm. In that case move the recorder away from the D variometer 9 mm. Check this distance by direct measurement.

385. If the recorder has been moved as above, repeat torsion observations, recording several tests photographically, at the new recording distance; then make a final computation of the D scale value using the new torsion factor, the new effective recording distance, and the field factor. Calculate also the gamma scale value from equation (301).

INSTALLING THE H VARIOMETER

386. ***Torsion and compensation.***—These directions apply to H variometers designed for magnetic temperature compensation. Observations for the torsion constant, k', of the H fiber are highly desirable though not strictly necessary.

387. Examine the H nomogram, figure 141, appendix VI. Select a combination of size of fiber, magnetic moment, M_s, of the recording magnet, and a suitable sensitivity control magnet. Determine and record all of the constants and miscellaneous data for the H variometer as outlined in table 15 as the work progresses.

388. Remove all magnets from the variometer, install the H fiber, and remove torsion as for the D variometer. Adjust the damping chamber for proper damping as described in paragraph 375 for a D variometer, and then make tests for residual torsion as described in the same place. Adjust the mirror face (central mirror if a triple-face mirror is used) approximately parallel to the axis of the torsion weight as for a D variometer, and then adjust the torsion head until the H spot falls near the center of the recording drum when the system is at rest and the axis of the torsion weight is in the magnetic meridian through H, as judged by eye. Note the scale reading on the recorder scale, or on a paper millimeter scale attached to the drum. Replace the torsion weight with the selected H recording magnet and allow the system to come to rest. Note the position of the H spot. If it is close to n_0 the H magnet is in the magnetic meridian with practically no torsion in the fiber. If not in the magnetic meridian within 2°, readjust the magnet relative to the frame by use of the mirror adjusting apparatus, figure 87. When this apparatus is used with the H variometer the shorter focus lens should be installed. Continue adjustments until the H spot falls near the original ordinate, n_0, with no torsion in the fiber, magnetic conditions quiet, and no appreciable stray fields. Read the torsion head. This is R_0, table 16. Deter-

TABLE 15.—*H Variometer; miscellaneous data*

OBSERVATORY: Cheltenham, Md. DATE: December 14, 1948
VARIOMETER: Toepfer No. XII OBSERVER: J. H. D.

LINE	SYMBOL	ITEMS	EXAMPLES
1		SECTION I. OPTICAL SCALE VALUE	
2	L	Front of H lens to the H spot on the magnetogram	1180 mm
3	ϵ	Approximate optical scale value, $\frac{1}{2L}$ radians per mm	0.000 424
4		SECTION II. VARIOMETER MAGNETS	
5	M_s	Magnetic moment of H recording magnet (Alnico II; cylindrical; 2 x 10 mm).	5.0 cgs
6	M_a	Magnetic moment of H temperature magnet (Alnico II; cylindrical; 5 x 25 mm).	78.5 cgs
7	M_a	Magnetic moment of sensitivity magnet (Alnico II; cylindrical; 5 x 25 mm).	50.0 cgs
8	f_E	Field of sensitivity magnet at 11.4 cm; N end west	−0.0677
9	P_A	Distribution coefficient (temperature and recording magnets)	1.52
10	S_H^*	H scale value (approximate), γ/mm	2.66
11		SECTION III. TEMPERATURE COEFFICIENTS	
12	q_1	Temperature coefficient of H recording magnet	0.000 50
13	q_2	Temperature coefficient of temperature-compensation magnet	0.000 48
14	q_3	3 times the coefficient of thermal (linear) expansion of the bar	0.000 06
15	q_5	Temperature coefficient of the torsion constant of the quartz fiber	0.000 16
16	$\Sigma q=$	q_1+q_5 (see equation 251)	0.000 66
17	$\Sigma' q=$	$q_1+q_2+q_3+q_5$	0.001 20
18	Q_H	Temperature coefficient of the H variometer (from equation 249)	0
19	Q_H	Temperature coefficient of the H variometer (by test)	0
20		SECTION IV. TEMPERATURE COMPENSATION	
21	H	Horizontal intensity	0.183 cgs
22	C/H	Ratio of compensation field to H (from equation 253)	−0.55
23	C	Field of temperature magnet at M_s (from equation 253)	−0.101
24	r_1	Estimated correct distance for temperature magnet (equation (346))	11.58 cm
25	r_0	Calculated distance for temperature magnet (from equation (347))	11.63 cm
26		SECTION V. QUARTZ FIBER	
27	l	Effective length of the fiber	15.0 cm
28	d	Diameter of fiber; give several measurements at uniformly spaced intervals. Calculate mean as explained in par. 188.	0.0040
29	T	Period of system with torsion weight (I=1.83)	10.5
30	k^r	Torsion constant (approximate) from dimensions (equation (148))	0.474
31	k'	Torsion constant from oscillations (equation (150))	0.655
32	k'	Torsion constant from torsion observations (equation (151))	0.641
33	k'	Torsion constant from scale value observations (equations (153) and (154)).	0.652
34	k'	Adopted value	0.655
35		SECTION VI. *a* FACTOR DETERMINATIONS	
36	n	Ordinate of undeflected spot in mm −37. 8 +15. 0 75. 6	
37	$2u_H$	Double deflection of H spot in mm 135. 5 130. 4 126. 1	
38	S_H^*	Observed H scale value in γ/mm 2. 56 2. 66 2. 75	
39	ΔS_H	Increment in H scale value 0. 10 0. 09	
40	Δn	Increment in H ordinate, in mm 52. 8 60. 6	
41	$\frac{\Delta S_H^*}{\Delta n}$	*a* factor in γ/mm/mm 0. 0019 0. 0015	
		Mean 0. 0017	
42		SECTION VII. TORSION OBSERVATIONS (OPTIONAL)	
43	L	Recording distance, H variometer, in mm	1180
44	ϵ	Optical scale value, H variometer	0.000 424
45	L_1	Recording distance, auxiliary scale, in mm	1000
46	ϵ_1	Optical scale value, auxiliary scale	0.000 500
47	n_0	H ordinate, no torsion in fiber	15. 0
48	n	H ordinate, magnet deflected by torsion	112.2
49	$n-n_0$	Change of H ordinate	97.2
50	s_0	Scale reading, no torsion in fiber	0.2
51	s	Scale reading, magnet deflected by torsion	200.2
52	$s-s_0$	Change in scale reading	200.0
53	h	Angular motion, in radians, of H magnet, $\epsilon(n-n_0)$	0.0412
54	f	Angular motion, in radians, of torsion head, $\epsilon_1(s-s_0)$	0.1000
55	$f-h$	Torsion in the fiber	0.0588
56	$\frac{f}{f-h}$	Torsion factor	1.70
57		Mean torsion factor	1.70
58	$\frac{h}{f-h}$		0.70
59		SECTION VIII. RESERVE DISTANCES	
60	H^R-H	Distance in mm on magnetogram, regular spot to upper reserve	182.0
61	$H-H_R$	Distance in mm on magnetogram, regular spot to lower reserve	181.5

mine the torsion factor by the method described for low-sensitivity variometers, paragraph 417. Adjust the H base-line spot so that increasing H ordinate corresponds to increasing horizontal intensity and so that there may be few negative ordinates on quiet days. Test the H spot for parallax. If the amount is appreciable, reduce it by raising or lowering vertically the entire H variometer (see par. 176). If this adjustment results in reflecting the recording spot too high or too low in relation to the cylindrical lens of the recorder, the variometer mirror must be tilted slightly by adjusting the mirror screws, or by bending the mirror frame in older instruments.

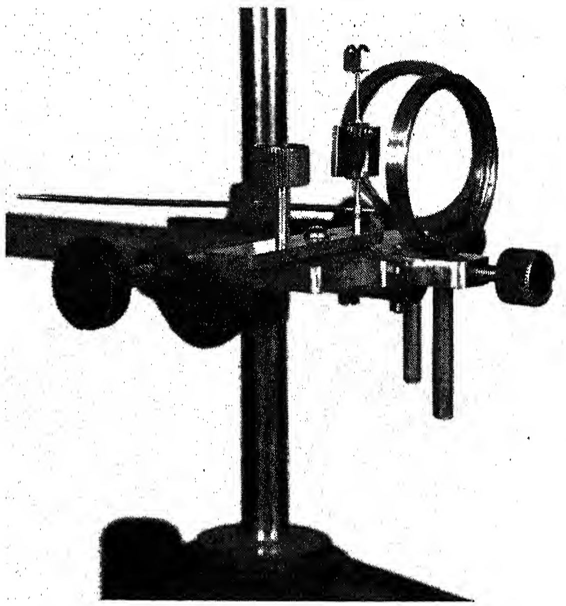

Figure 87.—Mirror-adjusting apparatus, for adjusting mirror relative to recording magnet.

TABLE 16.—*H variometer, torsion in the fiber*

Figure	Estimated torsion (τ)	C field	Magnet (θ)	R_0	Torsion-head circle $R=R_0+\theta+\tau$	Remarks about M.
	°	γ	°	°	°	
88(a)	$\tau_0=0$	0	0	75	$R_0=75$	N end N.
88(b)	$\tau_1=145$	0	90	75	$R_1=310$	N end E.
88(c)	$\tau_2=65$	10100	90	75	$R_2=230$	N end E.

389. Calculate the torsion constant, k', of the H fiber from its dimensions (eq. 148); from the period observations (eq. 150); from torsion observations (eq. 151); and from scale-value observations (eq. 153 and 154). Compare the results. The torsion constant determined from oscillations is quite likely the most reliable.

390. Calculate the factor, $\frac{C}{H}$, from equation (250) or (253), or estimate its value from table 10 (p. 117), and then compute C, the value of the compensating field necessary to make the temperature coefficient,

Q_H, of the variometer equal to zero. From the nomogram, figure 137, appendix VI, estimate the magnetic moment of a temperature compensation magnet that will provide the required C field at a distance, $r_1=12$ cm, along its magnetic axis produced, and select a magnet of this approximate moment. Assuming $P_A=0$, calculate the approximate value, r_1, for the temperature-compensation magnet from the relation,

$$r_1=\left[\frac{2M_a}{-C}\right]^{\frac{1}{3}}. \tag{346}$$

Calculate the distribution coefficient, P_A, from the dimensions of the magnets or estimate its value from table 21a, and then the distribution factor, $1+\frac{P_A}{r_1^2}$. Finally, calculate the more precise value, r_0, from the relation

$$r_0=r_1\left[1+\frac{P_A}{r_1^2}\right]^{\frac{1}{3}}. \tag{347}$$

391. ***Uncompensated H variometer.***—Clamp the fiber, remove the mirror frame, and turn the magnet until its long axis is approximately perpendicular to the regular mirror face, N end to the rear of the mirror face. Suspend the mirror frame and turn the torsion head clockwise, looking down, until the magnet is in the magnetic prime vertical as estimated by eye. If the H spot does not fall at the desired ordinate repeat the adjustment of the magnet relative to the frame until satisfactory. Use the mirror adjusting apparatus for this operation. Preliminary orientation tests may be made at this point if desired. If these tests indicate that the H magnet is closely in the prime vertical (say within one degree), and the H spot is at the desired ordinate, the variometer is adjusted for routine operation but it is uncompensated for temperature. The torsion-head reading is R_1, table 16.

392. ***Compensated H variometer.***—Attach a variometer deflection bar to the north or south side of the variometer parallel to the magnetic meridian and set the temperature-compensation magnet (center of magnet) at the calculated correct distance, r_0, to give the required C field to make $Q_H=0$. A more uniform C field may be obtained by using two temperature-compensation magnets as shown in figure 88 (c). In this case the deflection distances should be equal so that each magnet provides half of the C field. The N end of the compensation magnet should be to the south to make C negative. Reduce the torsion in the fiber by turning the torsion head counterclockwise, looking down, until the H spot comes to rest at the desired ordinate, n_0, making allowance for possible change of H during the interval. Observe the torsion head reading, R_2. The variometer is now compensated for temperature. The example given in table 16 is shown graphically in figure 88. Make orientation observations as described in chapter 12.

393. ***Adjustment of the sensitivity-control magnet.***—If the scale value is too high or too low, attach a variometer deflection bar to the H variometer on the side near the recorder. Note the scale reading of the H spot. Mount a sensitivity magnet on the bar with its N end toward the variometer, say in this case N end west, and

then move it slowly along the bar toward M_s until r is about 10 cm. If M_s shows no appreciable deflection during this operation, the bar is closely parallel to M_s and the sensitivity magnet remains practically parallel to M_s throughout the range of r. If M_s is deflected through an appreciable angle for small changes of r, it signifies maladjustment of M_s, or of the bar, or that the axis of M_a is not parallel to the bar or to M_s.

394. Determine the H scale value by deflections with no sensitivity magnet attached to the variometer bar and for several equally spaced positions along the bar, first with the N end of M_a to the west and then with the N end to the east, noting the *away* positions of the H

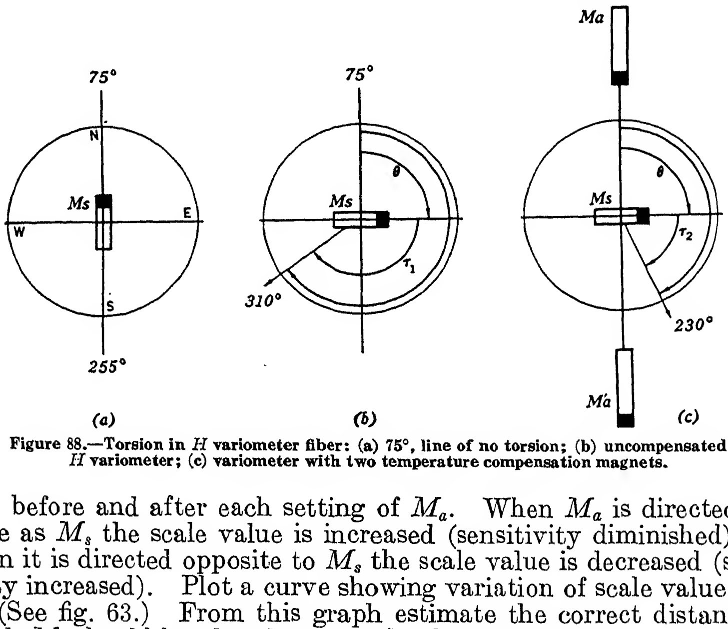

Figure 88.—Torsion in *H* variometer fiber: (a) 75°, line of no torsion; (b) uncompensated *H* variometer; (c) variometer with two temperature compensation magnets.

spot before and after each setting of M_a. When M_a is directed the same as M_s the scale value is increased (sensitivity diminished) and when it is directed opposite to M_s the scale value is decreased (sensitivity increased). Plot a curve showing variation of scale value with r. (See fig. 63.) From this graph estimate the correct distance at which M_a should be placed to give the desired scale value. If the H spot is not at the desired ordinate (within a few mm) adjust the temperature magnet by a very small distance, just sufficient to bring the H spot to the desired ordinate. (See par. 301 regarding the effect of this adjustment on the temperature coefficient of the variometer.) Some sensitivity-magnet holders are equipped with a slow-motion device for making fine adjustments of the sensitivity magnet. Determine the scale value again and if satisfactory, clamp the control magnets in place. Note: For most precise work the observations with the sensitivity magnet in different positions should be made with the away position of the H spot the same for all cases, since the scale value varies with ordinate. However, since these observations are made for the purpose of simplifying the process of adjustment of the sensitivity magnet, the change of scale value with ordinate may be ignored for that particular operation.

395. ***Computation of the f_E field (optional).***—Compute the f_E field from equation (11), omitting a distribution factor, and then compute the scale value from equation (144) and compare the result with the observed value.

396. ***Experimental determination of the a factor.***—Change the H ordinate by 50 to 75 mm by deflecting the H recording magnet by means of a large (auxiliary) deflector placed in the magnetic meridian on the opposite side of the variometer from the scale value shelf. The deflection distance should be quite large, say 2 or 3 meters. Make scale-value observations at this ordinate. Reverse the auxiliary deflector and repeat the scale-value deflections at low ordinate. The difference in the scale values divided by the difference in away ordinate, in mm, is the a factor. Some experimental determinations of the a factor are given in section VI of table 15.

397. ***Damping.***—Adjust the damping chamber in elevation relative to the H magnet so that the system is rather highly damped but not critically so.

INSTALLING THE Z VARIOMETER

398. ***Preliminary work.***—The directions in this section apply to vertical-intensity variometers equipped with assembled recording-magnet systems (Schmidt type) as distinguished from those made from one piece of steel (la Cour type).

399. ***Instrumental data.***—From table 11, page 134, select a scale value at which the variometer should be operated. Determine and record as in table 17 all of the constants and pertinent instrumental data, as the work progresses.

400. From table 10 estimate the value of $\frac{C}{Z}$, or estimate the value of this ratio from the q coefficients, and then calculate C, the required vertical field to be applied opposite to Z, for effective temperature compensation.

401. From the nomogram, figure 137, appendix VI, estimate the magnetic moment of a temperature-compensation magnet that will provide the required C field at a distance, r_1, along its axis for r_1 equal to approximately 13 cm. Select a magnet that has approximately this moment.

402. Determine the magnetic moment, M_s, of the recording magnet by deflections on a magnetometer at a place where H is known.

403. Unless such data are furnished with the instrument, determine the temperature coefficients of the magnetic moments of M_a and M_s and calculate more precise values of $\frac{C}{Z}$ and C.

404. From the equation $r_1^3 = \frac{2M_a}{-C}$, calculate the approximate value of r_1 and then allowing for a distribution factor, $1+\frac{P_A}{r_1^2}+\frac{Q_A}{r_1^4}$, calculate a more precise value, r_0, from the relation, $r_0 = r_1\left(1+\frac{P_A}{r_1^2}+\frac{Q_A}{r_1^4}\right)^{\frac{1}{3}}$, with due regard for the signs of P_A and Q_A. This value, r_0, is the distance at which the temperature magnet, M_a, must be set vertically above or below the center of M_s, N end of M_a up, to provide the necessary C field for effective compensation. For south magnetic latitudes the N end of M_a should be down.

TABLE 17. *Z variometer; miscellaneous data*

OBSERVATORY: Cheltenham, Md. DATE: December 15, 1948
VARIOMETER Toepfer No. XIII OBSERVER R. X. R.

LINE	SYMBOL	ITEMS	EXAMPLES
1		SECTION I. OPTICAL SCALE VALUE	
2	L	Center of front of Z lens to Z spot on the magnetogram	2322 mm
3	ϵ	Optical scale value, $\frac{1}{2L}$ radians per mm	0.000 215
4		SECTION II. VARIOMETER MAGNETS	
5	M_s	Magnetic moment of recording magnet (cobalt steel)	877
6	M_a	Magnetic moment of temperature magnet (Alnico II)	404
7		Magnetic moment of N sensitivity magnet (if used)	100
8		Magnetic moment of S sensitivity magnet (if used)	100
9	L_s	Length of recording magnet	8.1 cm
10	L_a	Length of temperature magnet	3.5 cm
11		Length of N sensitivity magnet (if used)	4.0 cm
12		Length of S sensitivity magnet (if used)	4.0 cm
13	P_A	Distribution coefficient, recording magnet and temperature magnet (from table 21a).	−28
14	Q_A	Distribution coefficient, recording magnet and temperature magnet (from table 21a).	+320
15		SECTION III. TEMPERATURE COEFFICIENTS	
16	q_1	Temperature coefficient of magnetic moment of recording magnet	0.000 34
17	q_2	Temperature coefficient of magnetic moment of temperature magnet	0.000 28
18	q_3	3 times the coefficient of thermal (linear) expansion of the bar	0.000 06
19	q_4	Temperature coefficient of mechanical couple (lever arm a)	0.000 01
20	Σ_q	q_1+q_4	0.000 35
21	Σ'_q	$q_1+q_2+q_3+q_4$	0.000 69
22	Q_Z	Temperature coefficient of the variometer (calculated) (Eq. 233)	0
23	Q_Z	Temperature coefficient of the variometer from tests	0
24		SECTION IV. TEMPERATURE COMPENSATION	
25	Z	Vertical intensity at the Z variometer	0.538 cgs
26	$\frac{C}{Z}$	Ratio of required compensating field to vertical intensity (Eq. 238)	−0.507
27	C	Required compensating field to make $Q_Z=0$; $C=-0.507\ Z$	−0.273
28	r_1	Calculated approximate distance (see par. 404) assuming $P_A=0$ and $Q_A=0$.	14.36
29	r_0	Calculated distance (see par. 404) when $P_A=-28$ and $Q_A=+320$	13.72
30		SECTION V. SCALE VALUE	
31	S_Z	Z scale value from deflections, at average ordinate; γ/mm	4.52
32		SECTION VI. RESERVE DISTANCES	
33	Z^R-Z	Distance in mm on magnetogram, regular spot to upper reserve spot	175.6
34	$Z-Z_R$	Distance in mm on magnetogram, regular spot to lower reserve spot	175.2
35		SECTION VII. SHRINKAGE	
36		Shrinkage gauge; distance between points	100.0 mm
37		SECTION VIII. MISCELLANEOUS DATA (OPTIONAL)	
38	m	Mass of Z recording magnet including sensitivity poise but without balancing poises.	37.55 grams
39	m_2	Mass of sensitivity poise (without set screw)	1.397 grams
40	m_3	Mass of south latitude poise	2.396 grams
41	m_4	Mass of north latitude poise	2.396 grams
42	m_t	Mass of temperature poise	2.921 grams
43	m_1	Mass of assembled magnet system	42.34
44		Pitch of sensitivity thread, $\frac{1}{40}$-inch (nominal)	0.0638 cm
45		Pitch of invar spindle thread, $\frac{1}{64}$ inch (nominal)	0.0397 cm
46		Pitch of temperature spindle thread	0.0397 cm

405. ***Installation and major adjustments.***—Set up the Z variometer at the Z position on the variometer pier and mount the temperature magnet in its holder at the calculated distance r_0. This is the distance in cm from the center of M_a to the center of M_s, when M_s is in operating position. The center of M_s may be taken as the knife-edge

(or pivot) support. Level the instrument and adjust it in azimuth so that the axis of the Z lens is approximately normal to the drum. With a camel's-hair brush remove any dust particles from the recording magnet, knife edges, bearings, and the damping chamber. Clean the optical parts and the magnet with lens cloth. Set the sensitive poise quite low (large scale value), and the counterpoises at their midpositions. Place the magnet in its cradle and lower it carefully by means of the cradle mechanism so that the knife edge (or pivots) will take the load gently. Quite likely the magnet will not be balanced. Lift the magnet by the cradle mechanism, adjust one or both counterpoises until the recording magnet remains horizontal (as estimated by eye) when it is resting on its bearings, N end to the north. Turn the whole variometer, slowly, 180° in azimuth so that the N end of M_s is to the south. At this azimuth, M_s is more sensitive. Rebalance if necessary, and continue these tests until the magnet remains practically horizontal (as estimated by eye) for any azimuth, N, E, S, or W. Finish the test with the N end to the north and with the Z spot at the desired ordinate. Readjust the Z base-line spot so that increasing Z ordinate corresponds to numerical increase of Z.

406. Examine the Z spot and time line, and if there is appreciable parallax between them, raise or lower the Z variometer until the parallax is negligible.

407. Determine the scale value by deflections. Raise the sensitivity poise several turns, rebalance if necessary by adjustment of a latitude poise so that the Z spot is at approximately the same ordinate as in the first test. Make scale-value deflections for this position of the sensitivity poise. By interpolation or extrapolation, estimate the number of turns of the sensitivity poise that will be necessary to produce the scale value desired. Make this adjustment, repeat scale-value deflections, and continue this process until the deflections show that the proper sensitivity has been attained, say within 5 percent. (Note: By observing the period of the system for each position of the counterpoise and then plotting a curve of period or turns of counterpoise vs. double deflection or scale value, the value of a period which will correspond to a desired scale value or value of $2\ u_Z$ may be easily determined.)[1]

408. Make orientation tests, and calculate the exlevel angle. If necessary, readjust the latitude poises to correct for a large exlevel angle. Small adjustments in level may be made by slight adjustments to the temperature-magnet distance. Continue orientation tests and adjustments until the exlevel angle is 1° or less.

409. Unless the variometer is provided with extra prisms or mirrors for independent adjustment of the ordinate of the Z spot, turn the variometer in azimuth until the Z spot is at the desired ordinate for routine operation. Again adjust the Z base-line mirror so that increasing Z ordinate corresponds to numerically increasing vertical intensity and so there may be no negative ordinates on quiet days.

410. Repeat orientation tests and scale-value observations, recording final values as in figures 72 and 84. The variometer is now adjusted for routine operation.

411. ***Final check.***—Remove all tools and superfluous magnetic materials from the variation room. Examine all of the recording spots and the time line for parallax. Also, see that all of the spots are record-

[1] H. E. McComb, The sensitivity of magnetic variometers, Terr. Mag., 33, 65, 1928.

ing at the desired ordinates and positions on the magnetogram, when magnetic conditions are quiet. If the instruments seem to be operating satisfactorily, attach the foot plates to the marble blocks or metal spacers with plaster of paris and attach the spacers or blocks to the pier in the same manner, exercising great care not to move the foot plates or blocks from their proper places.

412. Make scale-value deflections and orientation tests for all the variometers, recording all deflections photographically.

413. ***Report on the installation.***—Furnish a complete, detailed report of the project including sketches that show all pertinent dimensions (see fig. 85 and fig. 131, page 211) and, if possible, a photograph of the complete magnetograph made ready for operation.

LOW-SENSITIVITY MAGNETOGRAPHS

414. ***Object.***—In high geomagnetic latitudes, where daily variations are large and the amplitudes of the recording spots may be very high during magnetic storms, variometers must be operated at low sensitivities. Even in low latitudes, many observatories operate both high and low-sensitivity instruments to guard against loss of record during magnetic storms.

415. ***Procedure.***—From table 11, determine the approximate scale values at which the variometers should be operated. Using the D nomogram, figure 140, appendix VI, select a combination of size of quartz fiber and magnetic moment of recording magnet that will give the desired D scale value. (The use of sensitivity-control magnets on a D variometer is not recommended.) Using the H nomogram, figure 141, appendix VI, select a combination of size of fiber and magnetic moment that will give the desired H scale value. A sensitivity-control magnet may be used if necessary for final adjustment to the desired value. The sensitivity of the Z recording magnet may be adjusted to any desired low value by means of the sensitivity poise.

416. In estimating the C fields for effective temperature compensation of the H and Z variometers, and in measuring the magnetic moments of the temperature magnets and other magnets of the magnetograph, follow the same procedures used for sensitive variometers.

417. ***Auxiliary optical lever.***—The torsion factors of large fibers may be determined as follows: With a piece of laboratory wax, mount a planoconvex mirror (fig. 45) centrally on the torsion head of the D variometer so that the reflecting surface is vertical. Set up a straight-filament incandescent lamp and a millimeter scale at the same elevation as the mirror and at the proper distance for sharp focus of the image. Using this optical lever as a means of estimating accurately the angular motion of the torsion head (in radians), make torsion observations as described in paragraph 382, page 152. Record all observations and data, as shown in table 15, section VII, for the sensitive H variometer. Calculate the torsion factor, $\frac{f}{f-h}$, and the field factor, $\frac{H+C}{H}$, and then the D scale value from equation (122). It is desirable, though not strictly necessary, to determine likewise the torsion factor for the H variometer.

418. Make orientation tests, scale-value observations, parallax tests, and all other adjustments in the manner prescribed for sensitive variometers except that the deflectors for scale-value and orientation deflections should have larger magnetic moments or should be used at shorter deflection distances.

419. Record all pertinent data as in tables 14, 15, 16, and 17. Furnish a complete report of the installation with sketches showing all pertinent dimensions as in figures 85 and 131, appendix V, and if possible, a photograph of the complete magnetograph as it will be operated.

THE LA COUR MAGNETOGRAPH

420. ***Description of the instruments.***—Complete descriptions of the instruments and directions for their installation are contained in publications of the Danish Meteorological Institute.[2] Only a brief

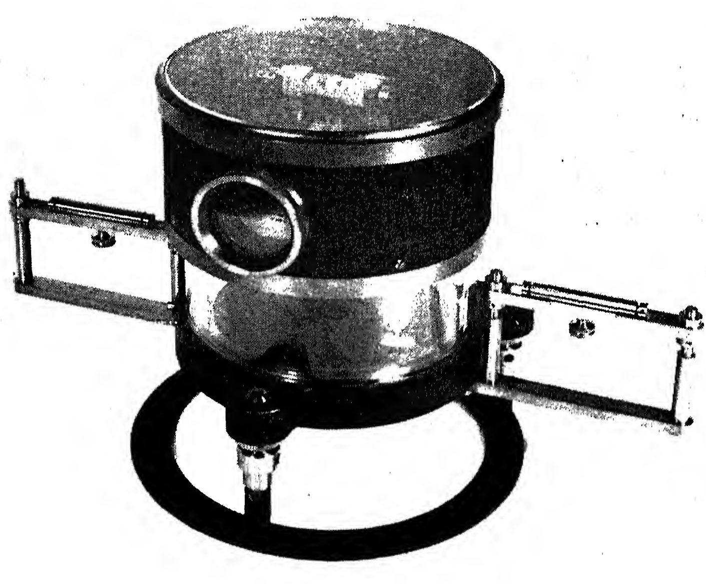

Figure 89.—La Cour vertical-intensity variometer equipped with sensitivity-control magnets, as operated at Honolulu Magnetic Observatory.

summary will be given here. The general principles involved in the selection of quartz-fiber suspensions and magnets are similar to those outlined elsewhere in this manual. The use of adjustable prisms on the la Cour *D* and *H* variometers simplifies many of the problems of installation, such as adjustment of the azimuth and inclination of the rays reflected from the mirrors of the recording magnets. La Cour's method of attachment of fibers to the torsion-head stem and to the magnet mirror frame is indicated briefly in paragraph 179. The *Z* recording magnet, magnet mirror, and knife edges are fabricated from

[2] La Cour, Danske Met. Inst. Pubs. (see items 8 and 11 of bibliography).

one piece of magnet steel. The magnet is ground, polished, and balanced by the manufacturer for a particular magnetic latitude and sensitivity. Final balancing and/or adjustment of the sensitivity may be accomplished at the observatory where it is to be operated, by the simple process of careful honing of the magnet with a fine-grained carborundum hone. To increase the sensitivity it is honed below the longitudinal axis of symmetry that is normal to the knife edges, and for balancing it is honed on one end. The knife edges are ground and polished to a definite small radius, and at the same time the two edges are made collinear by a simple and ingenious process. The polished steel mirror is on the upper side of the magnet, and an adjustable 90° prism mounted above the mirror affords a means of controlling the azimuth and inclination of the incident and reflected light. The magnet is balanced on a pair of agate cylinders and is in a sealed chamber which is partially evacuated. At the Honolulu Magnetic Observatory the la Cour Z variometer is equipped with a pair of sensitivity-control magnets (see fig. 89.) Its performance for a number of years has been quite satisfactory.

421. ***Optical compensation.***—The 90° prisms on the H and Z variometers are mounted on bimetallic (silver-platinum) strips. As the temperature changes the strip bends, causing angular motion of the prism. By proper facing of the strip and adjustment of its length, the linear motion of the recording spot due to a change in temperature of the recording magnet (and the resulting change in its magnetic moment) may be just compensated by the angular motion of the 90° prism through which the incident and reflected rays pass.

422. ***Reserve spots.***—By the use of a series of 90° prisms mounted in front of a straight-filament recorder lamp, two or more reserve images are produced for each variometer, as mentioned in paragraph 170, page 66.

CHAPTER 14. PROCESSING OF DATA AT THE OBSERVATORY

PROGRAM OF OBSERVATORY WORK

423. ***Routine duties.***—After the observatory has been established and the magnetograph and other instruments are functioning satisfactorily, most of the regular magnetic work is of a routine nature. However, to carry on this routine and to achieve continuously the objectives for which the observatory has been established requires great care on the part of the observer in operating delicate instruments, considerable skill in the observing programs, and perseverance in keeping the routine work on a current basis. Some of the principal items are: (a) absolute observations for magnetic declination, usually with a magnetometer, (see fig. 90); (b) absolute observations for horizontal intensity with a magnetometer, sine galvanometer (fig. 37), Quartz Horizontal Magnetometer (fig. 91), or other equally precise instrument; (c) absolute observations for inclination (dip) with an earth inductor (fig. 92), or (d) observations for vertical intensity with a standard magnetic field balance or Magnetometric Zero Balance (BMZ),[1] or some equally precise instrument; (e) scale-value observations; (f) time observations for comparison of chronometers and pendulum clocks; (g) changing the traces daily on the magnetograph recorder and developing the magnetograms; (h) applying legends on the magnetograms (see fig. 93); (i) scaling of hourly ordinates of H, D, and Z, from the magnetograms (see fig. 100); (j) computations of absolute values of D, H, and I from absolute observations; (k) computations of D, H, and Z base lines; (l) estimation of character figures; (m) scaling of K indices; (n) tabulation of times of sudden commencements of magnetic storms, times of natural disturbances of the variometers (seismic or other); (o) solar flare studies; (p) observations for orientation of variometer recording magnets; (q) operation of auxiliary magnetographs, such as low-sensitivity variometers, photoelectric magnetographs,[2] etc.; (r) abstracting of pertinent records and monthly transmittal of data and magnetograms to the home office.

424. ***Conversion to absolute values.***—In order to determine the absolute value of D, H, or Z for any moment from the magnetogram it is necessary to know: (a) the base-line value, that is, the absolute value of the element when its trace and base line coincide; (b) the scale value; and (c) in the case of H and Z, the temperature coefficient of the variometer. Let d, h, and z denote the ordinates in mm at the temperature t, corrected for shrinkage of the magnetogram (increasing ordinates corresponding to increasing D, H, and Z). Let S_D', S_H, and

[1] La Cour, Danske Met. Inst. Pub. 19 (see item 10 of bibliography).

[2] R. E. Gebhardt, T. J. Hickley, and T. L. Skillman, A photoelectric magnetograph, Trans. Amer. Geoph. Union, 32, 322, 1951 (abstract only). This arrangement is illustrated in figure 94.

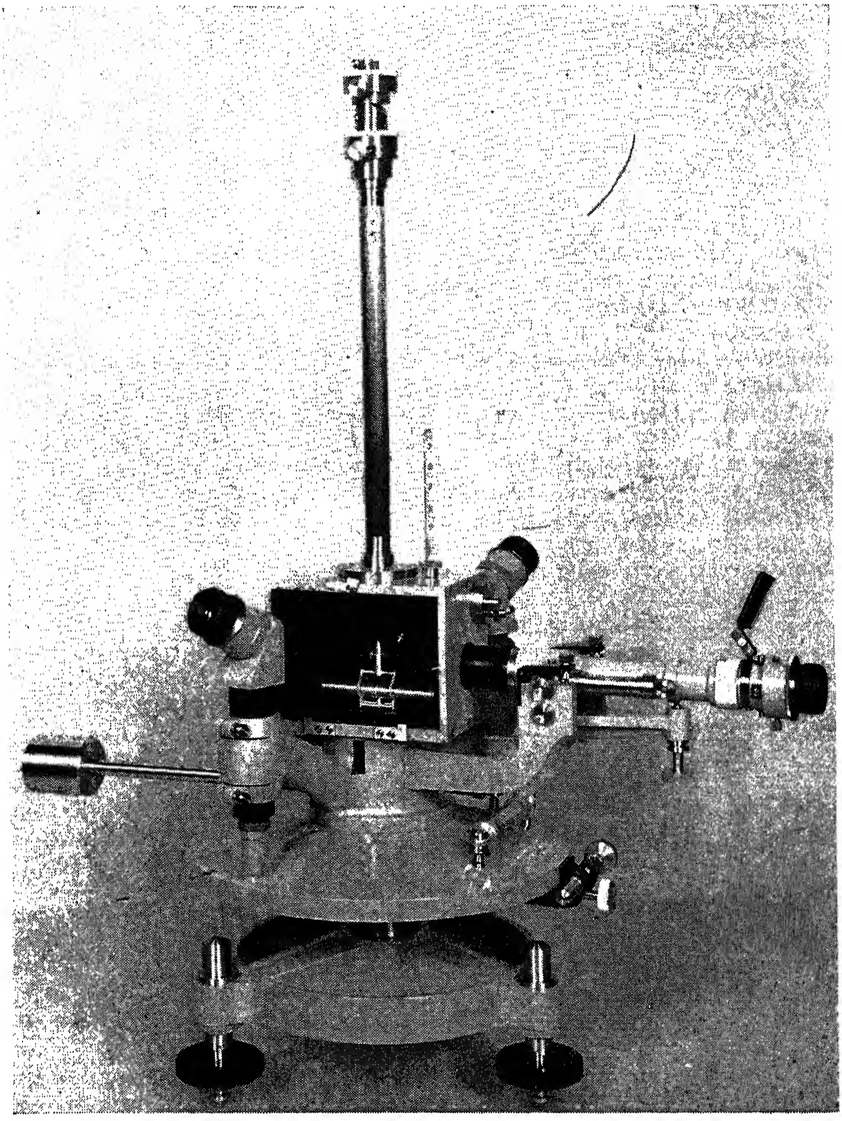

Figure 90.—Observatory magnetometer (Ruska type) with telescope having modified Gaussian eyepiece.

S_Z represent the scale values of the D, H, and Z traces respectively, and let

S_0 = the base-line scale value for H;

a = the a factor for H;

$\overline{S}_H = S_0 + \frac{1}{2}\, a\, h$ (see par. 259);

t_0 = the standard temperature, usually 20° C.

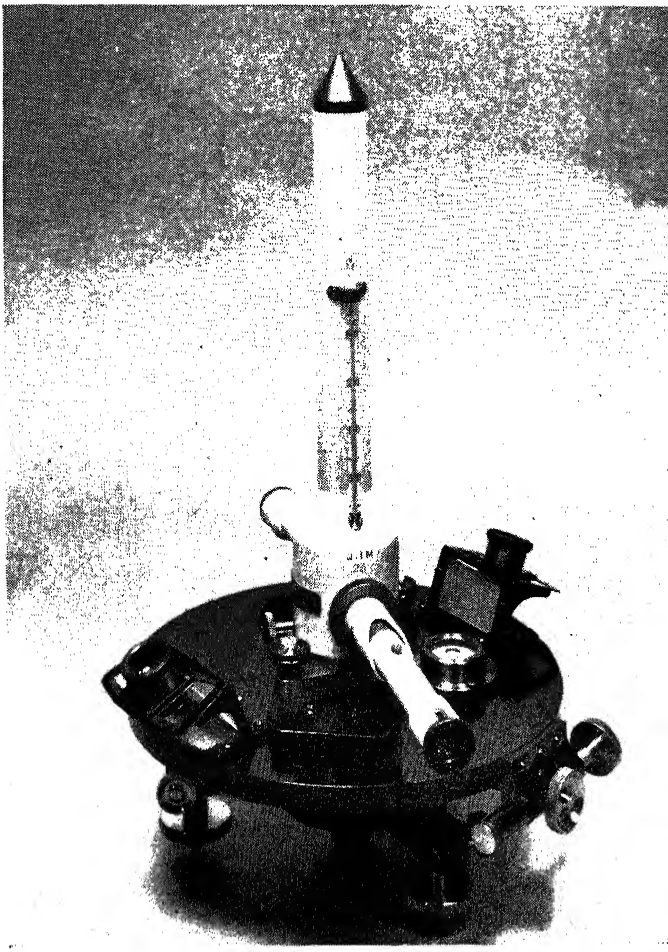

Figure 91.—Quartz Horizontal Magnetometer (QHM) mounted on a special base with divided circle and verniers.

Finally, Q_H and Q_Z are the temperature coefficients of the H and Z variometers (par. 288); and B_D, B_{H0}, and B_{Z0} are the base-line values for D, H, and Z (in the case of H and Z, reduced to the standard temperature t_0).

Then
$$D = B_D + S_D' d \tag{348}$$

$$H = \mathrm{B}_{H0} + \overline{S}_H h + Q_H (t - t_0) \tag{349}$$

$$= \mathrm{B}_{H0} + (S_0 + \tfrac{1}{2} a h) h + Q_H (t - t_0) \tag{350}$$

$$Z = \mathrm{B}_{Z0} + S_Z z + Q_Z (t - t_0). \tag{351}$$

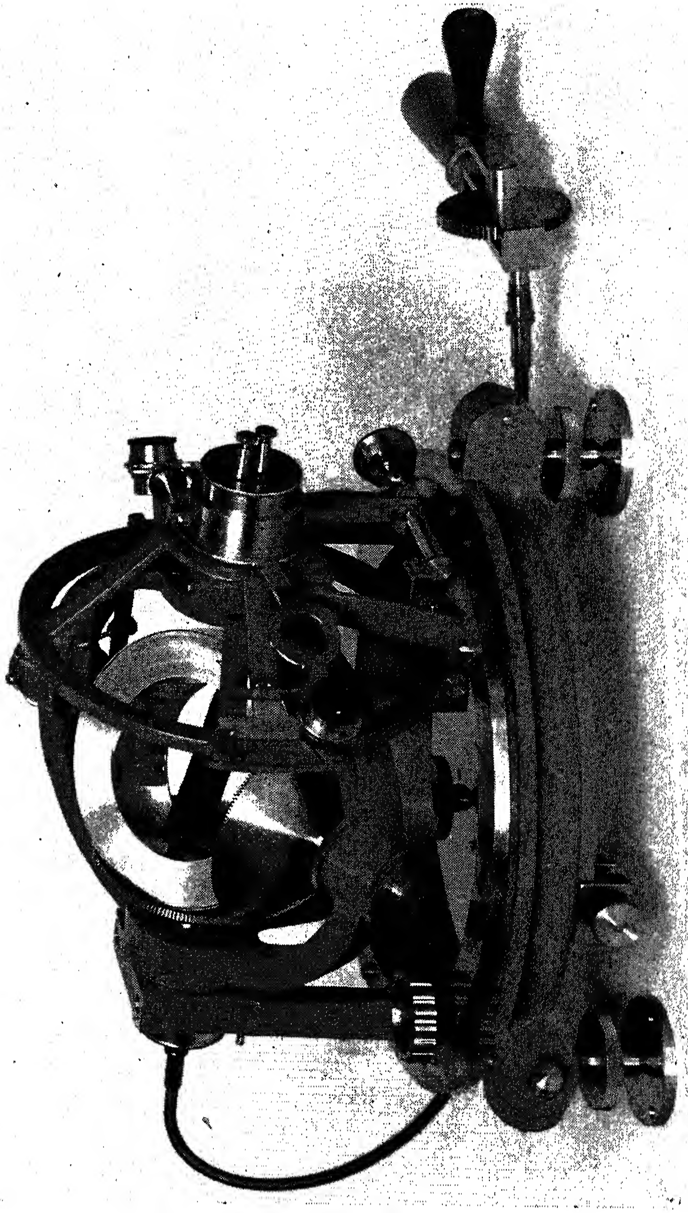

Figure 92.—Earth inductor with transverse axial driving gear.

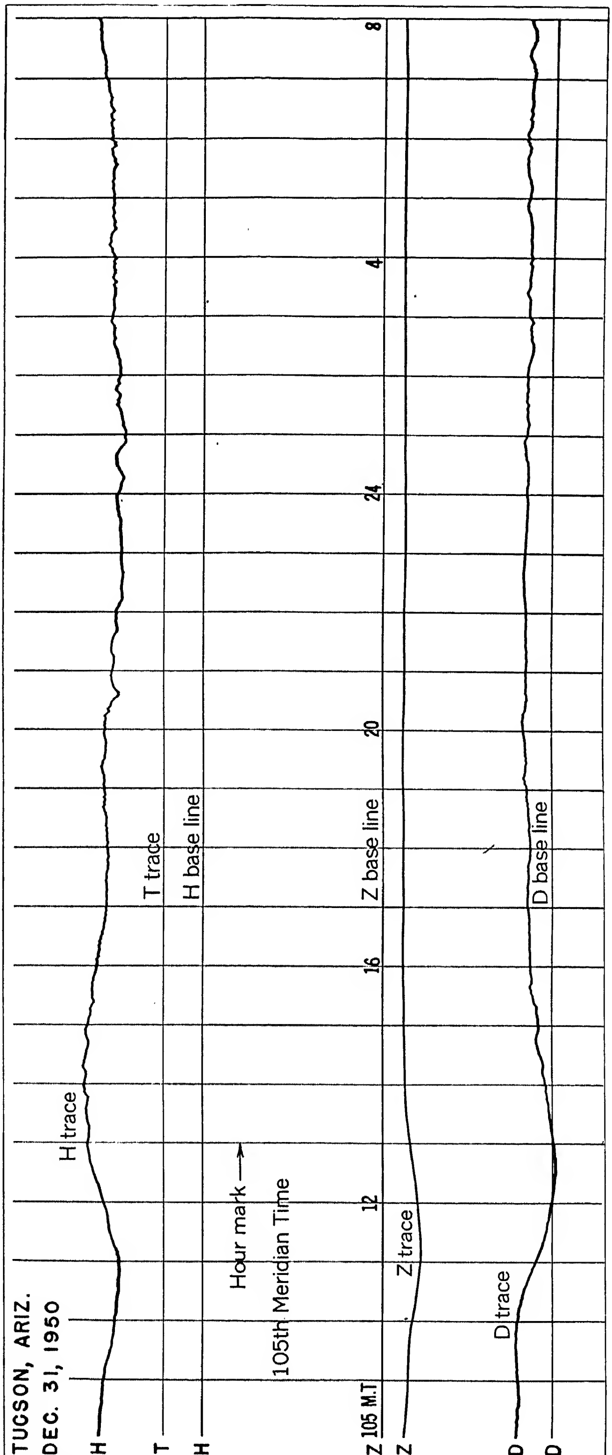

Figure 93.—Magnetogram as recorded at Tucson Magnetic Observatory.

210111—53——12

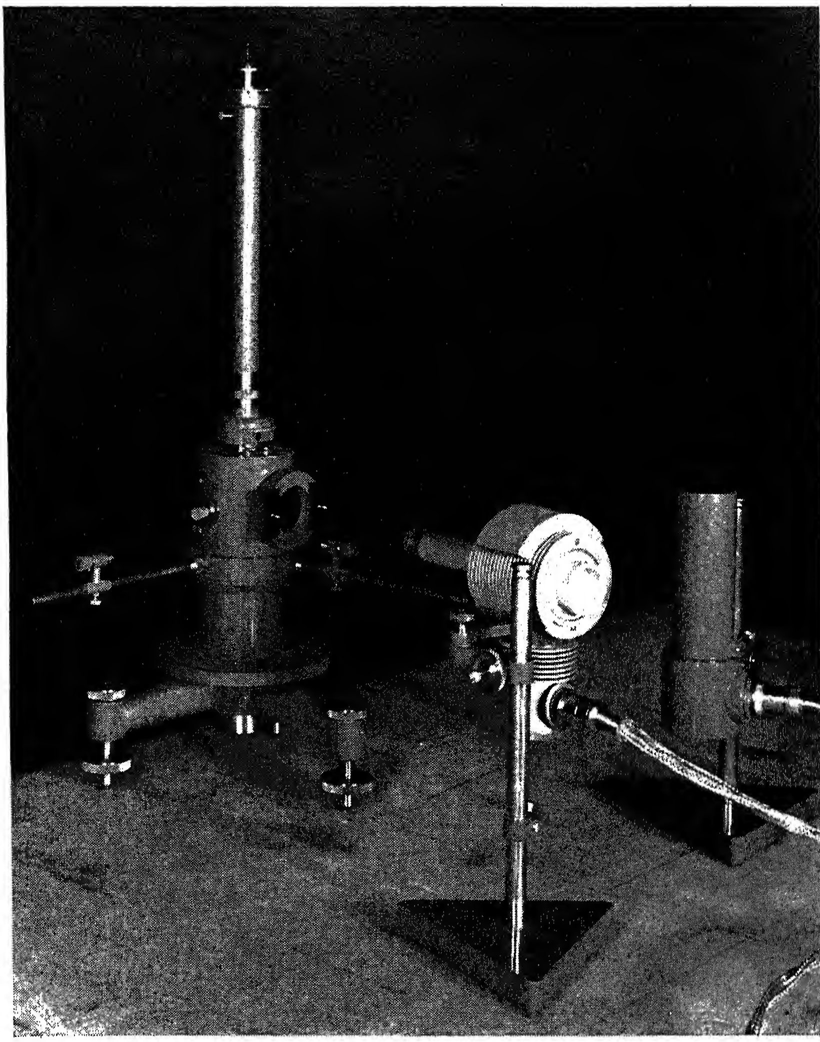

Figure 94.—Special light source and photoelectric cell used with a horizontal-intensity variometer for visual-recording magnetograph.

425. ***Base-line values.***—For the determination of base-line values, absolute observations are made at least once a week. From an inspection of the above formulas it will be seen that if the ordinates d, h, and z be read for the times at which absolute observations have been made, the base-line values may be computed, provided the scale values and the temperature coefficients are known. Generally the absolute value of the vertical intensity must be computed, however, from values of H and I. It is, in general, not feasible to make simultaneous absolute observations of H and I, but the value of H at the time of the absolute observations of dip may be determined from the magnetogram after a preliminary H base-line value has been com-

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form No. 357
Ed. July 1948

Magnetic Observatory COLLEGE, ALASKA

JAN. 1951
(Month) (Year)

DECLINATION BASE-LINE VALUE

Magnetograph ESCHENHAGEN *Magnetometer No.* 25

DATE	150 M. Time	MEAN SCALING *	COMPUTED ORDINATE †	OBSERVED D	BASE-LINE VALUE	OBSERVER	REMARKS
		mm.	′	° ′	° ′		
Jan 3	10 28 35	11.6	11.4	29 14.5	29 03.1	VAT	
3	10 38 45	12.4	12.1	16.0	03.9	VAT	
3	13 38 45	08.8	08.6	12.1	03.5	MLC	
3	13 47 54	08.8	08.6	11.8	03.2	MLC	
8	10 46 53	11.3	11.1	14.9	03.8	MLC	
8	10 55 62	13.5	13.2	17.4	04.2	MLC	
8	13 51 58	09.9	09.7	13.6	03.9	VAT	
8	14 01 08	09.8	09.6	13.7	04.1	VAT	
Mean					29 03.7		

* Corrected for shrinkage to 100.0 mm.
† Assigned scale value = 0.98 ′/mm. at zero shrinkage.

Scaled by V A Thomas *Checked by* M L Cleven
Computed by V A T *Checked by* M L C *Abstracted by* M L C

U. S. GOVERNMENT PRINTING OFFICE 19—56307-1

Figure 95.—Declination base-line computations.

puted from equation (355). Z is computed from this value of H and the observed I by the relation

$$Z = H \tan I. \tag{352}$$

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form No. 358
Ed. Oct. 1942

Magnetic Observatory TUCSON, ARIZONA

Sept 1951
(Month) (Year)

HORIZONTAL INTENSITY BASE-LINE VALUE

Magnetograph la Cour *Magnetometer No.* 31

DATE	4			4			11		
	105 M. TIME		MEAN SCALING	TIME		MEAN SCALING	TIME		MEAN SCALING
	Begin	End		Begin	End		Begin	End	
	h. m.	h. m.	mm.	h. m.	h. m.	mm.	h. m.	h. m.	mm.
Oscill. E.	09 50	09 57	14.1	13 38	13 45	21.8	09 37	09 44	26.2
Defl. E.	10 03	10 13	13.4	13 54	14 08	24.2	09 53	10 07	25.4
Defl. I.	10 16	10 26	13.9	14 12	14 26	25.0	10 12	10 26	25.0
Oscill. I.	10 33	10 40	14.1	14 35	14 42	24.1	10 38	10 45	19.4
Mean Ord.			13.88			23.78			24.00
Shrinkage	100.6		-.08	100.6		-.14	100.2		-.05
Corr. Ord.*			13.8			23.6			24.0
ϵ_h (γ/mm.) 2.95	2.97			2.98			2.98		
h (γ)			41			70			72
H (Observed)			25982			26009			26001
H Base Line			25941			25939			25929
Observer	J B Campbell			L R Southwick			G E Arthur		

DATE									
	TIME		MEAN SCALING	TIME		MEAN SCALING	TIME		MEAN SCALING
	Begin	End		Begin	End		Begin	End	
	h. m.	h. m.	mm.	h. m.	h. m.	mm.	h. m.	h. m.	mm.
Oscill. E.									
Defl. E.									
Defl. I.									
Oscill. I.									
Mean Ord.									
Shrinkage									
Corr. Ord.*									
ϵ_h (γ/mm.)									
h (γ)									
H (Observed)									
H Base Line									
Observer									

Parallax allowed for. Scaled interval lies 0.0 mm. to right left (cross out one) of interval shown by time flashes.

* Ordinate corrected to 100.0 mm. shrinkage distance.

Scaled by L R S R L R *Checked by* G E A

Computed by GEA, RLR *Checked by* JBC, LRS *Abstracted by* GEA

U. S. GOVERNMENT PRINTING OFFICE 16—50439-1

Figure 96.—Computation of horizontal-intensity base-line value.

We have for the computation of the base-line values from the observed absolute values, D_A, H_A, and I_A,

$$B_D = D_A - S'_D d \tag{353}$$

$$B_{H0} = H_A - \overline{S}_H h - Q_H\ (t - t_0) \tag{354}$$

$$= H_A - (S_0 + \tfrac{1}{2}h)\ h - Q_H\ (t - t_0) \tag{355}$$

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form No. 359
(Rev. Oct. 1942)

Magnetic Observatory HONOLULU, T. H.
July 1951
(Month) (Year)

VERTICAL INTENSITY BASE-LINE VALUE

Magnetograph 4 *Earth Inductor No.* 4

Date	5			5			10		
	165 M. Time	Mean Scalings H	Mean Scalings Z	Time	Mean Scalings H	Mean Scalings Z	Time	Mean Scalings H	Mean Scalings Z
	h. m.	mm.	mm.	h. m.	mm.	mm.	h. m.	mm.	mm.
1st half	10 36 38	17.2	30.0	13 44 46	12.7	32.7	10 46 48	27.8	25.0
2d half	10 43 46	17.0	29.9	13 50 53	12.7	32.9	10 50 53	28.1	25.0
Mean Ord.		17.10	29.95		12.70	32.80		27.95	25.00
Shrinkage	99.5	.09	.15	99.5	.06	.16	99.8	.06	.05
Corr. Ord.*		17.2	30.1		12.8	33.0		28.0	25.0
ϵ_h and ϵ_z		2.76	3.29		2.76	3.29		2.78	3.29
t and h_t									
Δt and Δh									
h			47			35			78
H base line			28504			28504			28504
H			28551			28539			28582
I 38°			51.81			53.27			48.98
log H			4.45562			4.45544			4.45609
" tan I			9.90625			9.90663			9.90552
" Z			4.36187			4.36207			4.36161
Z			23008			23018			22994
t and z_t									
Δt and Δz									
z			99			109			82
Z base line			22909			22909			22912
Observer	T H Pearce			R F White			D C McGowan		

Scale mean ordinates separately for the two halves of the period of observation. Parallax allowed for. Scaled interval lies ...0.3... mm. to right ~~left~~ (cross out one) of interval shown by time flashes for H and ...0.2... mm. to ~~right~~ left for Z.

* Ordinate corrected to 100.0 mm. shrinkage distance.

Scaled by D C M *Checked by* R F W

Computed by D C M *Checked by* R F W

Abstracted by R F W

Z revised ...., 194...., using differential formula ΔZ= .... ΔH+ ....ΔI.

U. S. GOVERNMENT PRINTING OFFICE 16—50438-1

Figure 97.—Computation of vertical-intensity base-line value.

and

$$B_{Z0} = H \tan I_A - S_Z\, z - Q_Z\, (t - t_0) \tag{356}$$

$$= [B_{H0} + (S_0 + \tfrac{1}{2} a\, h)\, h + Q_H\, (t - t_0)] \tan I_A - S_Z\, z - Q_Z\, (t - t_0). \tag{357}$$

Figures 95, 96, and 97 will illustrate how D, H, and Z base-line values are computed for the cases where the temperature coefficients of the H and Z variometers are zero. Note: In figures 96 and 97 the symbol ϵ is used in place of S for the scale value.

IMPORTANCE OF ADEQUATE CONTROL

426. ***Control observations for deriving absolute values.***—Directions for making absolute observations with a magnetometer and earth inductor are given by Hazard.[3] Directions for operating the magnetic field balance are well covered by Joyce[4] and by Heiland and Pugh.[5] A complete description of the sine galvanometer is given in the Researches of the Department of Terrestrial Magnetism.[6] The publications of the Danish Meteorological Institute[7] give complete details on the operation of the la Cour instruments.

427. In making absolute observations at a magnetic observatory greater care must be exercised in the operations and a greater degree of accuracy is required than is the case for work in the field. However, it is not possible, as a rule, to achieve in the individual observations an accuracy on a par with the short-term stability of the variometers. Usually the base-line values resulting from a series of observations, when plotted against time, will show more or less spread due to accidental error in the absolute observations, or to the application of temperature or other corrections that are made with faulty coefficients. They may also show a progressive change or drift with time. This drift may result from loss of magnetism of the suspended magnets, from the seasonal effect of error in the adopted temperature coefficient of the variometer, or (for H) from changes in the elastic properties of the quartz-fiber suspension. Hence, in determining what base-line values to adopt, it is necessary to adjust the observed values with due regard for these progressive changes. For any particular set of instruments it can be determined only by experience how closely the adopted values should correspond to those resulting from actual observations.

428. ***Time observations.***—Chronometers, pendulum clocks, or other timepieces used in any part of the observing program should be compared daily with radio time signals broadcast by the U. S. Naval Observatory, the National Bureau of Standards, or other reliable source. Where a time-flashing mechanism is used to place time marks on the magnetogram, it is convenient to keep the time correction of the time-marking clock small so that it will not be necessary to allow any time correction in scaling values from a magnetogram. The method of keeping a record of the performance of a chronometer or clock is shown in figure 98.

DIRECTIONS FOR PROCESSING RECORDS

429. ***Producing the magnetograms.***—Change the paper on the magnetograph drum daily at the same hour, on the hour, noting on the magnetograph record (fig. 99) any pertinent facts, however trivial, regarding any adjustments of the instruments, natural or artificial mechanical disturbances, or other unusual events related to the operation of the magnetograph and of the absolute instruments. Develop the traces as soon as possible after exposure, and examine them carefully for quality and for possible malperformance of any of

[3] D. L. Hazard, Dir. for Mag. Meas. (see item 4 of bibliography).
[4] J. W. Joyce, Manual on Geoph. Pros. with the Mgr., U. S. Dept. Int., Bur. of Mines, 1937 (see item 7 of bibliography).
[5] C. A. Heiland and W. E. Pugh. Am. Inst. Min. and Met. Eng., Tech. Pub. 483, 1932 (see item 5 of bibliography).
[6] Barnett, DTM, CIW, Pub. No. 175 (see item 1 of bibliography).
[7] La Cour, Danske Met. Inst. Pubs. (see items 8–11 of bibliography).

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 698—Ed. Jan. 1950

GPO 16—29655-2

TUCSON, ARIZONA (Station) Chron. 1595 (Chron. or Clock No.)

Jan. (Month) 1952 (Year)

TIME RECORD

DATE	BROAD-CAST STATION	105 MER. TIME OF SIGNAL			105 MER. TIME BY CHRON. OR CLOCK			
		h.	m.	s.	h.	m	s.	
1	WWV	09	00	00	09	03	49.2	
2	"						52.8	
3	"						55.8	
4	"						58.9	
5	"					04	02.5	
6	"						06.3	
7	"						09.4	
8	"						13.2	
9	"						16.0	
10	"						19.3	

Figure 98.—Time comparisons between chronometer and continuous time signals from Station WWV, U. S. National Bureau of Standards.

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 247
Rev. Feb. 1938

MAGNETOGRAPH RECORD

Oct., 1951
(Month)

TUCSON Magnetic Observatory. Magnetograph No. LaCour

Day of Month	Time by Chron.* No. watch			Corr. to 105th Mer. Time		Temperature		Remarks
	Stop	Begin	Other†	Chron.*	Pend. Clock	H	Z	
	h. m.	h. m.	h. m.	m. s.	m. s.	°	°	
11			10 04 10 06					Parallax Test
			10 08 10 43					Scale Value
12	08 00					26.4		
		08 02						
13	08 00					26.6		
		08 02						
14	08 00					26.2		
		08 03						
			15 57					Check Operations
15	08 00					26.0		Wound Clock
		08 01						
			16 02 16 10					Visitors
16	08 00					25.9		
		08 02						
17	08 00					25.8		
		08 02						
18	08 00					25.8		
		08 02						
19	08 00					25.5		Cleaned paper clamp bar
		08 03						
20	08 00					25.4		
		08 02						

* Or watch.
† This column is to be used for single time entries, usually when the photographic trace is not interrupted.

U. S. GOVERNMENT PRINTING OFFICE 16—41990-1

Figure 99.—Magnetograph Record, Tucson Magnetic Observatory.

the variometers or of the time-marking system. Establish a systematic and fixed routine for this part of the work. Carefully apply all necessary legends on the magnetograms as shown in figure 93. This work should be done neatly and accurately, because the magnetograms are subsequently reproduced for publication.[8]

[8] U. S. Coast and Geodetic Survey, Magnetograms and Hourly Values MHV-Ch50 (in press, 1952).

SCALING OF ORDINATES

430. ***Types of scaling.***—Three varieties of scaling are customarily done on the magnetograms at the observatory, namely, those concerned with: (a) hourly values, or mean ordinate over each of the twenty-four hourly intervals during the day; (b) base-line scalings, or mean ordinate for the time interval during which an observation has been made for absolute value of a magnetic element; and (c) scale-value deflections.

431. ***Hourly values.***—For measuring the average ordinate for an hour a special scaling glass is used, accurately engraved with rulings

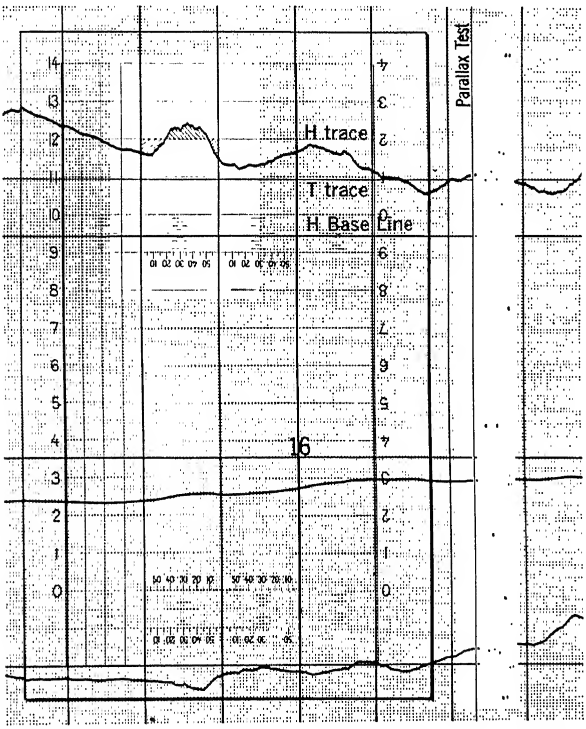

Figure 100.—Magnetogram reading scale superimposed on magnetogram for scaling ordinates. Cut-out portion, upper left, shows method of averaging hourly ordinates. Parallax test and scale-value deflection on right.

and graduations as shown in figure 100. Vertical or lengthwise lines on the scaling glass are 20 mm apart, corresponding to a time interval of one hour on the magnetogram. Horizontal or transverse lines are 1 cm apart, with finer divisions of 1 mm. The scaling glass is laid on the magnetogram with the ruled surface next to the paper, and with the vertical lines coinciding with the hour marks on the gram. The space divided into millimeters on the glass should be placed across the base line. The scale is then moved up or down until one of the traverse lines is set for the average ordinate of an hour-long interval. With a little practice this can be done rapidly and accurately by making equal the areas between the trace being scaled and the trans-

verse line, as illustrated by the shaded areas in the figure. The number of whole centimeters is read at the end of the transverse line, and the fraction of a centimeter is read to tenths of millimeters at the base line. The tabulations are ordinarily made directly in tenths of millimeters for the whole ordinate. Thus, for the example illustrated the tabular entry would be 257.

432. Occasionally during disturbed magnetic conditions it is difficult to judge accurately the proper setting of the scaling glass for the mean ordinate of the whole hour. It is then easier to use half-hour or quarter-hour intervals, calculating the mean of the two or four values thus scaled for use as the hourly value. One section of the scaling glass is divided into half-centimeter (quarter-hour) intervals. Under extremely disturbed conditions it may be found desirable to divide the hourly interval into even smaller time intervals by drawing auxiliary time lines directly on the magnetogram, but it is seldom that the increase in accuracy thus obtained for the hourly value can justify the extra time required by this procedure.

433. Tabulations of the hourly values may be made in any convenient form. Figure 101 illustrates the form now being used by the Coast and Geodetic Survey. This style of scaling sheet expedites the step of transferring the values to punch cards for further processing of the data at the Washington office, where the publications are compiled.

434. ***Base-line scalings.***—The same procedure described for hourly value scalings is used for base-line scalings, except that the interval of time is determined by the duration of the absolute observations with a magnetometer or earth inductor. Extra time lines are placed on the magnetogram, through a push-button circuit from the absolute building, when an observation is begun and again when it is finished. For instance, observations for declination usually require 9 minutes. The extra time lines would thus be 9 minutes, or 3 millimeters, apart on the magnetogram, and the mean D ordinate for that 9-minute interval would constitute the base-line scaling for that declination observation. Base-line scalings are customarily tabulated in millimeters, to the nearest tenth-millimeter.

435. ***Scale-value deflections.***—Figure 100 also illustrates a typical set of scale-value deflections for all three elements, D, H, and Z. In the illustration, deflections of the Z trace are made first (two deflections with the deflector magnet placed N end up, separated by a deflection made with the N end down), followed by deflections of the D trace, and finally deflections of the H trace. All three variometers are disturbed, of course, when the deflector magnet is in place for any deflection, so that care must be used in selecting the proper deflection spots on the magnetogram.

436. Scalings of the deflected spots are made, not to the base line but from one spot to the immediately succeeding spot—that is, from an up spot to a down spot and then from a down spot to the next up spot. (All measurements should be made in a direction perpendicular to the base line—it is the difference between ordinates of deflected spots that is being measured.) This is a measurement of $2u$, the effect caused by a complete reversal of the deflecting magnet, and the quantity used in computing the scale values of the several element traces on the magnetogram. The sample scale-value computation shown in figure 72 is derived from the deflections of figure 100.

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Division: Geo. & Seis.
Form 401e
(FEB. 1949)

COLLEGE, ALASKA (Observatory) 1951 (Year) OCT. (Month)

HOURLY MEAN ORDINATES FROM MAGNETOGRAMS

Average values for successive periods of one hour beginning at midnight 150 M.T.
Tenths of millimeters corrected to shrinkage distance of 100.0 mm
Signs are as follows unless otherwise noted: D + ; H + ; Z +
Signs reviewed by M.L.C.

Char.	0				1				0		
*Shr.	99.0				99.0				99.0		
Day	25				26				27		
Hr.	D	H	Z	Hr.	D	H	Z	Hr.	D	H	Z
01	101	375	71	01	110	26	82	01	53	447	98
02	97	372	72	02	197	199	14	02	94	473	95
03	102	364	71	03	106	446	51	03	121	328	82
04	103	366	69	04	109	383	75	04	130	241	32
05	110	367	69	05	146	362	73	05	110	292	5
06	113	368	70	06	153	352	65	06	133	367	40
07	125	365	70	07	144	361	62	07	132	362	61
08	130	368	70	08	163	363	61	08	125	358	64
09	121	368	67	09	138	372	58	09	111	352	67
10	112	362	68	10	146	356	62	10	98	343	68
11	102	363	67	11	99	332	59	11	92	338	70
12	89	366	65	12	48	311	77	12	83	345	72
13	82	368	67	13	-115	278	63	13	72	352	73
14	90	363	70	14	-62	341	68	14	71	351	71
15	81	365	70	15	10	378	89	15	71	355	72
16	82	363	70	16	31	391	104	16	70	360	70
17	81	370	69	17	30	402	92	17	77	370	71
18	84	373	69	18	-15	441	95	18	74	377	72
19	86	374	69	19	12	543	126	19	71	381	76
20	75	376	69	20	49	510	131	20	53	510	122
21	42	450	90	21	40	473	127	21	162	458	143
22	78	521	102	22	61	396	109	22	98	422	124
23	63	467	94	23	67	387	92	23	67	416	111
24	33	452	65	24	68	422	91	24	68	387	100
Sum	2182	9246	1733		1735	8825	1926		2236	8985	1859
Mean	91	385	72		72	368	80		93	374	77
* Scaled by		W.H.S.				W.H.S.				W.H.S.	
*Checked by		V.A.T.				V.A.T.				V.A.T.	

* This entry refers to magnetogram which begins on this day rather than to calendar day

Figure 101.—Tabulation of scaled hourly values.

437. Scalings are also made of the *away* positions of the trace—the ordinate, measured from the base line, of the last portion of the trace immediately preceding scale-value deflections, and the ordinate of the first recording of the trace immediately following scale-value deflections. These *away* ordinates serve to give an approximate value of the element during scale-value observations.

438. ***Scaling from low-sensitivity magnetogram.***—Some observatories are equipped with a low-sensitivity magnetograph, whose variometers are ordinarily from one-third to one-sixth as sensitive as the regular variometers. Except for the difference in scale values, the magnetograms produced are equivalent to the regular magnetograms. During severe magnetic storms the rate of movement of the recording spot may be so rapid on the regular magnetogram that no photographic impression is left on the paper, or the individual traces may overlap to such an extent that portions of them cannot be identified. In such cases a comparison of the regular and low-sensitivity magnetograms will often serve to disentangle the regular traces or assist in identification of the uncertain parts. However, for scaling hourly values where it is difficult to judge the proper position of the scaling glass on the regular gram, the following plan is often used: Hourly values are scaled on the low-sensitivity magnetogram for the most disturbed hours (scaling even these traces by quarter-hours if necessary); after the mean ordinate is measured on the low-sensitivity gram a salient or easily identified *point* can usually be found, within a few hours of the interval being scaled, whose ordinate is equal to the mean ordinate for the interval; the corresponding point is then identified on the regular magnetogram, its ordinate is measured with respect to the regular base line, and that value is used as the mean ordinate of the disturbed hour that could not be scaled directly. A rough check, for gross errors, can usually be made on the regular magnetogram even though accurate values cannot be scaled.

439. ***Shrinkage.***—Most photographic paper, when subjected to the processes of developing, washing, and drying, will undergo a marked change in its linear dimensions. Furthermore, some additional change of dimensions will occur from day to day as the humidity of the air fluctuates. The difference between the width of a magnetogram at any time and its width at the time of photographic exposure is called shrinkage, even though under some circumstances the shrinkage is negative, i. e., even though the paper actually shows an increase in size. All scaled ordinates (including deflections for scale value) are corrected for shrinkage effects by applying a shrinkage correction either before actual entries are made on the tabular form, or on the form itself if space is provided, as it is on the form for *H* base-line computations (fig. 96).

440. *Shrinkage points* are pricked in the unexposed photographic paper just before placing it on the recording drum and again immediately after removing it from the drum. At the time of scaling the magnetograms the distance between the pin pricks is measured, and the shrinkage correction is based on the change in the distance between the shrinkage points. The original distance is 100.0 mm; the shrinkage gauge itself consists simply of two needle points permanently fixed 100.0 mm apart in a brass bar. Since the shrinkage effect is not the same in different directions (along or across the grain of the paper),

the shrinkage gauge should always be applied along a line parallel to the direction of the ordinates to be measured.

441. ***Parallax test.***—Since the optical system that imprints the time marks on the magnetograms is different from the system that produces the magnetic traces, it is possible for the light spot recording *D*, *H*, or *Z* on the magnetogram not to fall on the time line. This produces an apparent time difference between trace and time mark that is called parallax, as explained in paragraphs 175 and 176 of chapter 5. By extinguishing the trace-recording lamp for about three minutes and, in the middle of that time interval, placing a time line on the gram, the amount of parallax can be recorded. A typical parallax test is shown in figure 100. In this example the time line is displaced not more than 0.1 mm from the center of the gap in the traces, indicating a parallax of less than 0.5 minute in time—a negligible amount for all practical purposes. In some installations, however, parallax might amount to a minute or more in time, and in such a case it may be necessary to take into consideration the shift of the time lines with respect to true time of recording the trace—particularly for base-line scalings. It has been mentioned that parallax between base lines and time marks is of no consequence; when a magnetograph is being installed it is well-nigh impossible to maintain the same parallax for each base line as for the corresponding active trace.

442. ***Magnetic-activity data.***—A magnetic observatory ordinarily provides data on magnetic activity, in addition to the routine variometer-control data and hourly means of *D*, *H*, and *Z*. In particular there are monthly tabulations of:

(a) K indices (values of the three-hour-range index), one for each three-hour interval of the Greenwich day, on a scale of 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9.

(b) C figures (character figures), one for each Greenwich day, on a scale of 0, 1, and 2.

(c) Time of sudden commencement (SC) of each magnetic storm.

(d) Times of beginning and ending of each solar flare effect (sfe), with a nonflare K value, denoted by K′.

(e) Magnetic-storm data, in addition to sudden commencements.

(f) Zone-time (local-day) character figures, one for each local day, on a scale of 0, 1, and 2.

Sample tabulations of (a), (b), (c), and (d) are shown in figure 102. The local-day character figures may be tabulated on the scaling sheets (par. 433, fig. 101), for convenience in preparing the hourly-value (HV) reports.

443. The requirements of research scientists, communications engineers, and others interested in magnetic activity are continually changing. The determination of magnetic activity from the magnetograms, while not particularly difficult, is a somewhat subjective and highly specialized work. For these reasons no attempt is made to describe in this manual the methods of deriving K, C, and the other activity data, but current instructions and references are to be observed.[9]

---

[9] J. Bartels, *et al.*, Terr. Mag. 44, 411-454, 1939; Intl. Assn. Terr. Mag. and Elect., Circular to the Observatories, Dec. 1951, K-indices, sudden commencements, and solar-flare effects; Intl. Assn. Terr. Mag. and Elect., Bull. No. 12c, Geomagnetic Indices K and C, 1950, (1951).

Cheltenham, Md.
(Observatory)
July 1951
(Month) (Year)

Magnetic Activity

(Greenwich civil time, counted from midnight to midnight)

Date	K-indices 00–03	03–06	06–09	09–12	12–15	15–18	18–21	21–24	Sum	Whole-day character 0, 1, or 2
1	4	2	1	2	2	3	3	6	23	0
2	7	7	6	5	3	4	3	5	40	1
3	4	4	4	3	5	4	4	5	33	1
4	4	4	4	2	2	4	3	4	27	1
5	3	3	3	2	2	3	1	3	20	0
6	2	3	3	2	2	2	3	3	20	0
7	2	2	2	1	1	3	3	4	18	0
8	3	3	1	1	1	2	3	4	18	0
9	4	4	2	2	3	3	4	3	25	1
10	3	2	3	2	1	2	3	2	18	0
11	1	1	2	2	2	2	3	2	15	0
12	2	2	1	2	2	2	2	4	17	0
13	3	2	1	2	1	1	2	3	15	0
14	2	1	2	2	1	1	3	3	15	0
15	2	1	3	2	2	3	3	6	22	0
16	3	4	2	3	3	3	3	3	24	0
17	4	3	3	4	3	3	4	4	28	1
18	3	3	5	3	3	4	4	3	28	1
19	3	2	4	4	2	3	3	3	24	0
20	3	3	3	2	2	2	2	3	20	0
21	4	3	4	3	1	1	2	1	19	0
22	1	4	6	4	3	4	3	5	30	1
23	6	3	4	4	2	2	3	3	27	1
24	2	3	4	2	3	1	1	2	18	0
25	1	2	4	2	2	3	3	3	20	0
26	3	4	5	3	3	4	3	4	29	1
27	4	4	4	5	1	3	2	4	27	1
28	5	6	5	4	3	3	4	4	34	1
29	3	4	4	4	2	2	2	3	24	0
30	2	2	2	3	3	2	3	3	20	0
31	4	5	5	4	3	4	5	4	34	1
									Sum	12

Time scale on magnetograms
20 mm/hr

Sudden commencements

d	h	m
1	22	27

Possible solar-flare effects based on inspection of grams alone (without reference to data from other sources).

Begin d	h	m	End d	h	m	
31	20	08	31	21	- -	K′ 4

K scale used:

	D	*H*	*Z*	
Lower limit for K=9	93.2	190.4	119.3	(mm)
Current scale value	5.4	2.6	4.0	(γ/mm)
Lower limit for K=9	500γ	500γ	480γ	(to nearest 10γ)

Scalings and computations have been checked. Approved: Ralph R. Bodle, Observer in Charge.

Figure 102.—Magnetic activity report.

APPENDIX I. SOME APPROXIMATIONS

444. For convenient reference there are listed below some relationships frequently used in geomagnetic computations.

N = any number;

e = the base of natural logarithms;

x = a small quantity, of the order of 0.002 or less;

A = any value such that the product, Ax, is small compared to unity;

$\ln N$ = the natural logarithm of N;

$\log N$ = the logarithm of N to the base 10 (common log);

$$\log e = 0.434\ \ldots;\ \frac{1}{\log e} = 2.303\ \ldots \tag{358}$$

$$\log N = 0.434 \ln N \tag{359}$$

$$\ln N = 2.303 \log N \tag{360}$$

$$\ln (1+x) \approx x \tag{361}$$

$$\log (1+x) \approx 0.434x \tag{362}$$

$$x \approx 2.303 \log (1+x) \tag{363}$$

$$(1+Ax) \approx (1+x)^A. \tag{364}$$

By equation (364),

$$\log (1+Ax) \approx \log (1+x)^A \tag{365}$$

$$\approx A \log (1+x) \tag{366}$$

and by equation (362)

$$\log (1+Ax) \approx 0.434\ Ax. \tag{367}$$

$$\frac{d}{dN} \ln N = \frac{1}{N} \tag{368}$$

$$d\ (\ln N) = \frac{1}{N} dN \tag{369}$$

$$d\ (\log N) \approx d\ (0.434 \ln N). \tag{370}$$

From equation (359),

$$d\ (\log N) \approx 0.434\ d\ (\ln N); \tag{371}$$

By equation (369),

$$d\ (\log N) \approx 0.434 \left(\frac{1}{N}\right) dN. \tag{372}$$

From equation (372),

$$\frac{dN}{N} \approx \frac{1}{0.434}\, d \log N \tag{373}$$

$$\approx 2.303\ d \log N. \tag{374}$$

From equation (373),

$$\frac{d \log N}{dN} \approx 0.434 \left(\frac{1}{N}\right). \tag{375}$$

$$\sin x \approx \tan x \approx x \text{ (in radians)} \tag{376}$$

$$\cos x \approx 1 - \frac{x^2}{2} \text{ (in radians)} \tag{377}$$

$$\sin 1' \approx \tan 1' = 0.000\ 2909 \tag{378}$$

$$\frac{1}{\sin 1'} \approx \frac{1}{\tan 1'} \approx \cot 1' = 3438. \tag{379}$$

Number of minutes in 1 radian$=\dfrac{360 \times 60}{2\pi}=3438.$

Then $$1' = \frac{1}{3438} \text{ radian.}$$

This is applied in scale-value analysis, where for example we use

$$\tan \Delta D = \frac{\Delta D'}{3438}$$

$$= (\Delta D')\ \tan 1'. \tag{380}$$

In general, if a small angle x is expressed in minutes and denoted by x', then

$$\sin x \approx 0.000291\ x' \tag{381}$$

$$\tan x \approx 0.000291\ x'. \tag{382}$$

445. ***Corrections to log T^2 in Oscillations.***—To obtain a closer approximation to the true value, (T), of the time of one oscillation

in the determination of H or K with a magnetometer, corrections should be applied to the observed time T for the following:

ITEM	REF. (PAR.)
(a) Rate of chronometer	446
(b) Change of magnetic moment due to temperature change, $\frac{\Delta M}{\Delta t}$	447
(c) Change with temperature of moment of inertia of magnet, $\frac{\Delta K}{\Delta t}$	449, 450
(d) Change of moment of inertia of inertia weight with temperature, $\frac{\Delta K_1}{\Delta t}$	448, 450
(e) Torsional couple of the suspension fiber or filament, $\frac{h}{f}$	451
(f) Change of H during observations, ΔH	452
(g) Damping (probably negligible)	---
(h) Reduction to infinitesimal arc (probably negligible)	---
(i) Induction (inductive effect of H on magnetic moment)	---

446. ***Corrections to $(T)^2$ for rate of chronometer.***—Let

d=the daily rate of the chronometer

=the number of seconds lost by the chronometer in one day, plus (+) if chronometer is losing, minus (−) if gaining;

(T)=the true time in seconds of one oscillation;

T=the observed time of one oscillation in *chronometer* seconds;

86,400=the number of seconds in 1 day;

$\frac{d}{86,400}$=the fractional part of a second lost by the chronometer in 1 second;

$(T)\frac{d}{86,400}$=the seconds lost by the chronometer in (T) seconds.

Then

$$(T)=T+(T)\frac{d}{86,400} \tag{383}$$

$$T=(T)-(T)\frac{d}{86,400}$$

$$=(T)\left(1-\frac{d}{86,400}\right)$$

$$(T)=\frac{T}{1-\frac{d}{86,400}}=\frac{T}{1-0.000\ 0116d}$$

$$\log\ (T)=\log\ T-\log\ (1-0.000\ 0116d). \tag{384}$$

Compare log $(1-0.000\ 0116d)$ with equation (362), in which x may be equal to $-0.000\ 0116d$. By equation (362),

$$\log (T)=\log T-0.434\times(-0.000\ 0116d)$$
$$=\log T+0.000\ 0050d$$

$$\log (T)^2=2\ (\log T+0.000\ 0050d)$$
$$=2\log T+0.000\ 01d. \tag{385}$$

That is, to obtain log $(T)^2$, add to log T^2 the correction, d, in the fifth decimal of logs. The log $(T)^2$ will be increased for a losing rate (d positive) and decreased for a gaining rate (d negative).

447. ***Correction to $(T)^2$ for change of magnetic moment with temperature.***—Let

M_{20}=magnetic moment of the magnet at standard temperature, 20° C;
$\Delta M=M_t-M_{20}$=change in magnetic moment when the temperature changes from standard temperature to t° C;
$\Delta t=t-20$;
$q=-\frac{\Delta M}{M\Delta t}$=temperature coefficient of the magnetic moment, by definition;
t=temperature at which the observations are made;
(T)=time of one oscillation at standard temperature;
T=observed time of one oscillation at t° C.

We have

$$(T)^2=\frac{\pi^2 K}{HM_{20}}\ (K,\ H,\text{ and } M\text{ constant}) \tag{386}$$

and at a temperature t°,

$$T^2=\frac{\pi^2 K}{H(M_{20}+\Delta M)}\ (\text{When } M \text{ changes by } \Delta M). \tag{387}$$

Dividing equation (386) by equation (387),

$$\frac{(T)^2}{T^2}=\frac{M_{20}+\Delta M}{M_{20}}=1+\frac{\Delta M}{M}$$
$$=1-q\Delta t.$$

For $(t-20)$ change in temperature,

$$\frac{(T)^2}{T^2}=1-q(t-20)$$

$$(T)^2=T^2\ [1+q(20-t)]$$

$$\log (T)^2=\log T^2+\log\ [1+q(20-t)].$$

Compare log $[1+q(20-t)]$ with equation (366) or equation (367); let $q=x$, and $(20-t)=A$. By equation (366),

$$\log (T)^2 = \log T^2 + (20-t) \log (1+q). \tag{388}$$

By equation (367),

$$\log (T)^2 = \log T^2 + 0.434\, q\, (20-t). \tag{389}$$

448. ***Change of moment of inertia of inertia weight with temperature.***

(a) For an inertia weight in the form of a *solid right circular cylinder*, the moment of inertia, K_1, at 20° C, about a transverse diameter through the center, is

$$K_{1,\,20} = m\left(\frac{l^2}{12} + \frac{d^2}{16}\right) \tag{390}$$

in which m is the mass of the cylinder in grams; l and d are the length and diameter in cm at 20° C. When the temperature changes to $t°$, $\Delta t = (t-20)$; l and d will change by Δl and Δd. At $t°$ C,

$$K_{1,\,t} = m\left(\frac{(l+\Delta l)^2}{12} + \frac{(d+\Delta d)^2}{16}\right). \tag{391}$$

Expanding, and neglecting $(\Delta l)^2$ and $(\Delta d)^2$, since they are extremely small,

$$K_{1,\,t} = m\left(\frac{l^2}{12} + \frac{d^2}{16}\right) + m\left(\frac{2l\Delta l}{12} + \frac{2d\Delta d}{16}\right). \tag{392}$$

Subtracting equation (390) from equation (392),

$$K_{1,\,t} - K_{1,\,20} = \Delta K_1 = m\left(\frac{2l\Delta l}{12} + \frac{2d\Delta d}{16}\right). \tag{393}$$

Let α' = the coefficient of thermal expansion (linear)
= 0.000 018 per deg. C, for a brass weight.

Then $\Delta l = l\alpha'\Delta t$ and $\Delta d = d\alpha'\Delta t$. (394)

Substituting these values in equation (393),

$$\Delta K_1 = m\left(\frac{2l^2\alpha'\Delta t}{12} + \frac{2d^2\alpha'\Delta t}{16}\right) \tag{395}$$

$$\Delta K_1 = (K_{1,\,20})(2\alpha'\Delta t) \tag{396}$$

and

$$\frac{\Delta K_1}{K_{1,20}} = 2\alpha'\Delta t. \tag{397}$$

But from equation (373),

$$\frac{\Delta K_1}{K_{1,20}}=\frac{1}{0.434}\ \Delta \log K_{1,20}$$

then
$$2\alpha'\Delta t=\frac{1}{0.434}\ \Delta \log K_{1,20}$$

and
$$\Delta \log K_{1,20}=0.434(2\alpha'\Delta t)$$
$$=0.868\alpha'\Delta t.$$

That is,
$$\log K_{1,t}-\log K_{1,20}=0.868\ \alpha'\Delta t. \tag{398}$$

For brass $\Delta \log K_{1,20}=0.000\ 016$ if $\Delta t=1°$ C.

If the dimensions of the inertia weight are determined at, say, $t=25°$ C, then from equation (398),

$$\log K_{1,20}=\log K_{1,25}-0.868\alpha'(25-20);$$

for brass:
$$\log K_{1,20}=\log K_{1,25}-0.000\ 078.$$

(b) *For an inertia weight in the form of a ring,* oscillating about its central axis,

$$K_{1,20}=\frac{m}{8}\ (d_1^2+d_2^2) \tag{399}$$

in which m is the mass in grams, and d_1 and d_2 are the inner and outer diameters of the ring in cm at 20° C.

At $t°$ C,

$$K_{1,t}=\frac{m}{8}\ [(d_1+\Delta d_1)^2+(d_2+\Delta d_2)^2]. \tag{400}$$

Following the same procedure as in 448 (a), we find that, *for a brass ring,*

$$\Delta \log K_1=0.000\ 016 \text{ if } \Delta t \text{ is } 1°\text{ C.}$$

Thus the change in the value of $\log K_1$ per degree C is the same for the ring as for the cylinder.

(c) *In general* the moment of inertia of a body of uniform composition will change with temperature in accordance with the approximate relation

$$\log K_0-\log K_t=0.868\ \alpha(t-t_0). \tag{401}$$

449. ***Correction to $(T)^2$ for change of moment of inertia of the magnet with change of temperature.***—When the temperature increases, the dimensions of the magnet, stirrup, and inertia weight increase, thus increasing both K and K_1, and lengthening the corresponding periods of oscillation. Consider the change in the moment of inertia K of the magnet system alone due to change in temperature,

neglecting all other changes which influence the period (such as changes in H, torsion factor, temperature coefficient of magnetic moment, etc.).

At 20° C, $$(T)^2=\frac{\pi^2 K_{20}}{HM_{20}}. \tag{402}$$

At a temperature t,

$$T^2=\frac{\pi^2(K_{20}+\Delta K)}{HM_{20}} \tag{403}$$

where $\Delta K=K_t-K_{20}$.

Divide equation (402) by equation (403),

$$\frac{(T)^2}{T^2}=\frac{K_{20}}{K_{20}+\Delta K}=\frac{1}{1+\frac{\Delta K}{K_{20}}}.$$

But, as in equation (397),

$$\frac{\Delta K}{K_{20}}=2\alpha\Delta t, \tag{404}$$

where $\Delta t=(t-20)$.

Then $$(T)^2=\frac{T^2}{1+2\alpha\Delta t} \tag{405}$$

$$\log\,(T)^2=\log\,T^2-\log\,(1+2\alpha\Delta t).$$

By equation (367),

$$\log\,(T)^2=\log\,T^2-0.868\alpha(t-20) \tag{406}$$

and, for $\alpha=0.000\,011$,

$$\log\,(T)^2=\log\,T^2+0.000\,01\,(20-t). \tag{407}$$

The factor $0.000\,01\,(20-t)$ is the correction that must be applied to $\log T^2$ to give the value of $\log\,(T)^2$ at 20° C. In this case $\alpha=0.000\,011$ the coefficient of thermal expansion (linear) of steel. The coefficient is slightly larger for aluminum-nickel-steel alloys.

450. ***Correction for change in*** $(K+K_1)$ ***due to temperature change (loaded system).***—It may be shown that the treatment is the same as for changes in the dimensions of the magnet (or the inertia weight) due to temperature change, except that in this case the effective coefficient of expansion is taken as approximately the mean of the coefficients for steel and brass. Taking $\alpha_1=0.000\,014$ as this mean coefficient, and letting $\Delta K(=K_t-K_{20})$ represent the total increase in the moment of inertia of the magnet, stirrup, and inertia weight combined (due to an increase in temperature $\Delta t(=t-20)$, with all other coefficients equal to zero), and substituting in equation (406),

$$\log\,(T_1)^2=\log\,T_1^2-0.868\times 0.000\,014\,(t-20) \tag{408}$$

$$=\log\,T_1^2+0.000\,012\,(20-t), \tag{409}$$

in which (T_1) is the time of one oscillation at 20° C (loaded system), and T_1 is the observed time of one oscillation at t° C (loaded system). The term, 0.000 012 $(20-t)$, is the correction that must be applied to log T_1^2 to give the value of log $(T_1)^2$ at the standard temperature, 20° C, under the imposed conditions.

451. ***Correction to T^2 for torsion factor.***—Let

f =angular motion of torsion head in minutes, in torsion test;
h =angular deflection of magnet in minutes, for $(f-h)$ minutes of twist;
(T) =true time of one oscillation (torsionless fiber; H, M_s, K, chronometer rate, etc. constant);
T =observed time of one oscillation, torsion effective; (H, M_s, K, etc., constant).

For a small amplitude and torsionless fiber,

$$(T)^2=\frac{\pi^2K}{HM_s}; \qquad (410)$$

and with torsion effective,[1]

$$T^2=\frac{\pi^2K}{HM_s+k'}. \qquad (411)$$

By equation (151), when $C=0$,

$$k'=HM_s\,\frac{h}{f-h} \qquad (412)$$

and

$$T^2=\frac{\pi^2K}{HM_s+HM_s\,\frac{h}{f-h}}. \qquad (413)$$

Divide equation (410) by equation (413):

$$\frac{(T)^2}{T^2}=1+\frac{h}{f-h}=\frac{f}{f-h}. \qquad (414)$$

Letting $f=5400'$, the usual angular motion of the torsion head in magnetometer work,

$$\frac{f}{f-h}=\frac{5400'}{5400'-h}=1+\frac{h}{5400'-h}. \qquad (415)$$

Since h is usually small compared to 5400′, equation (414) may be written:

$$\frac{(T)^2}{T^2}\approx 1+\frac{h}{f}. \qquad (416)$$

[1] Dir. for Mag. Meas., pp. 16, 17 (see item 4 of bibliography).

By equation (362),

$$\log\,(T)^2 \approx \log\,T^2 + 0.434\,\frac{h}{5400'} \tag{417}$$

$$\approx \log\,T^2 + h\left(\frac{0.434}{5400}\right)$$

$$\approx \log\,T^2 + 0.00008h. \tag{418}$$

Note: Even when h is as large as 60′, which is unusual, equation (418) holds with sufficient precision in this work.

452. ***Correction for change in H (all other factors constant).***— Let $(T)^2$ be the time of one oscillation, reduced to a standard H $(=H_0)$;

Then
$$(T)^2 = \frac{\pi^2 K}{H_0 M_s}. \tag{419}$$

Now let H change by ΔH, such that ΔH is small compared to H_0; H is now $H_0+\Delta H$, and

$$T^2 = \frac{\pi^2 K}{(H_0+\Delta H)M_s}. \tag{420}$$

Divide equation (419) by equation (420):

$$\frac{(T)^2}{T^2} = \frac{H_0+\Delta H}{H_0} = 1 + \frac{\Delta H}{H_0} \tag{421}$$

$$(T)^2 = T^2\left(1+\frac{\Delta H}{H_0}\right) \tag{422}$$

$$\log\,(T)^2 = \log\,T^2 + \log\left(1+\frac{\Delta H}{H_0}\right). \tag{423}$$

From equation (362),

$$\log\,(T)^2 \approx \log\,T^2 + 0.434\,\frac{\Delta H}{H_0}. \tag{424}$$

When the observations are made at a magnetic observatory it is proper and convenient to choose the H base line value (B_H) for H_0. Then ΔH becomes the mean ordinate of the H trace during the time of a set of oscillations. Thus

$$H = H_0 + \Delta H \tag{425}$$

becomes
$$H = B_H + h_\gamma \tag{426}$$

$$= B_H + h_{mm} S_H \tag{427}$$

and
$$\frac{\Delta H}{H_0} = \frac{h_{mm} S_H}{B_H}. \tag{428}$$

Then, from equation (424),

$$\log (T)^2 = \log T^2 + 0.434 \frac{S_H}{B_H} h_{mm}. \tag{429}$$

Example: Let

$$B_H = 18{,}200$$

$$S_H = 2.60.$$

Then

$$0.434 \frac{S_H}{B_H} = 0.000\ 062.$$

It may be shown that any value of B_H within about one-half percent of the value of H is sufficiently accurate to reduce T^2 to the value $(T)^2$ that would be observed if the H ordinate were zero.

453. ***Correction for induction in oscillations.***—Let

ΔM = the change in the magnetic moment due to induction;

μ = the induction factor (see par. 110, chap. 4).

Then $\mu = \frac{\Delta M}{H}$, and $\frac{\Delta M}{M} = \mu \frac{H}{M}$.

In oscillations,

$$(T)^2 = \frac{\pi^2 K}{HM} \text{ (everything constant, } \mu = 0). \tag{430}$$

When induction *is* effective and everything else is constant,

$$T^2 = \frac{\pi^2 K}{H(M + \Delta M)}. \tag{431}$$

Divide equation (430) by equation (431),

$$\frac{(T)^2}{T^2} = \frac{M + \Delta M}{M} = 1 + \frac{\Delta M}{M}. \tag{432}$$

As explained in paragraph 110, ΔM may be replaced by μH, and equation (432) becomes

$$\frac{(T)^2}{T^2} = 1 + \mu \frac{H}{M};$$

then

$$\log (T)^2 = \log T^2 + \log \left(1 + \mu \frac{H}{M}\right). \tag{433}$$

From equation (362),

$$\log (T)^2 = \log T^2 + 0.434 \mu \frac{H}{M}. \tag{434}$$

In practice it is convenient to use a table of $\log\left(1+\mu\frac{H}{M}\right)$ against $\log\frac{H}{M}$ for a given value of μ, to get $\log\left(1+\mu\frac{H}{M}\right)$. In the lower part of figure 35, page 56, such a table is illustrated, showing critical values of $\log\frac{H}{M}$ for values of $\log\left(1+\mu\frac{H}{M}\right)$, for $\mu=3.5$.

454. ***Variation of log C with temperature in magnetometer deflections.***—In the deflection equation

$$\frac{H}{M}=\frac{C}{\sin u} \tag{435}$$

the constant C is made up primarily of $\frac{2}{r^3}$, which varies with temperature because r varies with temperature.

That is,

$$C=\frac{2}{r_{20}^3} \tag{436}$$

$$\frac{dC}{dr}=-\frac{3C}{r_{20}} \tag{437}$$

and

$$dC=-\frac{3C}{r_{20}}\,dr. \tag{438}$$

But

$$dr=r_{20}\alpha dt \tag{439}$$

in which α is the coefficient of thermal expansion (linear) of the deflection bar, and dt is a small increment in the temperature. When everything is constant at 20° C,

$$C_{20}=\frac{2}{r_{20}^3}. \tag{440}$$

When the temperature changes by Δt and everything else remains constant except r,

$$C_t=\frac{2}{(r_{20}+\Delta r)^3}. \tag{441}$$

Divide equation (440) by equation (441),

$$\frac{C_{20}}{C_t}=\frac{(r_{20}+\Delta r)^3}{r_{20}^3}=\left(1+\frac{\Delta r}{r}\right)^3 \tag{442}$$

$$\log C_{20}=\log C_t+3\log\left(1+\frac{\Delta r}{r}\right). \tag{443}$$

From equation (362),

$$\log C_{20} \approx \log C_t + 1.30\,\frac{\Delta r}{r}. \tag{444}$$

But, from equation (439), $\dfrac{\Delta r}{r} = \alpha \Delta t$;

hence

$$\log C_{20} \approx \log C_t + 1.30\,\alpha \Delta t \tag{445}$$

$$\approx \log C_t + 1.30\,\alpha (t-20) \tag{446}$$

and

$$\begin{aligned} \log C_t - \log C_{20} &= \Delta \log C \\ &\approx -1.30\,\alpha (t-20) \\ &\approx 1.30\,\alpha (20-t) \end{aligned} \tag{447}$$

$$\frac{\Delta \log C}{\Delta t} = -1.30\,\alpha. \tag{448}$$

Examples:

For brass, $\alpha = 0.000\ 018$;

$$\log C_{20} = \log C_t + 0.000\ 023 (t-20). \tag{449}$$

For duralumin, $\alpha = 0.000\ 022$;

$$\log C_{20} = \log C_t + 0.000\ 029 (t-20). \tag{450}$$

APPENDIX II. TEMPERATURE COEFFICIENT OF MAGNETIC MOMENT

455. In Chapter 4, the temperature coefficient of magnetic moment was taken as a constant over a temperature range of 50° C. However, the relation between temperature and magnetic moment is not strictly linear. A more precise expression is given by

$$M_t = M_0(1 - qt - q't^2) \tag{451}$$

in which M_t = the magnetic moment at a temperature t° C,

M_0 = the magnetic moment at t_0° C,

and q and q' are coefficients which must be evaluated from experimental data for each magnet.

Differentiating equation (451) with respect to t,

$$\frac{dM_t}{dt} = -M_0(q + 2q't) \tag{452}$$

$$\Delta M = -M_0(q + 2q't)\Delta t. \tag{453}$$

Equation (452) gives the rate of change of the magnetic moment at the temperature t° C. In general we are concerned with the change in magnetic moment for a particular change in temperature, hence the average rate, $\overline{\frac{\Delta M}{\Delta t}}$, between the reference temperatures must be used in calculating ΔM from equation (453).

456. It may be shown that the average rate of change over the range, t_0 to t_1, is

$$\overline{\frac{\Delta M}{\Delta t}} = -M_0\left(q + 2q'\,\frac{t_1 + t_0}{2}\right) \tag{454}$$

and

$$\Delta M = -M_0\left(q + 2q'\,\frac{t_1 + t_0}{2}\right)\Delta t. \tag{455}$$

457. Example: Let

$$M_0 = 1000 \text{ cgs at } t_0;$$

$$t_0 = 0° \text{ C};$$

$$t_1 = 50° \text{ C};$$

$$q = 0.000329;$$

and

$$q' = 0.000\,000\,91.$$

Then from equation (454),

$$\frac{\overline{\Delta M}}{\Delta t}=-1000\left[0.000\ 329+2(0.000\ 000\ 91)\left(\frac{50°+0°}{2}\right)\right]$$

$$=-0.3745 \text{ cgs units per degree C, for } M_0=1000.$$

When $M_0=1$, $$\frac{\overline{\Delta M}}{\Delta t}=-0.000\ 3745=\overline{q}.$$

Then $\overline{q}$ is the mean temperature coefficient between 0° and 50° C. From equation (455)

$$\Delta M=-1000\times 0.000\ 3745\times 50=-18.7 \text{ cgs.}$$

Equation (451) will give the same result.

458. ***Summary.***—This analysis may be summarized as follows:

(a) Equation (451) gives the magnetic moment at a temperature $t°$ C when M_0 and the q coefficients are known;

(*b*) Equation (452) gives the temperature-rate of change of magnetic moment *at* the temperature t;

(c) Equation (454) gives, for the mean temperature between t_0 and t_1, the temperature rate of change of the moment of the magnet whose moment is M_0 at t_0 or 0° C.

(d) Equation (455) gives the approximate change of the magnetic moment, ΔM, when the temperature of the magnet changes by Δt;

(e) The process is reversible; that is, to go from a higher temperature to a lower temperature, change the sign of Δt.

459. ***Errors in the computed value of E_x.***—The formulas for computing E_x for D, H, and Z variometer recording magnets assume that the deflector is placed properly and that the deflection, $2u$, if it exists at all, is due entirely to the misorientation of the recording magnet. However, if the deflector is improperly placed, there will be a deflection of the recording spot despite correct orientation of the variometer magnet. If the usual orientation formula is applied, using the $2u$ recorded on the magnetogram, the apparent (computed) value of E_x will be made up of two parts: first, that due to the real misorientation of the recording magnet; and second, that due to improper placing of the deflector. There is no way to separate these two parts. It can only be assumed that the second part can be kept small by using great care in properly installing the stops or guides that hold the deflector in place while the orientation deflections are being made. The effects of improper placing of the deflector are summarized in table 18, page 203.

460. ***Discussion of errors.***—In this discussion it is assumed that:

(a) The deflector, or its holder, is a rectangular parallelepiped;

(b) the deflector is symmetrically magnetized with its magnetic axis approximately coincident with its geometric axis or with the geometric axis of the holder;

(c) the test deflections are made by placing the deflector (or holder) against the fixed stops or guides which definitely fix the direction of the geometric axis of the deflector (or holder) and the geometric center of the deflector or holder with respect to the stops or guides;

(d) a sufficient number of deflections are made with the deflector in the four positions shown on figure 84, lines 6–9 (see p. 145) to eliminate the residuary effect of noncoincidence of magnetic and geometric axes;

(e) errors in fixing the guides or stops are small, say of the order of one degree or less in azimuth and level; and in any direction normal to the magnetic meridian the departure of the geometric center of the deflector does not exceed ½ of one percent of the value of the deflection distance, r (5 parts in 1000); and

(f) the recording magnet is in perfect orientation.

Note: It can be shown that the error equations derived below also hold approximately for the condition that the recording magnet under test is improperly oriented by a small angle of the same order of magnitude as the error angles introduced in setting the deflector.

461. A perfectly adjusted deflector will not deflect a perfectly oriented recording magnet. Imperfect adjustment of the deflector, M_a, will cause a deflection of the recording magnet, M_s, when M_a produces an effective field perpendicular to M_s, in the plane of rotation of M_s.

Let

f_p=component of the test field parallel to the recording magnet;

f_n = component of the test field normal to the recording magnet (in the horizontal plane for D and H; in the vertical plane for Z);

u_1 = the real "up" deflection of the spot (taken as +);

u_2 = the real "down" deflection of the spot (taken as +);

$2u = u_1 + u_2$;

S = the normal scale value (gamma scale value);

$(E_x)_c$ = the calculated exorientation angle of M_s;

and E_x = the real exorientation angle of $M_s = 0$.

Then from equation (334), page 139,

$$\tan\ (E_x)_c = \frac{2uS}{2\ f_p}. \tag{456}$$

But by definition of S $\qquad f_n = u_1\ S;$

and upon reversal of M_a $\qquad f_n = u_2\ S.$

Hence $\qquad 2uS = 2\ f_n$

and

$$\tan\ (E_x)_c = \frac{f_n}{f_p}. \tag{457}$$

Therefore, part of the analysis reduces to a comparison of f_n with f_p.

462. By introducing small known errors of placement of the deflector, the effect of such errors on the calculated exorientation angles may be computed. In the following paragraphs the errors $(E_x)_c$ in E_x, for critical and noncritical errors of placement, are derived for 5 possible cases for each variometer. The results are summarized in table 18, page 203.

463. ***D variometer.***—In all the cases, M_s is assumed in perfect adjustment.

Case (a), figure 103.—M_a is in perfect orientation except that it makes a small horizontal angle, ρ, with the magnetic meridian through M_s.

$$f_p = f_r = \frac{2M_a}{r^3} \cos\ \rho \tag{458}$$

$$f_n = f_\theta = \frac{M_a}{r^3} \sin\ \rho; \tag{459}$$

$$\tan\ (E_x)_c = \frac{f_n}{f_p} = \frac{\frac{M_a}{r^3} \sin\ \rho}{\frac{2M_a}{r^3} \cos\ \rho} = \tfrac{1}{2} \tan\ \rho \tag{460}$$

$$(E_x)_c \approx \tfrac{1}{2} \rho \text{ (for small values of } \rho). \tag{461}$$

Case (b), figure 104.—M_a is in perfect orientation except that the geometric center of M_a is displaced a small distance, y, east or west of the magnetic meridian through M_s. Angle $AOB=\delta$.

$$f_p=f_{\parallel}=\frac{M_a}{r^3}\,(3\cos^2\delta-1)=\frac{2M_a}{r^3},\ \text{since } \delta \text{ is small;} \tag{462}$$

$$f_n=f_{\perp}=\frac{3M_a}{r^3}\sin\delta\cos\delta=\frac{3M_a}{r^3}\sin\delta,\ \text{since } \delta \text{ is small;} \tag{463}$$

$$\tan\,(E_x)_c=\frac{f_n}{f_p}=\frac{\dfrac{3M_a}{r^3}\sin\delta}{\dfrac{2M_a}{r^3}}=\frac{3}{2}\sin\delta \tag{464}$$

$$(E_x)_c\approx\frac{3}{2}\,\delta\approx\frac{3}{2}\,\frac{y}{r}. \tag{465}$$

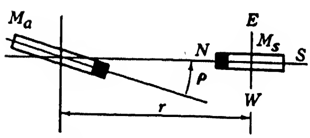

Figure 103.—Looking down.

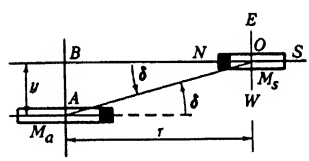

Figure 104.—Looking down.

Case (c), figure 105.—M_a is in perfect adjustment except that it is above or below the horizontal plane through M_s by a small distance, h. There is no component normal to M_s in a horizontal plane, hence no deflection of M_s, and $(E_x)_c=0$.

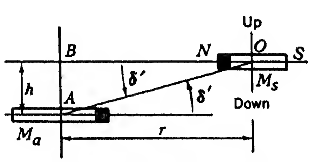

Figure 105.—Looking east.

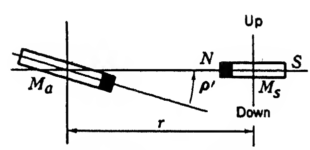

Figure 106.—Looking east.

Case (d), figure 106.—M_a is in perfect adjustment except that it is tilted in the magnetic meridian plane through a small vertical angle, ρ'. There is no component normal to M_s in a horizontal plane, hence no deflection of M_s, and $(E_x)_c=0$.

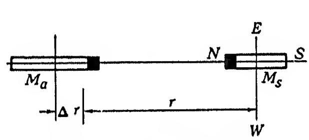

Figure 107.—Looking down.

Case (e), figure 107.—M_a is in perfect adjustment except that the deflection distance, r, is uncertain by a small distance, Δr. M_s will not be deflected and $(E_x)_c=0$.

464. ***H variometer.***—In all the cases, M_s is assumed in perfect adjustment.

Case (a), figure 108.—M_a is in perfect adjustment except that it makes a small horizontal angle, ρ, with the magnetic prime vertical through the center of M_a.

$$f_n = f_r = \frac{2M_a}{r^3} \cos \theta$$

$$= \frac{2M_a}{r^3} \sin \rho; \tag{466}$$

$$f_p = f_\theta = \frac{M_a}{r^3} \sin \theta$$

$$= \frac{M_a}{r^3} \cos \rho; \tag{467}$$

$$\frac{f_n}{f_p} = \frac{\dfrac{2M_a}{r^3} \sin \rho}{\dfrac{M_a}{r^3} \cos \rho} = \tan (E_x)_c \tag{468}$$

$$\tan (E_x)_c = 2 \tan \rho$$

$$(E_x)_c \approx 2\rho, \text{ since } \rho \text{ is small.} \tag{469}$$

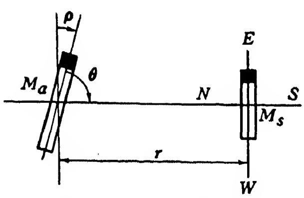

Figure 108.—Looking down.

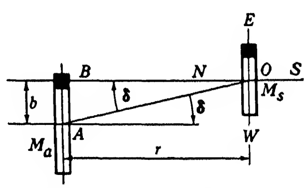

Figure 109.—Looking down.

Case (b), figure 109.—M_a is in perfect adjustment except that its center lies to the east or west of the magnetic meridian through M_s, by a small distance, b, such that the angle $AOB = \delta$.

$$f_n = f_\perp = \frac{3M_a}{r^3} \cos \delta \sin \delta$$

$$\approx \frac{3M_a}{r^3} \delta, \text{ since } \delta \text{ is small.} \tag{470}$$

$$f_p = f_\parallel = \frac{M_a}{r^3} (3 \cos^2 \theta - 1)$$

$$= \frac{M_a}{r^3} (3 \sin^2 \delta - 1)$$

$$\approx \frac{-M_a}{r^3}, \text{ since } \delta \text{ is small.} \tag{471}$$

$$\frac{f_n}{f_p} = \tan\,(E_x)_c \approx \frac{+\dfrac{3M_a}{r^3}\,\delta}{-\dfrac{M_a}{r^3}} \tag{472}$$

$$(E_x)_c \approx -3\delta \approx -3\,\frac{b}{r}. \tag{473}$$

Case (c), figure 110.—M_a is in perfect adjustment except that it is above or below the horizontal plane through M_s by a small distance, h. There will be no component normal to M_s in a horizontal plane, hence no deflection, and $(E_x)_c=0$.

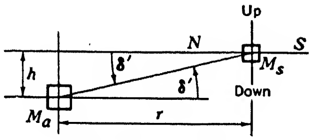

Figure 110.—Looking east.

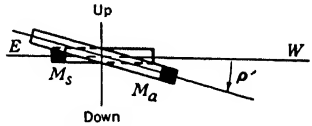

Figure 111.—Looking south.

Case (d), figure 111.—M_a is in perfect adjustment except that it is not level—that is, it makes a small vertical angle, ρ', with the horizontal plane through M_s (in the prime vertical). There will be no component normal to M_s in a horizontal plane, hence no deflection, and $(E_x)_c=0$.

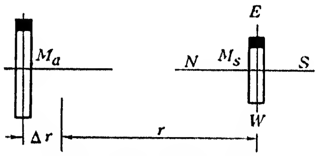

Figure 112.—Looking down.

Case (e), figure 112.—M_a is in perfect adjustment except that the deflection distance, r is uncertain by a small distance, Δr. There will be no deflection and $(E_x)_c=0$.

465. ***Z variometer.***—In all the cases, M_s is assumed in perfect adjustment.

Case (a), figure 113.—M_a is in perfect adjustment except that it dips through a small angle, ρ, in the magnetic meridian plane through M_s; that is, M_a is not level.

$$f_n = f_\theta = \frac{M_a}{r^3}\sin\rho \tag{474}$$

$$f_p = f_r = \frac{2M_a}{r^3}\cos\rho \tag{475}$$

$$\frac{f_n}{f_p} = \tan\,(E_x)_c = \frac{\dfrac{M_a}{r^3}\sin\rho}{\dfrac{2M_a}{r^3}\cos\rho} = \tfrac{1}{2}\tan\rho \tag{476}$$

$$(E_x)_c \approx \tfrac{1}{2}\rho, \text{ since } \rho \text{ is small.} \tag{477}$$

210111—53——14

Case (b), figure 114.—M_a is in perfect adjustment except that it is above or below the horizontal plane through M_s by a small distance, h, such that the angle $AOB=\delta$.

$$f_p=f_{\parallel}=\frac{M_a}{r^3}(3\cos^2\delta-1)$$

$$\approx\frac{2M_a}{r^3}\text{, since } \delta \text{ is small.} \tag{478}$$

$$f_n=f_{\perp}=\frac{3M_a}{r^3}\sin\delta\cos\delta$$

$$\approx\frac{3M_a}{r^3}\delta\text{, since } \delta \text{ is small.} \tag{479}$$

$$\frac{f_n}{f_p}=\tan(E_x)_c\approx\frac{\frac{3M_a}{r^3}\delta}{\frac{2M_a}{r^3}} \tag{480}$$

$$(E_x)_c\approx\frac{3}{2}\delta\approx\frac{3}{2}\frac{h}{r}. \tag{481}$$

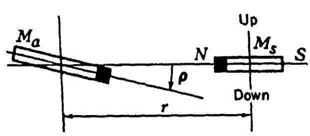

Figure 113.—Looking east.

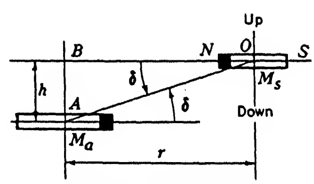

Figure 114.—Looking east.

Case (c), figure 115.—M_a is in perfect adjustment except that it is displaced in the horizontal plane through M_s by a small distance, y, such that the angle $AOB=\delta'$. There will be no component normal to M_s in the vertical plane, hence no deflection, and $(E_x)_c=0$.

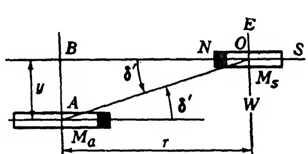

Figure 115.—Looking down.

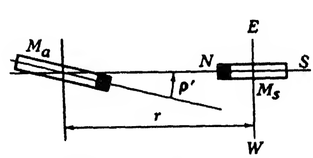

Figure 116.—Looking down.

Case (d), figure 116.—M_a is in perfect adjustment except that it makes a small horizontal angle, ρ', with the magnetic meridian through M_s. There will be no component normal to M_s in a vertical plane, hence no deflection, and $(E_x)_c=0$.

Case (e), figure 117.—M_a is in perfect adjustment except that the deflection distance, r, is uncertain by a small distance, Δr. There is no component of M_a perpendicular to M_s, hence no deflection, and $(E_x)_c=0$.

466. ***Practical applications.***—It is estimated that the deflector stops or guides can be set to an accuracy of 1 mm in 200 mm, so that $\rho=0.005$ radians$=17'$. Also, it is estimated that the guides can be placed so that the center of the deflector will be on the magnetic meridian or at the proper elevation within 2 mm. At $r=100$ cm, δ

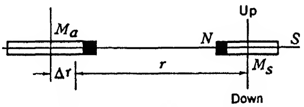

Figure 117.—Looking east.

should not exceed 0.002 radians or approximately 7′. Then referring to table 18 for these critical errors, assuming the recording magnet is perfectly oriented, the apparent exorientation angles would be:

$$\text{For } D\text{: } E_x \text{ (apparent)}=\frac{3}{2}\delta+\frac{1}{2}\rho=\frac{3}{2}\times 7'+\frac{1}{2}\times 17'=19'$$

$$\text{For } H\text{: } E_x \text{ (apparent)}=3\delta+2\rho=3\times 7'+2\times 17'=55'$$

For Z: Same as D.

It is obvious that the utmost care must be taken in fixing the positions of the stops or guides for the critical adjustments, otherwise the *calculated* exorientation errors may be quite large even though the *recording magnets* may be in excellent adjustment.

TABLE 18.—*Critical adjustments of the deflector in orientation tests.*

VARIOMETER	ERRORS IN PLACING THE DEFLECTOR	ERRORS IN COMPUTED E_x	
		$(E_x)_e$	Nominal value
D	Deflector rotated 1° about a vertical axis, par. 463 (a).	$\frac{1}{2}\rho$	30′
	Deflector moved 1 cm east or west, horizontally, par. 463 (b) ($r=100$ cm).	$\frac{3}{2}\delta=\frac{3y}{2r}$	52′
H	Deflector rotated 1° about a vertical axis, par. 464 (a).	2ρ	2°
	Deflector moved 1 cm east or west, horizontally, par. 464 (b) ($r=100$ cm).	$3\delta=\frac{3b}{r}$	1° 43′
Z	Deflector tilted 1° in the magnetic meridian plane, par. 465 (a).	$\frac{1}{2}\rho$	30′
	Deflector moved 1 cm up or down, par. 465 (b) ($r=100$ cm).	$\frac{3}{2}\delta=\frac{3h}{2r}$	52′

467. ***Errors in scale values arising from maladjustment of the deflector.***—In order to simplify the analyses and to concentrate on the physical picture, it will be assumed that the undeflected variometer magnets are in perfect adjustment, that is, that the D magnet is in the magnetic meridian, the H magnet is in the magnetic prime vertical, and the Z magnet is level; that there are no appreciable errors in the magnetic moment of the deflector and in the values of $2u$ (these errors are not relevant here); that operational procedures in scale-value observations are performed in the manner described in chapter 11; that the deflector is an ideal magnet; and that all other conditions are ideal, aside from maladjustment of the deflector. From the definition of scale value, the deflecting fields should be directed as follows (see fig. 118):

For H: Magnetic north or magnetic south.

For Z: Vertically upward or downward.

For D: Magnetic east or magnetic west.

468. Maladjustment of the deflector will, in general, change the magnitude of the component of the field in which we are interested. At the same time this maladjustment will introduce a small field parallel to the recording magnet which will affect the scale value; however, this latter effect will cancel out on reversal of the deflector.

469. Therefore, under the assumed conditions, the maladjustment of the deflector will affect the calculated field, f_c, insofar as this field differs from the real deflector field, f (f being the *effective* field—the field which actually produces the deflection), as described in the last part of paragraph 467. In computing scale values we assume ideal conditions and compute a deflector field, f_c, as in the examples which follow.

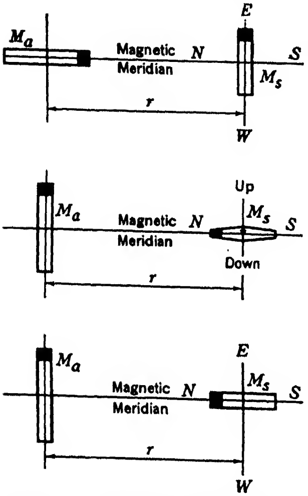

Figure 118.—Positions of deflector relative to recording magnet in observations for scale values by magnetic method. H deflections at top, Z in center, D at bottom.

470. In this discussion, the deflection, u, is the deflection of the recording spot on the gram caused by the deflector in one position only, and the double deflection, $2u$, is the sum of the deflections caused by the deflector direct and reversed. $2u$ is always considered positive (+).

471. ***Magnetic meridian and prime vertical.***—The magnetic meridian plane at a point is the vertical plane containing a magnet freely suspended at the point and acted on only by the earth's magnetic field. In this discussion the magnetic meridian will be taken as the horizontal line in the magnetic meridian plane and passing through the point.

472. The magnetic prime vertical plane at a point is the vertical plane perpendicular to the magnetic meridian plane, and containing the point. In this discussion the magnetic prime vertical will be taken as the horizontal line perpendicular to the magnetic meridian at the point.

473. The magnetic meridians in the variation building and absolute building are assumed to be parallel.

474. ***Calculated scale value.***—The calculated scale value is

$$S_c=\frac{2f_c}{2u}. \tag{482}$$

The real (true) scale value is

$$S=\frac{2f}{2u}. \tag{483}$$

The difference is

$$S_c-S=\frac{2f_c}{2u}-\frac{2f}{2u}$$

$$=\frac{2}{2u}(f_c-f). \tag{484}$$

The fractional error in the scale value will be

$$\frac{S_c-S}{S_c}=\frac{\frac{2}{2u}(f_c-f)}{\frac{2f_c}{2u}}=\frac{f_c-f}{f_c}; \tag{485}$$

and the percent error will be

$$\frac{S_c-S}{S_c}\times 100=\frac{f_c-f}{f_c}\times 100. \tag{486}$$

475. ***D scale value.***—(a) Consider first the deflector correctly oriented (fig. 119); deflector level; center on the magnetic meridian

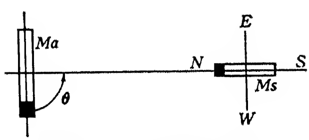

Figure 119.—Looking down.

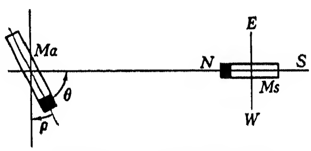

Figure 120.—Looking down.

through the recording magnet, M_s; axis normal to the magnetic meridian through M_s; same elevation as M_s. In figure 119, the effective field is $f_E=f_\theta=f_c$; $\theta=90°$; $\sin\theta=1$.

$$f_c=\frac{M_a}{r^3}\sin\theta=\frac{M_a}{r^3}. \tag{487}$$

This is the value of f_c in the following cases.

(b) Deflector correctly oriented except that it makes a small angle, ρ, with the prime vertical. In figure 120, θ is positive. $\theta=90°-\rho$. Effective field: $f_E=f_\theta$.

$$f_\theta=\frac{M_a}{r^3}\sin(90-\rho)$$

$$=\frac{M_a}{r^3}\cos\rho=f_c\cos\rho \tag{488}$$

$$f_c-f_\theta=f_c(1-\cos\rho). \tag{489}$$

The fractional error is

$$\frac{f_c-f_\theta}{f_c}=1-\cos\rho. \tag{490}$$

(c) M_a is correctly oriented except that its center is horizontally displaced from the magnetic meridian through M_s by a small amount, y (fig. 121). Effective field: $f_E=-f_\parallel$. Angle $AOB=\delta$; $\theta=90-\delta$.

$$\cos^2(90-\delta)=\sin^2\delta.$$

$$f_\parallel=+\frac{M_a}{r^3}(3\cos^2\theta-1)$$

$$=+\frac{M_a}{r^3}(3\sin^2\delta-1)$$

$$=+f_c(3\sin^2\delta-1) \tag{491}$$

$$f_c-f=f_c+f_\parallel=f_c(1+3\sin^2\delta-1)$$

$$=f_c(3\sin^2\delta). \tag{492}$$

The fractional error is

$$\frac{f_c+f_\parallel}{f_c}=3\sin^2\delta. \tag{493}$$

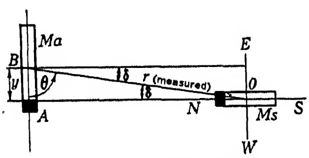

Figure 121.—Looking down.

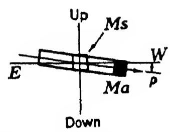

Figure 122.—Looking south.

(d) M_a is correctly oriented except that it makes a small angle, ρ', with the horizontal but remains in the magnetic prime vertical through its center (fig. 122). Effective field: Horizontal component of the f_θ field; call it f'_θ.

$$f_E=f'_\theta=f_\theta\cos\rho'=\frac{M_a}{r^3}\cos\rho'=f_c\cos\rho' \tag{494}$$

$$f_c-f'_\theta=f_c(1-\cos\rho'). \tag{495}$$

The fractional error is

$$\frac{f_c-f'_\theta}{f_c}=1-\cos\rho'. \tag{496}$$

(e) M_a is correctly oriented but its distance, r, from M_s is uncertain by a small amount, Δr (fig. 123). Effective field: $f_E=f_\theta$.

$$\Delta f=-\frac{3f_c}{r}\Delta r \text{ by eq (160).} \tag{497}$$

But $\qquad \Delta r=r-r_c$, hence $\Delta f=f_\theta-f_c$

and
$$f_\theta-f_c=-\frac{3f_c}{r}\Delta r \tag{498}$$

$$\frac{f_c-f_\theta}{f_c}=+\frac{3}{r}\Delta r. \tag{499}$$

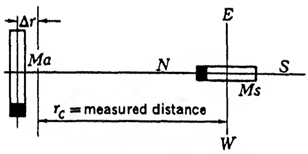

Figure 123.—Looking down.

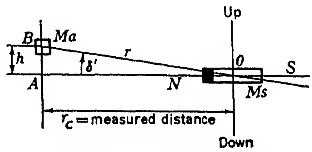

Figure 124.—Looking east.

(f) M_a is correctly oriented except that it is above or below the horizontal plane through M_s by a small amount, h (fig. 124). The effect is merely to increase the true deflection distance, r, by a small amount, Δr. In figure 124, OA is the measured distance, r_c; OB is the true distance, r; $AB=h$, the displacement above the horizontal plane. Effective field: $f_E=f_\theta$.

$$\Delta r=r-r_c=r-r\cos\delta'=r(1-\cos\delta').$$

As in case (e) above

$$\frac{f_c-f_\theta}{f_c}=\frac{3}{r}\Delta r=\frac{3}{r}[r(1-\cos\delta')]$$

$$=3(1-\cos\delta'). \tag{500}$$

If the measured distance, r_c, is OB rather than OA, then $f_c=f_\theta$ and $\Delta r=0$.

476. ***II scale value; maladjustment of the deflector.***—(a) M_a is in perfect orientation with deflector axis in the magnetic meridian through the center of M_s (fig. 125). Effective component producing deflection: $f_N=f_r=f_c$.

$$f_c=\frac{2M_a}{r^3}. \tag{501}$$

This is the value of f_c in the following cases.

(b) M_a is in perfect orientation except that it makes a small horizontal angle, ρ, with the magnetic meridian through M_s (fig. 126).

Effective component producing deflection is the radial component: $f_N = f_r$.

$$f_r = \frac{2M_a}{r^3} \cos \rho = f_c \cos \rho \tag{502}$$

$$f_c - f_r = f_c(1 - \cos \rho). \tag{503}$$

The fractional error in the scale value is

$$\frac{f_c - f_r}{f_c} = 1 - \cos \rho. \tag{504}$$

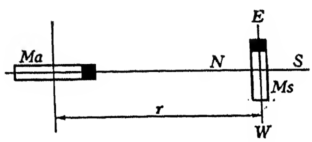

Figure 125.—Looking down.

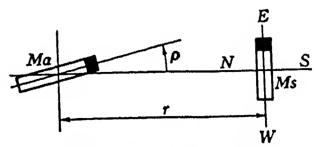

Figure 126.—Looking down.

(c) M_a is in perfect orientation except that it is east or west of the magnetic meridian through M_s by a small horizontal distance, y, such that the angle $AON = \delta$ (fig. 127). Effective component producing the deflection is the parallel component: $f_N = f_{\parallel}$.

$$f_{\parallel} = \frac{M_a}{r^3} (3 \cos^2 \delta - 1) = \frac{f_c}{2} (3 \cos^2 \delta - 1) \tag{505}$$

$$f_c - f_{\parallel} = \frac{f_c(2 - 3 \cos^2 \delta + 1)}{2} = \frac{f_c(3 - 3 \cos^2 \delta)}{2}$$

$$= \frac{3f_c(1 - \cos^2 \delta)}{2} = \frac{3f_c \sin^2 \delta}{2}. \tag{506}$$

The fractional error is

$$\frac{f_c - f_{\parallel}}{f_c} = \frac{3}{2} \sin^2 \delta. \tag{507}$$

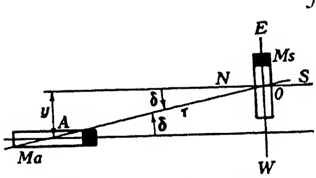

Figure 127.—Looking down.

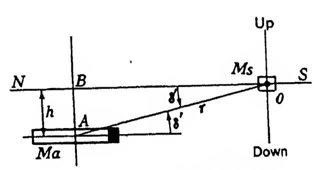

Figure 128.—Looking east.

(d) M_a is in perfect orientation except that the deflector is above or below the horizontal plane through M_s by a small distance, h, such that the angle $AOB = \delta'$ (fig. 128). Effective component producing the deflection: $f_N = f_{\parallel}$. The result is the same as in case (c). The fractional error in the scale value will be

$$\frac{f_c - f_{\parallel}}{f_c} = \frac{3}{2} \sin^2 \delta'. \tag{508}$$

(e). M_a is in perfect orientation except that it dips through a small angle ρ', while its magnetic axis remains in the magnetic meridian plane through M_s (fig. 129). Effective component producing the deflection is the radial component: $f_N = f_r$ as in case (b). The fractional error in the scale value is

$$\frac{f_c - f_r}{f_c} = 1 - \cos\rho'. \tag{509}$$

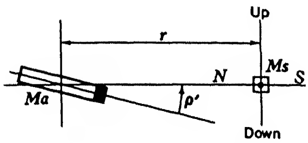

Figure 129.—Looking east.

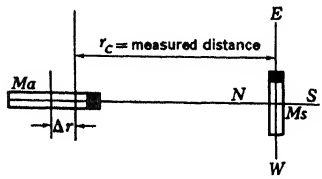

Figure 130.—Looking down.

(f). M_a is in perfect orientation except that the deflection distance, r, is uncertain by a small distance $+\Delta r$ (fig. 130). Effective component producing deflection: $f_N = f_r$. The fractional error in the scale value is

$$\frac{f_c - f_r}{f_c} = +\frac{3}{r}\,\Delta r. \tag{510}$$

477. ***Z scale value; maladjustment of the deflector.***—In Z scale-value deflections the deflector is used in the B position, just as in the D deflections. In deducing scale-value errors for the Z deflections, figures 119, 120, 121, and 123 may be taken as side elevations of the Z recording magnet and its deflector, looking east. Figure 122, if rotated 90° so that E means *up*, represents the condition that the deflector makes a small angle ρ' with the vertical in the magnetic prime vertical plane. Figure 124, if taken as a view looking down, represents the Z recording magnet and its deflector when the latter is displaced horizontally to the east by a small amount but its magnetic axis remaining vertical. In all cases the pertinent functions and estimated fractional errors are the same for both D and Z variometers.

478. ***Summary of errors.***—Tables 19 and 20 summarize the errors in the scale value deflections for all variometers due to small maladjustment errors of the deflector. It is assumed that with reasonable care the deflector holder can be adjusted to the accuracy indicated in each case. For example: It should be possible to adjust M_a (length 20 cm) parallel to a magnetic meridian or to make it level to an accuracy of 1° or better. For a 20-cm deflector this would be equivalent to 3.5 mm in 200 mm, a maladjustment easily detectable by eye. Also, it should be possible to make lateral or vertical adjustments to an accuracy of 2 or 3 mm by the methods described in paragraphs 343 to 349. As indicated in the tables, an error of 1 cm in h or y will not cause appreciable error in the deflections.

479. The controlling error is in the deflection distance, r, assuming that the other adjustments are made to the precision indicated in the tables. In the tables, r is taken as 100 cm, M_a as 10,000 cgs. Usually r is much greater for scale value deflections. The scale-value error

TABLE 19.—*Summary of possible errors in D and Z scale values due to maladjustment of the deflector.*

DEFLECTOR ORIENTATION		INCREMENT OF VARIABLE	ERROR FACTOR	ERRORS	
Position	Figure			f_c-f	% Error, $\frac{f_c-f}{f_c}100$
				γ	
a	119	Perfect orientation			
b	120	$\rho=1°$	$1-\cos\rho$	0.15	0.015
c	121	y=1 cm; δ=0.01 rad	$3\sin^2\delta$	0.3	0.03
d	122	$\rho=1°$	$1-\cos\rho'$	0.15	0.015
e	123	r=100 cm; Δr=0.1 cm	$+\frac{3}{r}\Delta r$	3.0	0.3
f	124	h=1 cm; δ'=0.01 rad	$+3(1-\cos\delta')$	0.15	0.045

TABLE 20.—*Summary of possible errors in H scale value due to maladjustment of the deflector*

DEFLECTOR ORIENTATION		INCREMENT OF VARIABLE	ERROR FACTOR	ERRORS	
Position	Figure			f_c-f	% Error, $\frac{f_c-f}{f_c}100$
				γ	
a	125	Perfect orientation			
b	126	$\rho=1°$	$1-\cos\rho$	0.3	0.015
c	127	y=1 cm; δ=0.01 rad	$\frac{3}{2}\sin^2\delta$	0.3	0.015
d	128	h=1 cm; δ'=0.01 rad	$\frac{3}{2}\sin^2\delta'$	0.3	0.015
e	129	$\rho'=1°$	$1-\cos\rho'$	0.3	0.015
f	130	r=100 cm; Δr=0.1 cm	$+\frac{3}{r}\Delta r$	6.0	0.3

due to Δr may be positive or negative; all the others are always positive. Assuming the most unfavorable condition where all have the same sign, the maximum possible error would be the sum of all the individual errors, and in the examples given should not exceed 0.4 percent. Assuming that the angular and linear displacement errors will be the same for values of r up to 300 cm, the percent error in the deflections due to error in r would decrease as r increases.

480. ***Secular change in D and its effect on maladjustment of the deflector.***—After a number of years it may be necessary to readjust the stops laterally and in azimuth because of possible large secular change of declination. For example: Suppose r=200 cm and the declination changes 30′ over a period of years. Then the east-west displacement of the magnetic meridian through the center of M_s, at a distance of 2000 mm, would be $y=2000 \tan 30'=2000\times0.0087=$ 17.4 mm. Unless the stops were readjusted to the new magnetic meridian, a small systematic error would be introduced into the calculated scale values as a result of this large secular change. It is obvious that a similar error would be introduced if deflections were made at a time when the declination differs by 30′ from the mean for the day, assuming that the deflector stops had just been adjusted to the mean value of D for that particular day. This question of readjustment of the stops is discussed further in paragraph 346.

APPENDIX V. CHANGE OF ORIENTATION AND SCALE VALUE OF *D* VARIOMETER CAUSED BY OTHER MAGNETS OF THE MAGNETOGRAPH

481. Assuming that the *D* variometer is constructed of nonmagnetic materials, and that there is no gross torsion in the *D* fiber, an isolated *D* variometer will always be properly oriented. The *H* and *Z* variometers, however, contain magnets whose fields can modify not only the orientation but also the scale value of the *D* variometer magnet. It is possible to compute these effects, if the magnetic moments of the several magnets are known, with an accuracy probably greater in all cases than that obtainable by direct measurement.

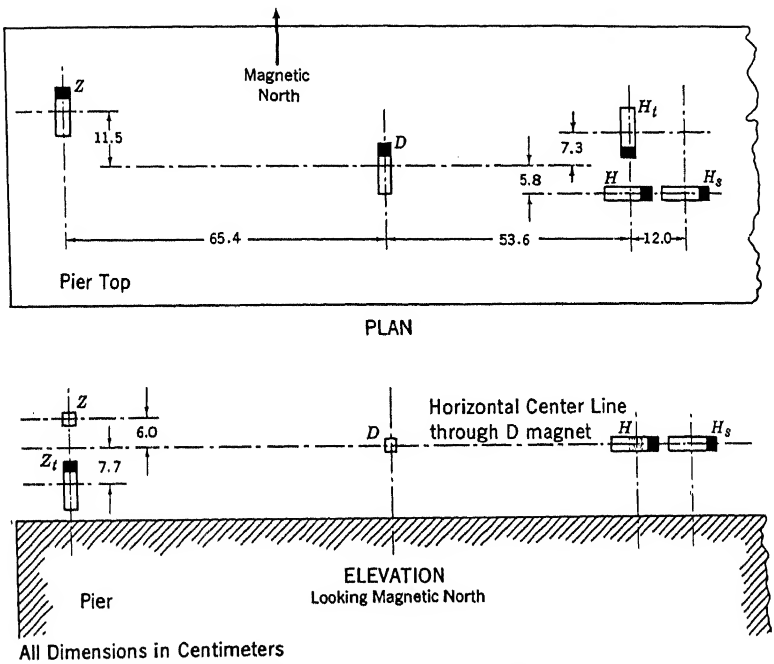

Figure 131.—Plan and elevation showing relative locations of all the magnets in the regular magnetograph at College, Alaska.

482. The example shown below applies to the sensitive magnetograph at College, Alaska, installed in February 1949. Figure 131 shows the relative positions in plan and elevation of all the magnets of the magnetograph. Dimensions are in centimeters. The following symbols are used, and the magnetic moments are as specified:

H is *H* variometer magnet; moment=8 cgs units;

H_t is *H* temperature-compensating magnet; moment=85 cgs units;

H_s is H sensitivity magnet; moment=83 cgs units;
Z is Z variometer magnet; moment=215 cgs units;
Z_t is Z temperature-compensating magnet; moment=450 cgs units;
$f_{\|}$=that component of the field of a magnet that is parallel to the axis of the magnet;
$f_{\perp}$=that component of the field of a magnet that is perpendicular to the axis of the magnet;
f_E=eastward horizontal component of the field of a magnet;
f_N=northward horizontal component of the field of a magnet.

483. Values of f_E and f_N at the D recording magnet will be computed separately for each of the five magnets; the resultant field will be the algebraic sum of the individual components. Magnitudes of f_E and f_N will be computed by means of equations (27) and (28); signs may be determined by examining the relative physical positions of the D magnet and of the magnet whose field is being computed, as illustrated in figures 132, 133, 134, 135, and 136. The figures are not drawn to scale. Slide-rule computations are used. Distribution coefficients (see par. 30) are neglected because the other magnets are far enough away from the D magnet that the effects of distribution will be small enough to ignore.

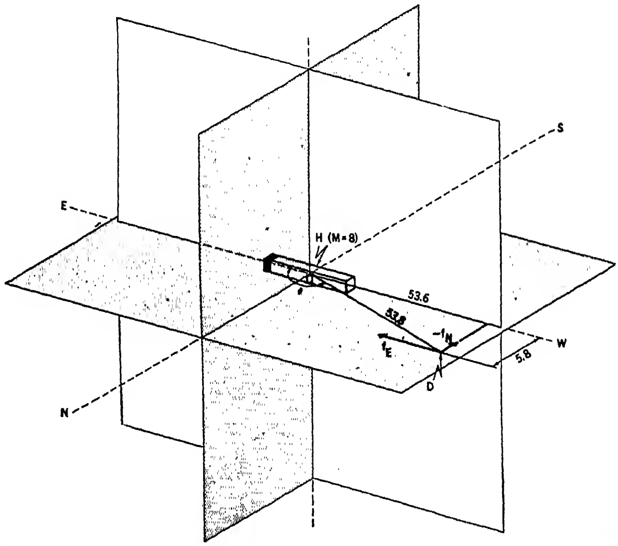

Figure 132.—Position of the *H*-variometer recording magnet relative to the *D* magnet.

484. *H* ***magnet.***—From figures 131 and 132, it is evident that the H magnet is 5.8 cm south and 53.6 cm east of the D magnet. Then, from equation (27),

$$f_E=f_{\|}=\frac{M}{r^3}(3\cos^2\theta-1) \tag{511}$$

$$=\frac{8\times10^5}{(53.8)^3}\left[3\left(\frac{-53.6}{53.8}\right)^2-1\right]$$

$$=5.14(2.978-1)$$
$$=10\gamma \text{ (eastward)};$$

and from equation (28),

$$f_N=f_\perp=\frac{M}{r^3}\,(3\sin\theta\cos\theta) \qquad (512)$$
$$=5.14\times3\times\frac{5.8}{53.8}\times\frac{-53.6}{53.8}$$
$$=-2\gamma \text{ (}2\gamma\text{ southward)}.$$

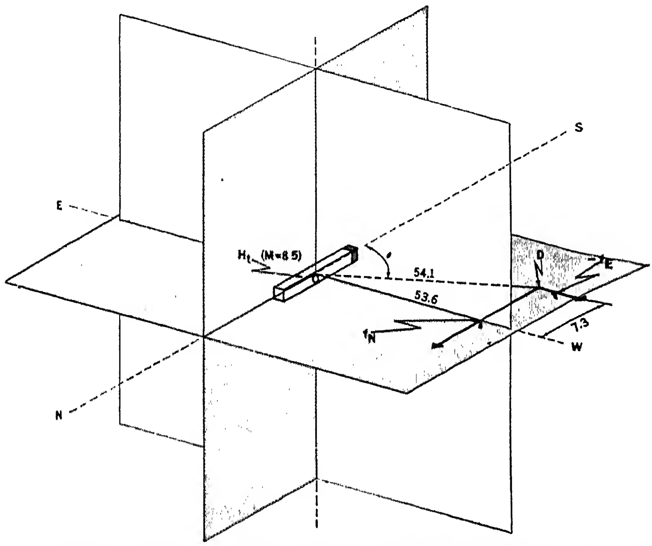

Figure 133.—Position of the H temperature-compensating magnet relative to the D magnet.

485. *H* ***temperature-compensating magnet.***—From figures 131 and 133, the H temperature-compensating magnet, H_t, is 7.3 cm north and 53.6 cm east of D. Then from equation (28),

$$f_E=f_\perp=\frac{M}{r^3}\,(3\sin\theta\cos\theta) \qquad (513)$$
$$=\frac{85\times10^5}{(54.1)^3}\times3\times\frac{53.6}{54.1}\times\frac{7.3}{54.1}$$
$$=53.7\times0.401$$
$$=22\gamma \text{ (westward)};$$

and from equation (27),

$$f_N=f_\parallel=\frac{M}{r^3}\,(3\cos^2\theta-1) \qquad (514)$$
$$=53.7\left[3\left(\frac{7.3}{54.1}\right)^2-1\right]$$
$$=53.7\times(-0.945)$$
$$=51\gamma \text{ (northward)}.$$

486. ***H sensitivity magnet.***—From figures 131 and 134, the H sensitivity magnet, H_s, is 5.8 cm south and 65.6 cm east of D. Then from equation (27),

$$f_E = f_{\parallel} = \frac{M}{r^3}\ (3\cos^2\theta - 1) \tag{515}$$

$$= \frac{83\times10^5}{(65.8)^3}\left[3\left(\frac{-65.6}{65.8}\right)^2 - 1\right]$$

$$= 29.1\times1.982$$

$$= 58\gamma \text{ (eastward);}$$

and from equation (28),

$$f_N = f_{\perp} = \frac{M}{r^3}\ (3\sin\theta\cos\theta) \tag{516}$$

$$= 29.1\times3\times\frac{5.8}{65.8}\times\frac{-65.6}{65.8}$$

$$= -8\gamma \text{ (}8\gamma\text{ southward).}$$

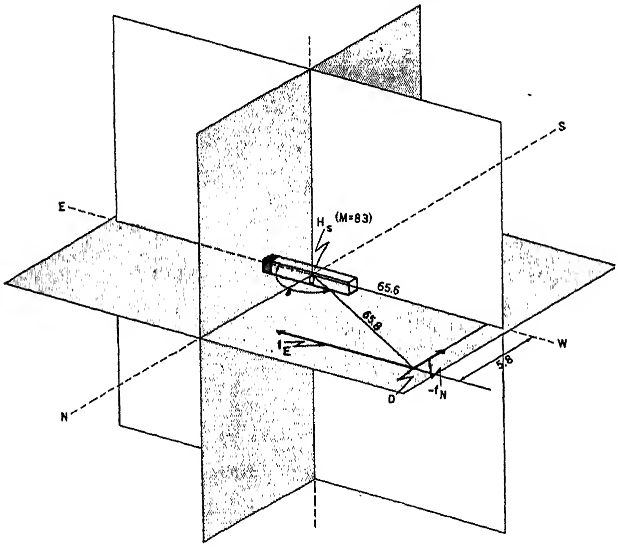

Figure 134.—Position of the *H* sensitivity magnet relative to the *D* magnet.

487. ***Z magnet.***—From figures 131 and 135, the Z variometer magnet, Z, is seen to be 11.5 cm north, 65.4 cm west, and 6.0 cm higher than D. The east component, which will affect D, is the horizontal projection of $f_{\perp}$.

$$f_E = f_{\perp}\cos\alpha$$

$$= \frac{M}{r^3}\ (3\sin\theta\cos\theta)\cos\alpha \tag{517}$$

$$=\frac{215\times10^5}{(66.7)^3}\times3\times\frac{65.7}{66.7}\times\frac{-11.5}{66.7}\times\frac{65.4}{65.7}$$

$$=-37\gamma \text{ (}37\gamma\text{ westward);}$$

and from equation (27),

$$f_N=f_{\parallel}=\frac{M}{r^3}\,(3\cos^2\theta-1) \tag{518}$$

$$=\frac{215\times10^5}{(66.7)^3}\left[3\left(\frac{-11.5}{66.7}\right)^2-1\right]$$

$$=-66\gamma \text{ (}66\gamma\text{ southward).}$$

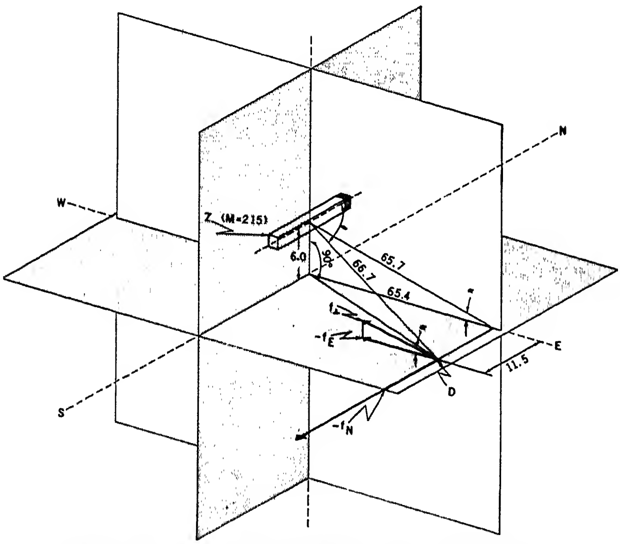

Figure 135.—Position of the *Z*-variometer recording magnet relative to the *D* magnet.

488. ***Z temperature-compensating magnet.***—From figures 131 and 136, the Z temperature-compensating magnet, Z_t, is 11.5 cm north, 65.4 cm west, and 7.7 cm lower than the D magnet. Again, as in the case of the Z magnet, the elevations of Z_t and D are different. f_E is the eastward component of $f_\perp$, and is therefore equal to $f_\perp \cos\beta$ (fig. 136); similarly f_N is the northward component of $f_\perp$, and is equal to $f_\perp$ sin β.

$$f_E=\frac{3M}{r^3}\sin\theta\cos\theta\cos\beta \tag{519}$$

$$=\frac{3\times450\times10^5}{(66.8)^3}\times\frac{66.4}{66.8}\times\frac{7.7}{66.8}\times\frac{65.4}{66.4}$$

$$=52\times0.985$$

$$=51\gamma \text{ (eastward);}$$

and $$f_N=\frac{3M}{r^3}\sin\theta\cos\theta\sin\beta \tag{520}$$

$$=52\times\frac{11.5}{66.4}$$

$$=9\gamma \text{ (southward).}$$

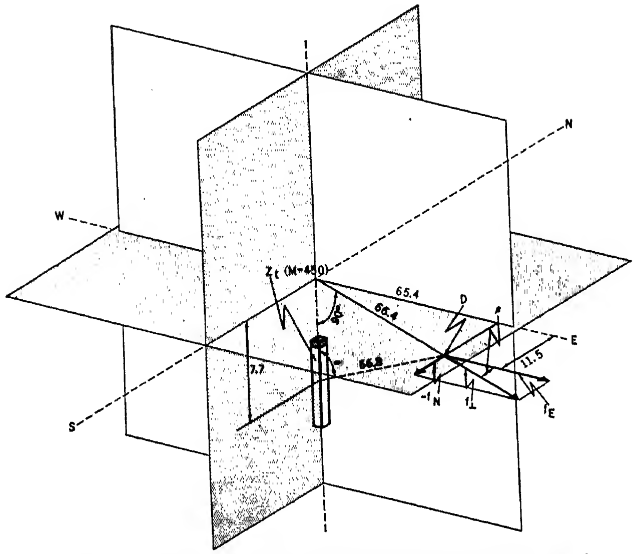

Figure 136.—Position of the Z temperature-compensating magnet relative to the D magnet.

489. The following table, then, summarizes the resultant field at the D variometer of all the other magnets in the magnetograph, westward and southward components being given a negative sign:

Field component	H	H_t	H_s	Z	Z_t	Total
f_E	+10	−22	+58	−37	+51	+60
f_N	−2	+51	−8	−66	−9	−34

490. When f_E is positive, therefore directed eastward, it causes the N end of the D magnet to lie in the northeast quadrant; the ex-orientation angle (par. 338) is arc tan $\frac{60}{12600}=16$ minutes of arc. (Horizontal component of the earth's field at College is 12600γ.)

491. When f_N is negative, therefore directed southward, it causes a change of scale value of the D variometer according to equation (122), where the C of the equation is equal to f_N derived above.

APPENDIX VI. NOMOGRAMS

492. The nomograms given in this appendix will be found useful in the preliminary studies associated with the establishment of a magnetograph. Beneath each illustration is a brief note explaining how it may be used and referring to the pertinent text paragraphs.

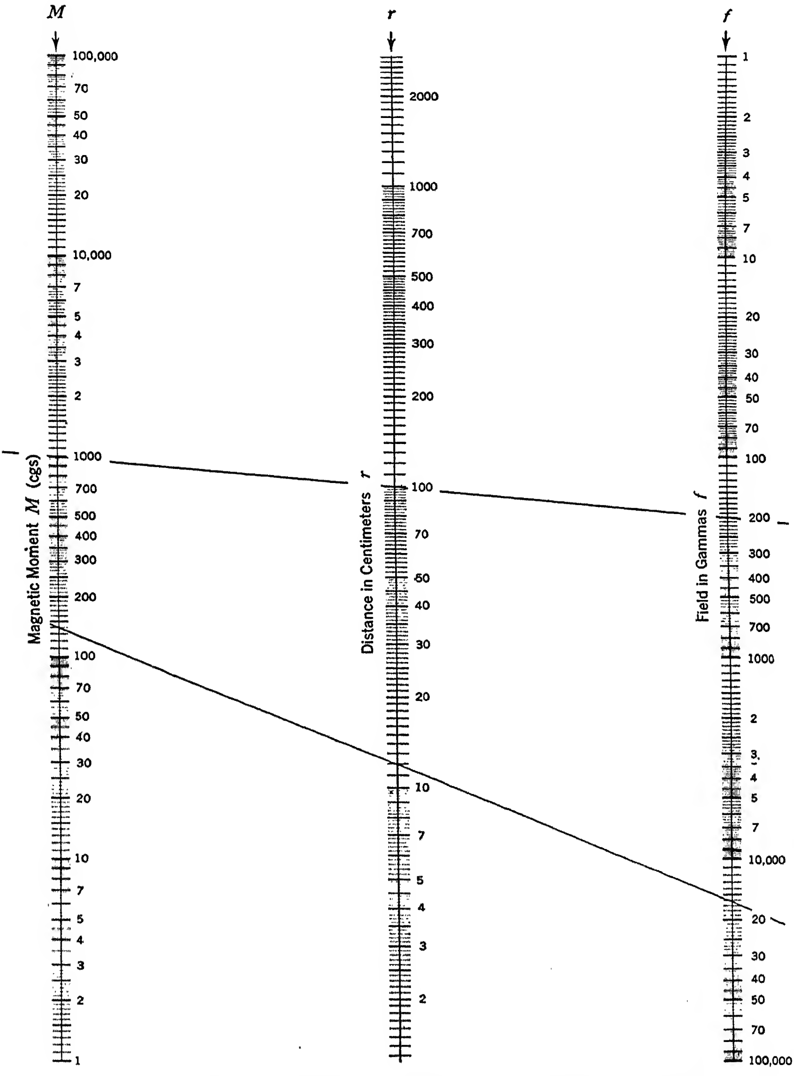

Figure 137. Field of a bar magnet at a point on its magnetic axis produced.

The above nomogram represents graphic solutions of the equation $f=\frac{2M\times10^5}{r^3}$, in which M is the magnetic moment and f the field in gammas at a distance r along the magnetic axis produced. Example: Given $M=1000$ and $r=100$ cm. Then a straight line from $M=1000$ on the M scale through $r=100$ on the r scale intersects $f=200$ on the f scale. The field is 200γ, the required field. Conversely, if it is desired to know the magnetic moment required to produce a field of 200γ at a distance of 100 cm along the magnetic axis, simply reverse the process (see par. 16).

210111—53 — 15

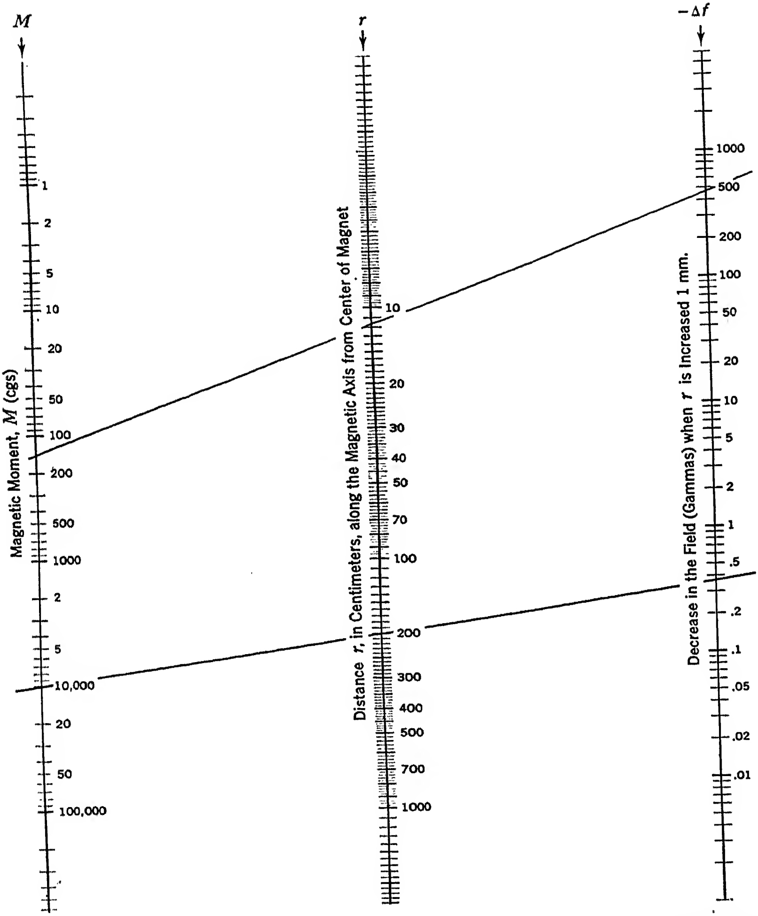

Figure 138.—Change in the field of a bar magnet, along its axis produced, with change of distance; magnetic moment known.

The above nomogram represents graphic solutions of the equation $-\Delta f=\frac{2\,M\times10^5}{r^3}\,\frac{3}{r}\,\Delta r=\frac{6M\times10^5}{r^4}\,\Delta r$.

Examples: Given $M=140$ cgs, $r=11.62$ cm, and $\Delta r=1$ mm. A straight line from $M=140$ on the M scale through $r=11.62$ on the r scale intersects the $-\Delta f$ scale at 460γ. That is, when r is increased from 11.62 cm to 11.72 cm the field will be smaller at $r=10.1$ by 460γ. Likewise if $M=10{,}000$ cgs, $r=200$ cm, and $\Delta r=1$ mm, then $\Delta f=-0.38\gamma$, approximately (see par. 16). In this nomogram Δr is fixed at 0.1 cm.

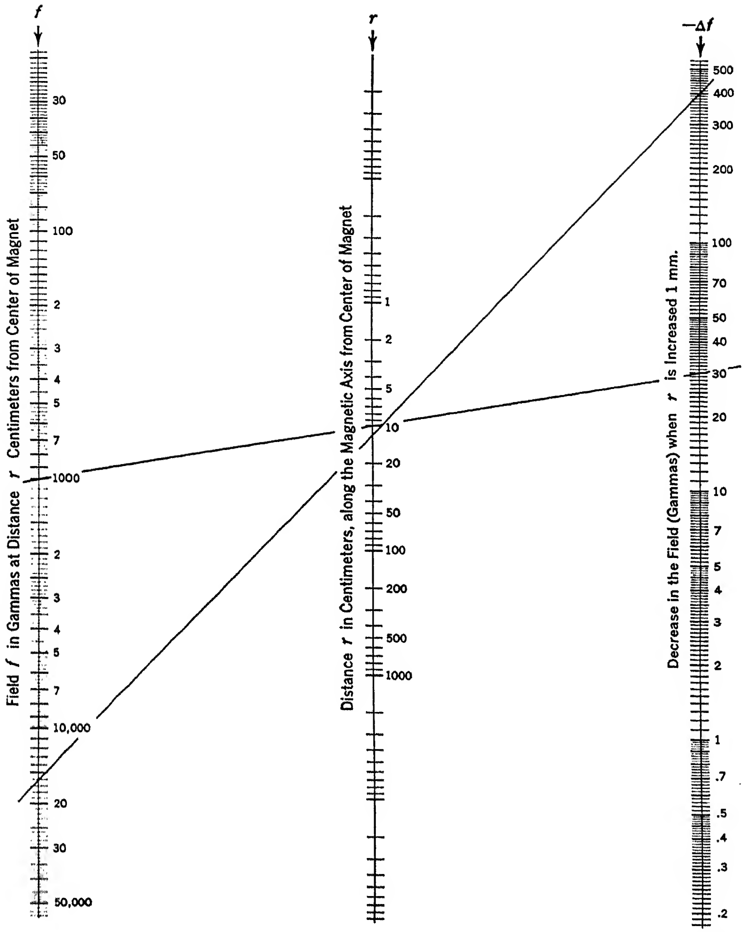

Figure 139. Change in the field of a bar magnet with change in distance along its magnetic axis produced; field at a distance r, known.

The above nomogram represents graphic solutions of the equation

$$\Delta f = -\frac{3f}{r}\Delta r$$

in which f is the field at a distance r from the center of the magnet, Δr is the increment in r, and Δf is the corresponding increment in f. Example: Given: $f=1000\gamma$ at a distance $r=10$ cm. Required Δf when $\Delta r=1$ mm. A straight line from $f=1000$ on the f scale through $r=10$ on the r scale intersects the $-\Delta f$ scale at 30. This means that if a field is 1000γ at a distance of 10 cm from the magnet and along its magnetic axis, the field will decrease approximately 30 gammas between the points $r=10.0$ cm and $r=10.1$ cm. (See par. 16). In this nomogram Δr is fixed at 0.1 cm.

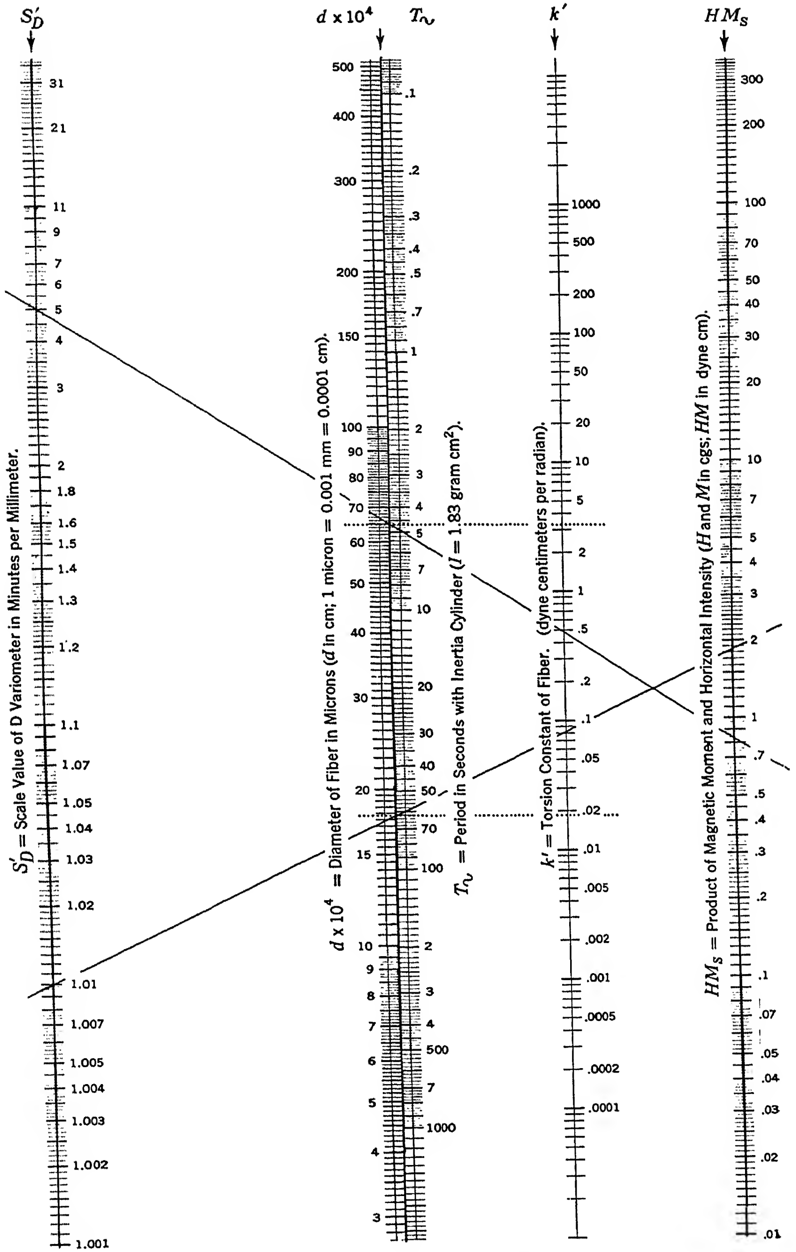

Figure 140.—The *D* Nomogram.

An alignment chart showing the relation between the product of the horizontal intensity and the magnetic moment of the recording magnet, the dimensions of the quartz fiber suspension, and the minute scale value; based on equations 124–126, with optical lever 2*R* taken as 3438 mm. See paragraph 215 for complete explanation and examples.

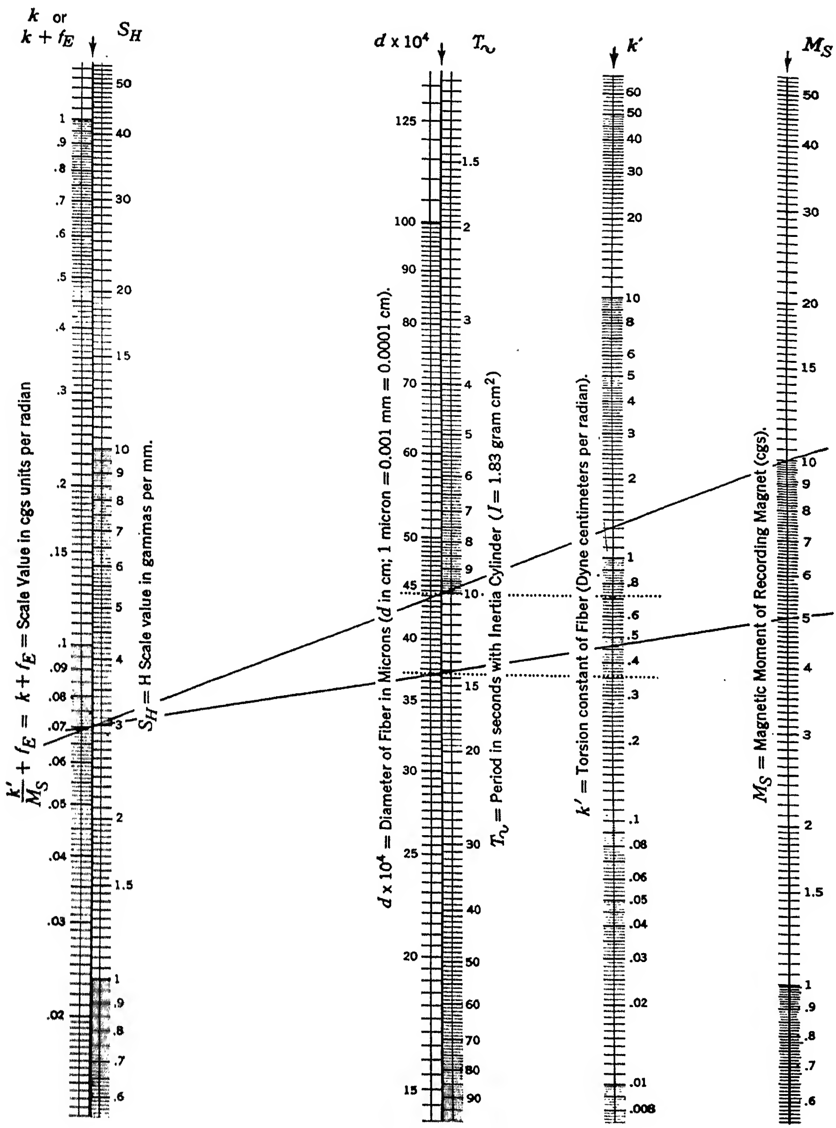

Figure 141. The H Nomogram.

An alignment chart showing the relation between the magnetic moment of the recording magnet, the dimensions of the quartz fiber suspension, and the H scale value. See paragraphs 251, 252, and 253, for additional details and examples.

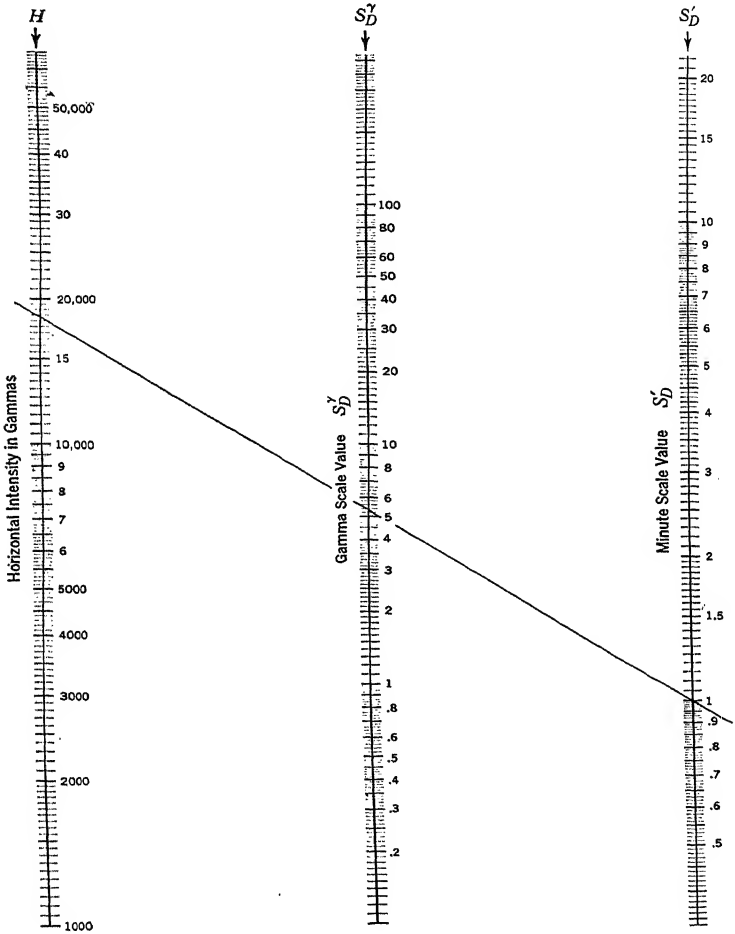

Figure 142.—Nomogram for conversion of *D* scale values.

This chart represents graphic solutions of equations (301) and (302). For example: Given a *D* scale value approximately equal to 1.00 minute per mm at a place where $H=18300\gamma$. Required the gamma scale value, $S_D\gamma$, of the *D* variometer. A straight line from $S_D'=1.00$ on the S_D' scale to $H=18300\gamma$ on the *H* scale intersects the $S_D\gamma$ scale at 5.3 which is the required gamma scale value, approximately. Likewise the minute scale value may be evaluated if the gamma scale value is given (see par. 323).

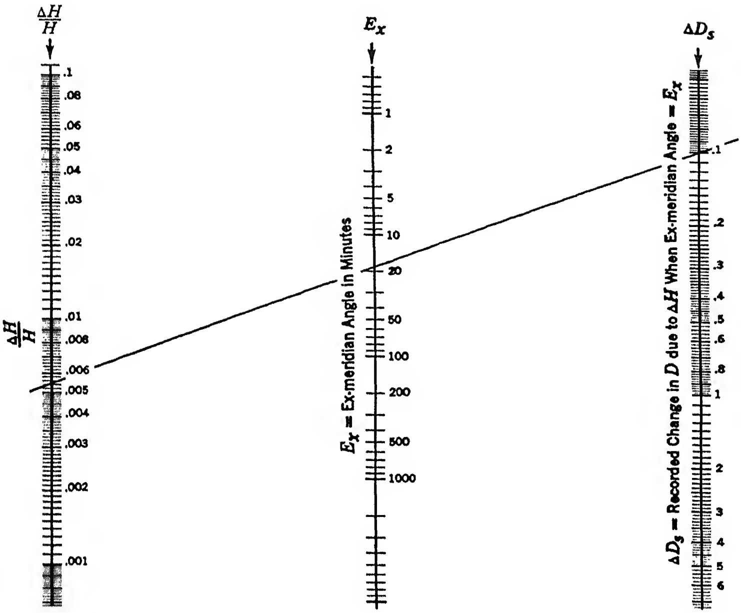

Figure 143. Nomogram showing spurious effects on recorded magnetic declination due to changes in the horizontal intensity (see par. 331).

This chart represents graphic solutions of the equation, $\Delta D_s = \frac{\Delta H}{H} \tan E_x$, in which H is the horizontal intensity in gammas; ΔH is the change in H in gammas, and E_x is the exmeridian angle of the D recording magnet. Example: Given, $H=18300\gamma$, $\Delta H=100\gamma$; and $E_x=18.6$ minutes. Then from the above equation, ΔD_s (in radians) $=\frac{100}{18300}\times 0.00532=0.00029$ radians $=0.1'$. The straight line from 0.00546 on the $\frac{\Delta H}{H}$ scale through $18.6'$ on the E_x scale intersects the ΔD scale at $0.1'$, the spurious change in recorded declination due to ΔH.

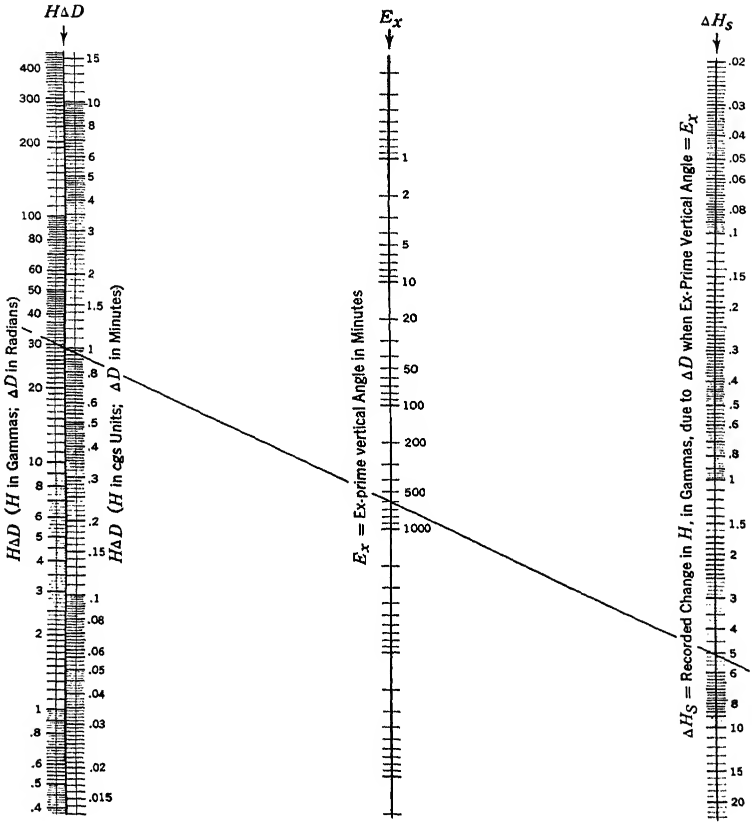

Figure 144.—Nomogram showing spurious effects on recorded horizontal intensity due to changes in magnetic declination (see par. 331).

This nomogram represents graphic solutions of the equation

$$\Delta H_s = \mathrm{H}\Delta D \tan E_x$$

in which ΔH_s is the recorded change in H, in gammas, due to a small declination change, ΔD, in radians, when the ex-prime vertical angle is E_x. Example: Given, $H=10{,}000\gamma$; $\Delta D=10'=0.00291$ radians; and $E_x=10°=600'$. Then $\Delta H_s=10{,}000\times 0.00291 \tan 10°=5.1\gamma$, the spurious effect. The straight line from the $H\Delta D=29.1$ on the appropriate $H\Delta D$ scale through $E_x=10°=600'$ on the E_x scale intersects the ΔH_s scale at 5.1γ.

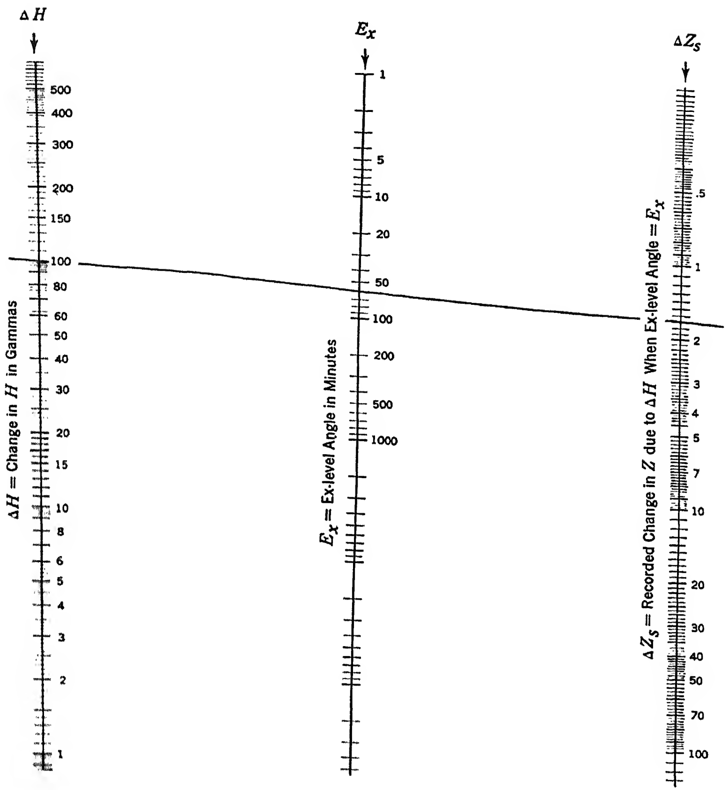

Figure 115. Nomogram showing spurious effects on recorded vertical intensity due to changes in the horizontal intensity (see par. 331).

This nomogram represents graphic solutions of the equation

$$\Delta Z_s = \Delta H \tan E_x$$

in which ΔH is a small increment in H; E_x is the ex-level angle of the Z recording magnet. The equation applies to a Z recording magnet operating in the magnetic meridian. Example; Given $\Delta H = 100\gamma$ and $E_x = 60'$. Then from the above equation, $\Delta Z_s = 1.84\gamma$, the spurious effect. The straight line from 100γ on the ΔH scale through $60'$ on the E_x scale intersects the ΔZ_s scale at 1.84γ.

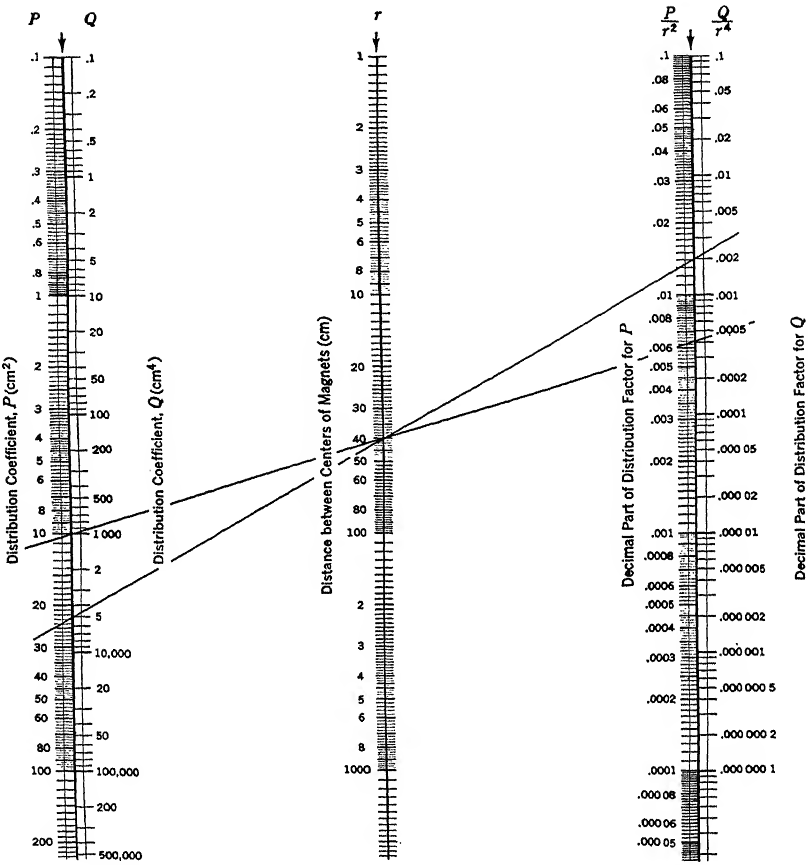

Figure 146.—Nomogram for distribution factors $\left(1+\frac{P}{r^2}+\frac{Q}{r^4}\right)$.

This nomogram represents graphic solutions of the terms $\frac{P}{r^2}$ and $\frac{Q}{r^4}$ for different values of the deflection distance r, for the conditions that P and Q are known. The coefficients P and Q may be calculated from the equations in table 2 or interpolated from the values in tables 21a, 21b, and 21c. Example: Given, for position A, $P=10$, $Q=5000$, and $r=40$ cm. Required: the factor $1+\frac{P}{r^2}+\frac{Q}{r^4}$. The straight line from 10 on the P scale through 40 on the r scale intersects the $\frac{P}{r^2}$ scale at 0.0064. The straight line from 5000 on the Q scale through 40 on the r scale intersects the $\frac{Q}{r^4}$ scale at 0.002. Then the distribution factor is 1.0000+0.0064 +0.0020=1.0084.

TABLE 21a.—*Approximate distribution coefficients for position A.*

$$P_A=2l_a^2-3l_s^2=0.32\ L_a^2-0.48\ L_s^2$$

$$l_a=0.4\ L_a \qquad l_s=0.4\ L_s$$

P_A

L_s \ L_a	0	1	2	4	6	8	10	12	14	16	18	20
0	0.00	+0.32	1.28	5.12	11.5	20.5	32.0	46.1	62.7	81.9	104	128
1	−0.48	−0.16	+0.80	4.64	11.0	20.0	31.5	45.6	62.2	81.4	103	128
2	−1.92	−1.60	−0.64	+3.20	9.60	18.6	30.1	44.2	60.8	80.0	102	126
4	−7.68	−7.36	−6.40	−2.56	+3.84	12.8	24.3	38.4	55.0	74.2	96.0	120
6	−17.3	−17.0	−16.0	−12.2	−5.76	+3.20	14.7	28.8	45.4	64.6	86.4	111
8	−30.7	−30.4	−29.4	−25.6	−19.2	−10.2	+1.28	+15.4	32.0	51.2	73.0	97.3
10	−48.0	−47.7	−46.7	−42.9	−36.5	−27.5	−16.0	−1.92	+14.7	33.9	55.7	80.0
12	−69.1	−68.8	−67.8	−64.0	−57.6	−48.6	−37.1	−23.0	−6.40	12.8	34.6	58.9

$$Q_A=3l_a^4-15l_a^2l_s^2+\frac{45}{8}\ l_s^4=0.0768\ L_a^4-0.384\ L_a^2\ L_s^2+0.144\ L_s^4$$

$$l_a=0.4\ L_a \qquad l_s=0.4\ L_s$$

Q_A

L_s \ L_a	0	1	2	4	6	8	10	12	14	16	18	20
0	0.00	+0.08	+1.23	19.7	99.5	315	768	1,590	2,950	5,030	8,060	12,300
1	0.14	−0.16	−0.16	+13.7	85.8	290	730	1,540	2,880	4,940	7,940	12,100
2	2.30	+0.84	−2.61	−2.61	+46.5	+219	617	1,370	2,650	4,640	7,570	11,700
4	36.9	30.8	+13.5	−41.8	−84.8	−41.8	+190	+745	1,780	3,500	6,110	9,870
6	187	173	133	−14.9	−212	−384	−428	−212	+427	+1,680	3,770	6,950
8	590	565	493	+216	−195	−668	−1,100	−1,360	−1,280	−668	+689	+3,050
10	1,440	1,400	1,290	845	+157	−703	−1,630	−2,500	−3,140	−3,360	−2,940	−1,630
12	2,990	2,930	2,770	2,120	1,090	−238	−1,780	−3,380	−4,900	−6,140	−6,870	−6,840

TABLE 21b.—*Approximate distribution coefficients for position B.*

$$P_B=-\frac{3}{2}\ l_a^2+6l_s^2=-0.24\ L_a^2+0.96\ L_s^2$$

$$l_a=0.4\ L_a \qquad l_s=0.4\ L_s$$

P_B

L_s \ L_a	0	1	2	4	6	8	10	12	14	16	18	20
0	0.00	−0.21	−0.96	−3.84	−8.61	−15.4	−24.0	−34.6	−47.0	−61.4	−77.8	−96.0
1	0.96	+0.72	+0.00	−2.88	−7.68	−14.4	−23.0	−33.6	−46.1	−60.5	−76.8	−95.0
2	3.84	3.60	2.88	0.00	−4.80	−11.5	−20.2	−30.7	−43.2	−57.6	−73.9	−92.2
4	15.4	15.1	14.4	+11.5	+6.72	0.00	−8.64	−19.2	−31.7	−46.1	−62.4	−80.6
6	34.6	34.3	33.6	30.7	25.9	+19.2	+10.6	0.00	−12.5	−26.9	−43.2	−61.4
8	61.4	61.2	60.5	57.6	52.8	46.1	37.4	+26.9	+14.4	0.00	−16.3	−34.6
10	96.0	95.8	95.0	92.2	87.4	80.6	72.0	61.4	49.0	+34.6	+18.2	0.00
12	138	138	137	134	130	123	114	101	91.2	76.8	60.5	+42.2

$$Q_B=\frac{15}{8}\ l_a^4-\frac{45}{2}\ l_a^2\ l_s^2+15\ l_s^4=0.048\ L_a^4-0.576\ L_a^2\ L_s^2+0.384\ L_s^4$$

$$l_a=0.4\ L_a \qquad l_s=0.4\ L_s$$

Q_B

L_s \ L_a	0	1	2	4	6	8	10	12	14	16	18	20
0	0.00	+0.05	+0.77	12.3	62.2	197	480	995	1,840	3,150	5,040	7,680
1	0.38	−0.14	−1.15	+3.46	+41.9	160	423	913	1,730	3,000	4,850	7,450
2	6.14	+3.89	−2.30	−18.4	−14.6	+55.3	+256	+670	1,400	2,560	4,300	6,760
4	98.3	89.1	+62.2	−36.9	−171	−205	−313	−233	+136	+885	+2,150	+4,090
6	498	477	415	+178	−187	−633	−1,100	−1,490	−1,720	−1,670	−1,180	−117
8	1,570	1,540	1,430	995	+308	−590	−1,630	−2,740	−3,810	−4,720	−5,330	−5,490
10	3,840	3,780	3,610	2,930	1,830	+350	−1,440	−3,460	−5,610	−7,760	−9,780	−11,500
12	7,960	7,880	7,630	6,650	5,040	2,850	+148	−2,990	−6,450	−10,100	−13,900	−17,500

TABLE 21c.—*Approximate distribution coefficients for position C.*

$$Pc=-\frac{3}{2}(l_a^2+l_s^2)=-0.24\,(L_a^2+L_s^2)$$

$$l_a=0.4\,L_a \qquad l_s=0.4\,L_s$$

Pc

L_s \ L_a	0	1	2	4	6	8	10	12	14	16	18	20
0	0.00	−0.24	−0.96	−3.84	−8.64	−15.4	−24.0	−34.6	−47.0	−61.4	−77.8	−96.0
1	−0.24	−0.48	−1.20	−4.08	−8.88	−15.6	−24.2	−34.8	−47.3	−61.7	−78.0	−96.2
2	−0.96	−1.20	−1.92	−4.80	−9.60	−16.3	−25.0	−35.5	−48.0	−62.4	−78.7	−97.0
4	−3.84	−4.08	−4.80	−7.68	−12.5	−19.2	−27.8	−38.4	−50.9	−65.3	−81.6	−99.8
6	−8.64	−8.88	−9.60	−12.5	−17.3	−24.0	−32.6	−43.2	−55.7	−70.1	−86.4	−105
8	−15.4	−15.6	−16.3	−19.2	−24.0	−30.7	−39.4	−49.9	−62.4	−76.8	−93.1	−111
10	−24.0	−24.2	−25.0	−27.8	−32.6	−39.4	−48.0	−58.6	−71.0	−85.4	−102	−120
12	−34.6	−34.8	−35.5	−38.4	−43.2	−49.9	−58.6	−69.1	−81.6	−96.0	−112	−131

$$Qc=\frac{15}{8}(l_a^2+l_s^2)^2=0.048\,(L_a^2+L_s^2)^2$$

$$l_a=0.4\,L_a \qquad l_s=0.4\,L_s$$

Qc

L_s \ L_a	0	1	2	4	6	8	10	12	14	16	18	20
0	0.00	0.05	0.77	12.3	62.2	197	480	995	1,840	3,150	5,040	7,680
1	0.05	0.19	1.20	13.9	65.7	203	490	1,010	1,860	3,170	5,070	7,720
2	0.77	1.20	3.07	19.2	76.8	222	519	1,050	1,920	3,240	5,160	7,830
4	12.3	13.9	19.2	49.2	130	307	646	1,230	2,160	3,550	5,550	8,810
6	62.2	65.7	76.8	130	249	480	888	1,560	2,580	4,090	6,220	9,120
8	197	203	222	307	480	786	1,290	2,080	3,240	4,920	7,230	10,300
10	480	490	519	646	888	1,290	1,920	2,860	4,210	6,080	8,630	12,000
12	995	1,010	1,050	1,230	1,560	2,080	2,860	3,980	5,550	7,680	10,500	14,200

BIBLIOGRAPHY

1. S. J. Barnett, A sine galvanometer for determining in absolute measure the horizontal intensity of the earth's field, Department of Terrestrial Magnetism, Carnegie Institution of Washington, Publication No. 175, vol. IV, pp. 373–394, Baltimore, 1921.
2. S. Chapman and J. Bartels, Geomagnetism, vols. I and II, Oxford, 1951.
3. G. Hartnell, Distribution Coefficients of Magnets, Special Publication 157, pp. 9–10, U. S. Coast and Geodetic Survey, Washington, 1930.
4. D. L. Hazard, Directions for Magnetic Measurements, Serial 166, 3rd ed., corrected 1947, U. S. Coast and Geodetic Survey, Washington, 1947.
5. C. A. Heiland and W. E. Pugh, The American Institute of Mining and Metallurgical Engineers, Technical Publication No. 483, New York, 1932.
6. H. H. Howe, On the theory of the unifilar variometer, Terr. Mag., **42**, 29–42, Baltimore, 1937.
7. J. W. Joyce, Manual on Geophysical Prospecting with the Magnetometer, U. S. Department of Interior, Bureau of Mines, Houston, 1937.
8. D. la Cour, La Balance de Godhavn, Danske Meteorologiske Institut, Communications Magnétiques, etc., Publication No. 8, Copenhagen, 1930.
9. ——— Le Quartz-Magnetometre QHM (Quartz Horizontal Force Magnetometer), Danske Met. Inst., Comm. Mag., etc., Pub. No. 15, Copenhagen, 1936.
10. ——— The Magnetometric Zero Balance, the BMZ, Danske Met. Inst., Comm. Mag., etc., Pub. No. 19, Copenhagen, 1942.
11. D. la Cour and V. Laursen, Le Variometre de Copenhague, Danske Met. Inst., Comm. Mag., etc., Pub. No. 11, Copenhagen, 1930.
12. J. Lamont, Handbuch des Erdmagnetismus, Berlin, 1849.
13. A. K. Ludy and H. H. Howe, Magnetism of the Earth, Serial 663, U. S. Coast and Geodetic Survey, Washington, 1945.
14. Smithsonian Physical Tables, Geomagnetism, Tables 495–511, 9th ed. (in press, 1952).

INDEX.

a factor, 87, 95, 97, 128, 166
  significance of, 96
— determination of, 159
— at C. & G. S. observatories, 97
Aberration, spherical, 67
Absolute control, importance of, 174
Absolute observations, 174
Absolute observatory, 21
Activity, magnetic, 181, 182
Alignment charts (*see* Nomograms)
Anomaly; tolerances in selecting site, 21
Approximations, 183
Aurora borealis, 20
Autocollimator (Gaussian ocular), 62
Auxiliary spots (la Cour), 66
Axis, magnetic (bar magnet), 1
— optical, 62
— of centered dipole, 16
Azimuth of *Z* magnet; scale value, 105
— true, 21

b factor; *H* scale value, 87
Balance, analytical, 104
Bar magnet; components of field, 8
Base-line mirror, 66
Base-line mirror, *D*, 74
Base line, scale value at, 165
Base-line value, *H*, 30
— drift of, 171
— scaling for, 178
Bench, orientation, 21, 141
Bimetallic strip, 120, 121, 123, 164
— on la Cour variometer, 164
Bifilar suspension, *H* variometer, 84
BMZ, 57

C FIELD:
  D variometer, 74, 77, 80
  H variometer, 95
  effective compensation, *H* variometer, 116
  computation, *H* variometer, 156
  Z variometer, 101
  evaluation of; *Z* variometer, 159
  estimation of; for *H* and *Z* variometers, 117
C figures, 181
C (instrumental constant of magnetometer), 27
— adjustment of, 55
  computation of, 58
Centered dipole, 16
Charts, isomagnetic, 16
Chronometer, 21
  rate of, 27
Clock, time marking, 21
Coefficient of thermal expansion, magnet and inertia weight, 29
Coil factor, induction tests, 50
Collimation, 62
Comparison observations, 148
Comparison of fields, bar magnet and earth, 18
Comparisons, summary of, 58, 60, 61
Compensated variometer, *H*, 157
Compensating field, variometers, 114, 116
Compensator distance, evaluation, 118
COMPONENTS OF FIELD:
  of a bar magnet, 8, 17
  for small angles, 10
CONSTANTS:
  of magnetograph, 148
  of magnetometer; summary, 55, 56
Control, absolute; importance of, 174
CONTROL MAGNET:
  D variometer, 81
  H variometer, 86, 90
  Z variometer, 104
Conversion factors, 30
Conversion to absolute units, 165
CORRECTIONS:
  index, 25
  instrumental, 25
Corrections to *H* observations, 59
Corrector, *D*, 152

Coulomb's equation for k', 88
COUPLES:
  in *H* variometer, 84, 85
  in *D* scale value equation, 78
Coupler, in variometers, 73
Cylindrical lens, thickness, 75

D nomogram, use of, 81
D scale-value, 74, 82, 83
D scale value equation, 78
D spot, 74
D VARIOMETER:
  C field of, 74, 77, 80
  control magnet, 80
  depth of focus of, 83
  east field at, 77
  effect of cylindrical lens, 75
  effective recording distance, 75
  evaluation of recording distance, 82
  field factor, 74
  gamma-scale value, 74
  la Cour, 65
  lens, thickness of, 75
  lens, specifications, 83
  line of no torsion, 76
  low-sensitivity, 80
  magnetic moment of recording magnet, 82
  measured distance, 75, 83
  nomogram, 81
  optical axis of *D* lens, 74
  optical lever, 75
  optical scale value, 75, 77
  orientation formulas, 136
  recording magnet, 74
  stray fields, 77
  torsion constant, 74, 81
  torsion factor, 76
  torsion in fiber, 75
  torsion tests, 76
Damping, 29, 74
— *H* recording magnet, 84, 159
— test, *D*, 150
Darkroom, 21
Declination, magnetic, 21
— standard value, 128
Deflection angle, *u*, 27
Deflection distances, 26, 27
— for orientation tests, 144
Deflection, double, in scale value, 128
— exmeridian, 135
— orientation, 144
Deflections, positions of magnets in, 5-8
— for *H* scale value, 126, 158
— for *Z* scale value, 161
Deflector orientation, 197
Deflector positions, effect on distribution coefficients, 12
Depth of focus, *D* variometer, 83
Developing processes, photographic, 69
Diaphragms, 67
Dip, magnetic (inclination), 16
Dipole, centered; representing earth's field, 16
Directions for performing operations (*see* supplement, page 232).
Distribution coefficients, 12, 13, 15, 26, 27, 227, 228
— formulas, 13
— special cases, 14
— corrections for, 5
Distribution effects, *H* scale value, 91
Distribution factors, 12
Disturbance, irregular, 20
— local, 21
Diurnal variation, 19
Dyne, 1

Earth inductor, 21
Earth's dipole field, 16
EARTH'S FIELD:
  changes in, 19
  east component, 16
  at various latitudes, 17

Earth's magnetic moment, 17
East field; at D variometer, 77, 151, 152
— at H variometer, 84, 86, 87, 88
Elements, magnetic, 16
Equivalent field; H variometer, 86, 87
Errors in orientation tests, 197-202
Errors in scale-value observations, 204
Exmeridian deflections, 135
Exorientation angles, evaluation, 84, 85, 145

Fiber, torsionless, 77
FIELD, MAGNETIC:
at a point; distribution considered, 14
around a bar magnet, 2, 17, 18
C, evaluation of, 159
dipole, 16
equivalent, 86
estimation from nomogram, 5
factor (D variometer), 74
in terms of magnetic moment, 5
of magnet, direction of, 11
p, at H variometer, 87, 158
resultant, 5, 8
strength, 1
uniform, 3
Focal length of lens, 62
Force, magnetizing, 1

Gamma scale value, D, 74, 79
Gauss, 1
Gaussian ocular, 62, 63
Geomagnetic latitude, recommended scale values for, 134
Geomagnetic poles, 17
Ghost images, 64

H MAGNET:
magnetic moment, 84, 87
effect on D variometer, 212
H nomogram; for H scale value, 93
H sensitivity control magnet; effect on D variometer, 214
— reversal of, 92
H spot, adjustment of, by temperature magnet, 158
H temperature compensating magnet; effect on D variometer, 213
H variometer, 84
— orientation formulas, 85, 139
— scale value, 85
Half period, in oscillations, 27, 29
Heat treatment of magnets, 33, 38
Helmholtz coil; external field, 134
Helmholtz-Gaugain coil, 132
Horizontal intensity, definition of, 16
Hourly values, publication of, 176
— scaling, 177, 178
Housing, nonmagnetic, 84

Image, character of, 67
Impurities, magnetic, in instruments, 25
Inclination (magnetic dip), definition of, 16
Index corrections, 25
Induction coefficient, 42
— Lamont's method, 44
— computations, 47
— Nelson's method, 48, 52, 54
Induction factor, 26, 27, 42
Inductor, earth, 21
Inertia cylinder, 71
Inertia weight, 29
Installation of magnetograph, report on, 162, 163
Instrumental corrections, 25
Instruments, quality of, 25
Intensity, horizontal, definition of, 16
Intensity, magnetic, 1
Intensity, vertical, definition of, 16
Intensity, total, definition of, 16
Intercomparison observations, 57
International standards, 57
Invar; coefficient of expansion, 110
Inverse square law, 1
Isomagnetic charts, 16

k, evaluation and dimensions of, 87-89
K indices, 181
Knife edge; Z recording magnet, 99
— care of, 161

La Cour fibers, 70, 163
— magnetograph, 70, 163
— variometers, optical compensation, 120
Latitude adjustment; Z variometer, 107
Latitude, geomagnetic and magnetic, 18
Lens, cylindrical, 65, 75
Lens, D variometer; specifications, 75, 83
— focal length, 62
Lenses, centering of, 62, 67
Lever, optical, 63, 64
Line of force, equation, 1, 18
Local disturbance, 21
Low-sensitivity variometers, 66, 80, 162
— orientation tests, 163
— scale values, 129

Magnet, length of, 4
— poles of, 1
— axis of, 1, 74, 85
Magnetic activity data, 181
Magnetic compensation, 116
Magnetic elements, 16
Magnetic field strength, 1
Magnetic impurities, 25
Magnetic meridian, 16, 74, 204, 205
— D variometer, 151
— in variation room; establishing, 142
Magnetic moment, 3, 27
— D magnet, 82
— equivalent for a coil, 50
— H magnet, 84, 88
— of the earth, 17
— temperature coefficient of, 195
Magnetic observatory, buildings and equipment of, 21
Magnetic prime vertical, 85, 87, 204, 205
— at H variometer, 87, 90, 95
Magnetic poles of the earth, 17
Magnetic standards, 57
Magnetic storms, 20
Magnetic temperature compensation, 96, 113, 117
Magnetism tester, 25
Magnetizing field, 42
Magnetizing force, 1
Magnetograms, 67, 69, 176
Magnetograph, 21, 23
— constants, 148
— installation, 148
— la Cour, 5
— optics, 63
— visual, 165
Magnetometer, 21
— equations, 26
— Quartz Horizontal (QHM), 21
— constants, 55, 56
Magnetometric Zero Balance (BMZ), 57
Magnets, fields of, 8-15
— bar; forces between, 1
MALADJUSTMENT:
of deflector in orientation tests, 198-202
of deflector in scale-value observations, 205-210
of deflector due to secular change in D, 210
of recording magnets, 135
Meridian, magnetic, 16
— true, 21
Milligauss, 2
Minute scale value, 74
Mirror-adjusting apparatus, 150, 154
MIRRORS:
base line, 66
front surfaced, 64
planoconvex, 64
time-flashing, 67
triple-face (three-face), 65, 150
Moment of inertia, 26, 27, 28, 30, 31
— directions for observing, 31
— corrections for change of temperature, 187
— of cylinder, 28, 89
— of weight, 29, 75
Moment, magnetic (*see* magnetic moment),
Monochromatic light, 62
Multiple reflections, 64

NOMOGRAMS:
D variometer, 81
H variometer, 93, 94
for estimating field of a magnet, 5

Observations, absolute, 174
Observatory, absolute, 21
Observatory, magnetic, 21
Observatory site, selection of, 21
Oersted, 1, 87

Optical axis, 62, 74
Optical compensation, 120, 164
— H variometer, 124
— la Cour variometer, 120, 123
Optical lever, 63, 64, 75
— auxiliary, 162
— double, 134
Optical scale value, 63, 75
— D variometer, 74, 75,77
— H variometer, 84, 87
Optical system; H variometer, 84
— magnetograph, 63, 148
Orientation, 135
Orientation bench, 21, 141
Orientation deflections, 144
Orientation errors, 197
— critical adjustment of deflector, 203
Orientation formulas, 136, 139
Orientation of variometer magnets, 135, 146
Orientation tests, 141, 145
— effects of maladjustment of deflector, 141, 144, 161, 198-202
— evaluation of E_x angles, 145
— low-sensitivity variometers, 163
Oscillations, 28, 29, 33
OSCILLATIONS, CORRECTIONS TO:
  for all effects, 184
  for change in H, 191
  for change in moment of inertia, 188, 189
  for change in magnetic moment, 186
  for torsion factor, 190
  for induction, 192
  for rate of chronometer, 185

p field at H variometer, 85, 87
— computation of, 158
p field at Z variometer, 104
Paper, recording, 67
Parallax, 67
Parallax tests, 156, 161, 163, 177, 181
Parallel component; field of bar magnet, 9
Parallel rays, 62
Period, torsion pendulum, 75
Perpendicular component; field of a bar magnet, 10
Photographic registration, 23, 62
Piers, 21
Pivots and knife edges, care of, 161
Poises, Z recording magnet, 161
Pole, unit, 1
Poles, earth's magnetic and geomagnetic, 17
Poles of a magnet, 1
Positions of magnets, 5-8, 12
Potentiometer, precision, 57
Prime vertical, magnetic, 85, 87, 204, 205
Prisms, 65, 163
Processing of records, 165, 174

q coefficients, 115, 116
QHM (Quartz Horizontal Magnetometer), 21, 57
Quality of instruments, 25
QUARTZ FIBERS: 70
  classification, 71, 72
  dimensions, 70, 75, 88
  galvanometer, 70
  installation, 72, 73
  la Cour type, 70
  measurement, 71
  modulus of rigidity, 70
  period of oscillation, 72, 75
  procurement, 70
  selection, 72, 154, 162
  torsion constant, 70, 72
Quartz Horizontal Magnetometer (QHM), 21, 57

Radial component; field of a bar magnet, 8, 10
Radian scale value; H variometer, 86
Rate of chronometer, 27, 185
Recorder, photographic, 23, 67
Recording distance, 82, 84
Recording paper, 67
Recording magnet, D, 74, 82
Recording magnet, H, 87, 88, 90, 95
— damping, 159
RECORDING MAGNET, Z:
  azimuth and scale value, 100, 101
  balancing poise center of gravity, 99, 100
  knife edge, 99
  la Cour, 163
  magnetic axis, 99, 104
  scale value, 101
  sensitivity poise, 99, 104
  supports, 99

Recording magnets, adjustment, 135, 145
Registration, photographic, 62, 67
Report of installation of magnetograph, 16[illegible], 1[illegible]3
Reserve distances, 155
Reserve spots, 65, 164
Resistance, standard, 57
Resolution of fields, resultant field, 5
Resultant field, D, 77
Resultant field, H, 87
Rigidity modulus of quartz, 75
Routine duties, 165

SCALE VALUE:
  a factor, significance of, 96
  a factor tests, 128, 159
  adjustment by control magnet, 90
  bimetallic strip, 121
  calculated, 205
  computations, 129
  correlated with geomagnetic latitude, 134
  double deflections, 128
  electromagnetic methods, 132
  errors due to maladjusted deflector, 204
  Helmholtz-Gaugain coil, 132
  low-sensitivity variometers, 129, 162, 163
  optical, 63, 75
  recommended, 134
  scale in magnet, 26
  scale in telescope, 26
  scaling of deflections, 177
  sensitivity-control magnet, 90
  shelves, 21
  temperature coefficient, 112
SCALE VALUE, D VARIOMETER:
  adjustment, 83
  computation, 154
  equations, 78, 82
  gamma, 74, 79
  gamma and minute, 129
  minute, 74, 231
  optical scale value, 75, 77
SCALE VALUE, H VARIOMETER:
  a factor, 87, 95
  adjustment by control magnet, 91, 92
  average, 97
  b factor, 87
  base-line, 97, 128
  cgs units per radian, 85, 86
  control magnet, 157
  correction for change in D, 97, 128
  deflections, determination, 126, 158
  dimensions of k, 87
  distribution effects, 91
  equation, 71, 128
  from constants, 90
  gammas per mm, 86
  graph, 158
  graphic solutions, 93
  optical, 86, 87
  reversal of control magnet, 92
  selection of, 90
  use of nomogram, 94
  variation with f_E, the east field, 91
  variation with k, 91
  variation with distance of sensitivity magnet, 93
  variation with ordinate, 97, 128
  with control magnet, 93
  with two control magnets, 92
SCALE VALUE, Z VARIOMETER:
  azimuth of recording magnet, 105, 106, 107
  center of gravity of recording magnet, 103
  deflections, 161
  equation, 101, 128
  example, 106
  effect of H, 105
  latitude and temperature compensation, 108
  N end south, 106
  parallel fields, 101
  p field, 104
  sensitivity poise, 99, 103, 106
Scaling, base line, 177, 178
— during a storm, 178
— hourly values, 177, 178
Scaling base line,
— low sensitivity magnetograms, 180
— ordinates, 33, 177
— scale value deflections, 178
Secular change, 20
Sensitivity, bimetallic strip, 121
— control magnet, 90, 91

Sensitivity—Continued
— double; D variometer, 162
— Z recording magnet, 104
Sensitivity magnet, adjustment of, H, 157
Sensitivity magnets, two, 92
Sensitivity magnets, la Cour Z variometer, 164
Shelves, scale value, 21
Shrinkage, 180
Sine galvanometer, 57
Site, observatory, selection of, 21
Solar flare effects, 181
Solenoids, axial field, 48
South magnetic latitude; temperature compensation, 112
Spherical aberration, 67
Spots, reserve, 65, 66
Spurious effects, 98, 135
Stabilization of magnets (*see* heat treatment)
Standard cells, 57
Standard resistance, 57
Standards, International Magnetic, 57
Storms, magnetic, 20
Stray fields at D variometer, 77, 211, 212, 216
Stray fields at Z variometer, 101
Sudden commencements, 181

Tangential component; field of a bar magnet, 8
Temperature bath, 34
Temperature coefficient and scale value, 112
Temperature coefficient, mechanical, 110
TEMPERATURE COEFFICIENT:
of D variometer, 119
of H variometer, 167
of magnetic moment, 26, 27, 29, 33, 195
of magnetic moment, alternate method, 39
of magnetic moment; directions, 34, 35
of magnetic moment; errors of scale reductions, 42
of magnetic moment; graph, 38
of magnetic moment; sensitivity, 41
of variometers, 109
of variometers; errors, 118
of variometers; meaning, 109
of variometers; optical compensation, 121
of Z spot, 109, 111
Temperature compensating field; C field, H variometer, 96, 156
Temperature compensating magnet; adjustment of H spot by means of, 158
TEMPERATURE COMPENSATION:
device for H variometer, 84
estimation of compensation distance, 118
H variometer, 154
H variometer by two magnets, 157
magnetic, 96, 113, 116–118
magnetic; south latitudes, 112, 113
magnetic, Z variometer; evaluation of q coefficients, 113, 115
mechanical; examples, 111, 112
mechanical; south magnetic latitudes, 112
optical, 120
Z variometer; evaluation of C field, 159
Temperature, effect of, on log C, 193
Temperature magnet, adjustment of, 119, 148
Temperature, standard, 166
Thermal expansion, coefficient of (magnet and inertia weight), 29
Time comparison, 174
Time flasher, 23, 32, 67, 153
Time signals, 174
Tolerances in local anomalies at observatory sites, 21
Torque, applied; H variometer, 84
Torsion constant; Coulomb's equation, 88
TORSION CONSTANT:
by oscillations, 89
by torsion observations, 89
calculations, 71, 72
D fiber, 74
equation, 71
evaluation from k' and M_s, 89
from nomogram, 94
from scale-value observations, 89, 90
H fiber, 84, 94
k', for D variometer, 81
definition, 88
estimation of; H variometer, 88
per unit magnetic moment; H variometer, 85
Quartz fiber, 70, 90
TORSION FACTOR:
D variometer, 74, 76, 152
H variometer, 154, 156
low-sensitivity variometer, 162
Torsion head, 150
Torsion in H variometer fiber, 84, 156, 157
Torsion, line of no; in variometer, 76, 85
Torsion in fiber, magnetometer, 27
Torsion observations, 30–32
Torsion pendulum, 71, 75
Torsion, removal of, 150, 154
Torsion tests, D, 76
Torsion, total; in D variometer fiber, 75
Torsion weight, variometer, 150
Torsionless fiber, 78, 87
Total field of bar magnet, 9
Total intensity, definition of, 16
True azimuth; true meridian, 21

Unifilar suspension, 84
Uniform field, 3
Unit field, 1
Unit pole, 1

Variation building, 21, 22
Variation, daily, 19
Variometers, 23
Variometer data, miscellaneous, 145
Variometer, D, 74
— installation, 150
— miscellaneous data, 152
Variometer, H, 84
— installation, 154
Variometer, low-sensitivity, Schmidt, 66
Variometer, positions on pier, 149
Variometer, Z, 99
— installation, 159
Vertical intensity, definition, 16
Visual magnetograph, 165
Z magnet, la Cour, 164
Z recording magnet, effect on D variometer, 214
Z scale value, effect of H on, 105
Z spot, adjustment of, 161
Z temperature-compensating magnet; effect on D variometer, 215
Z variometer equation, 99, 101

DIRECTIONS FOR PERFORMING OPERATIONS

a factor, H scale value; determination of, 159
Damping, H variometer; adjusting, 159
Disturbance, local, testing for, 148
Impurities, magnetic, testing for, 25
INDUCTION COEFFICIENT:
Lamont's method, 45
Nelson's method, 51
Magnetic meridian marking in variation room, 142
Magnetograph, installation of, 148
Mirrors, variometer; adjusting, 150
Optical lever, auxiliary, use of, 162
Optical system of magnetograph, testing of, 148
Orientation tests, 141
— critical adjustment of deflector, 141
Parallax, removal of, 67
Parallax test, recording of, 181
Processing records, 174
Quartz fibers, 71, 72
—installation of, 72
Recording distance, D variometer, evaluation of, 82
Scale values, coil method, 133
SCALE VALUE:
D, 129, 154
H, 126, 157
Z, 128, 161
low-sensitivity variometers, 129
scale in telescope, 26
scale in magnet, 26
Scaling ordinates, 177
Sensitivity-control magnet, adjustment of, 157
Shrinkage, determination of, 180
Stray east field, D variometer, determination of, 151
Temperature coefficient of magnetic moment, 35, 84
— alternate method of determination, 39
Temperature compensation, optical method, 164
Time flasher, adjustment of, 153
Torsion constant, determination of, 71
Torsion, D variometer; removal of, 150
Torsion, H variometer; removal of, 154
Torsion factor, D variometer, determination of, 152
Torsion factor, H variometer, determination of, 156
VARIOMETER INSTALLATION:
D, 150
H, 154
Z, 159
low-sensitivity, 162

U. S. GOVERNMENT PRINTING OFFICE: 1953